Economic Progress, Social Regress?
A Small World Approach.

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Keywords: social capital, modernization, small world, market and non-market interaction.

November 2003

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Abstract

A central issue in the debate regarding the relevance of social capital is whether the decline in social embeddedness that has attended modernization over the last forty years in the U.S. is as harmful as Putnam, among others, claim it to be. Critics of Putnam’s thesis argue that various arms-length institutions fulfill the roles performed by social capital thereby mitigating the negative impact of its recent decline. We develop a framework that provides insight into when such institutions may be adequate and when they might not. We find that if market (economic) and non-market (social) interactions differ in their payoffs but are interlinked through the modernization of the economy, the optimal level of modernization in market interactions will be higher than that in non market interactions. Further, market supporting institutions are likely to increase the divergence between economic and social interactions since analogs for market institutions that constrain opportunistic behavior are usually non-existent in social contexts. In this sense, economic progress may be accompanied by social regress.
1 Introduction

*Good neighbors make good fences, but burglar alarms may be a viable alternative.*

From “Can We Trust Social Capital?” by Joel Sobel (2002).

While Putnam’s (2000) influential treatise on social capital argues that the United States has experienced a sharp decline in virtually all forms of civic participation during a period of unprecedented growth and modernization, a central theme of Sobel’s critique is that documenting a change as Putnam has done is not equivalent to documenting a crisis. There are many ways to solve such problems, and a responsive police force may protect one’s home as effectively as vigilant neighbors. However, Sobel also notes that the effectiveness of a particular kind of network depends on the institutions available. For purely economic contexts, perhaps the availability of institutions such as a reliable judiciary and a police force are adequate to prevent problems of opportunism, but what about interactions that take place in a purely social context, such as marriage and child-rearing, where analogous institutions are non-existent and difficult, if not impossible, to mandate? In this paper we give structure to the idea that the value of social capital does depend on the nature of the underlying institutional environment and that if economic and social interactions are interlinked (in a way made more precise below), economic progress could be accompanied by social regress. In this sense, we provide an analytical framework that could be helpful to the social capital debate in terms of identifying the circumstances under which Putnam’s arguments or those of his critics are likely to be valid.

In *Bowling Alone* (2000) Putnam documents a decline in civic participation over the last forty years and argues that the decline is correlated with negative social outcomes such as mistrust, crime, divorce and diminished health, among others. His general thesis is that it has been during a period of unprecedented economic growth, mobility and modernization that the general decline in social capital he documents throughout his book has taken place. The theory that we develop in this paper provides a framework for thinking about this transition.

Our theoretical approach grafts a prisoners’ dilemma game onto the “small world” framework introduced in a pathbreaking recent paper by Watts and Strogatz (1998). The pris-
The “small world” framework, based on random graph theory, provides us with a novel way to model the transition of an economy from a situation of clustered, socially embedded interaction at one extreme, to anonymous random interaction at the other. In the paper we refer to this transition as the modernization of the economy. What we have in mind is the idea that the process of modernization can be represented in terms of a gradually increasing likelihood of interacting with individuals outside a person’s close relational neighborhood. Thus, if the economy is a traditional economy, interactions are mostly with neighbors; social embeddedness is strong and thus plays a vital role in governing transactions. However, as the economy modernizes, social embeddedness begins to weaken and consequently has less influence on economic behavior. A traditional economy is thus associated with a very low probability \( p \sim 0 \) of interacting with others outside one’s neighborhood. As modernization proceeds, this probability increases until finally we arrive at what could be called anonymous random interaction \( p \sim 1 \). Varying the probability of random interaction \( p \) in the economy permits the examination of the economy through intermediate stages of modernization.

By grafting a prisoners’ dilemma game onto this structure, we are able to examine cooperation (and its collapse) in the transition from highly clustered, group-based interaction at the one extreme, to anonymous random interaction at the other.

Thus, modernization is associated with an increasing likelihood for individuals to interact with others outside of their immediate relational neighborhood. In terms of our model, as this interaction probability increases, the average distance between any two individuals in the economy declines, implying a decrease in average search costs. However, as the interaction probability rises, social embeddedness, or the ability of society to oversee individual behavior, also declines. There is therefore a trade-off between social embeddedness and search costs as society modernizes: when modernization is low, social embeddedness is high, and the likelihood of cheating is low. But since search costs are high in this situation, the likelihood of finding the ideal partner will also be low. In the absence of any external institutions this trade-off leads to an optimal level of modernization for the economy which, in turn, leads to

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2The term *modernization* seems appropriate for this process because modernization is associated with development of infrastructure and adoption of technologies, such as roads and telecommunications, that are likely to increase random interactions in the economy.
an optimal level of social embeddedness.

We simultaneously model both market and non-market interactions taking place in this economy. The same individuals engage in separate market and non-market interactions thus interlinking the two. However, the benefits of search in an economic context clearly differ from the social benefits of having a more modern society. We show that the optimal level of modernization in market interactions can exceed that which is optimal from the perspective of non-market interactions. To the extent that the level of modernization in the economy is likely to be driven by market based concerns, one might argue that the phenomena presented by Putnam and other scholars reflects the fact that the level of modernization in the economy has exceeded the optimal level from the perspective of non-market interactions.

Moderating the negative impact of modernization on opportunism is the role of institutions. We consider two types of simple institutions: informational intermediaries who gather and transmit information regarding cheating to their clients, and enforcement intermediaries who increase the penalties for dishonest behavior. Informational intermediaries could be considered similar to credit reporting bureaus, trade associations and auditing firms. Enforcement intermediaries could be considered along the lines of the police force and judicial system. When modernization is low, the marginal gains from enforcement intermediaries are relatively higher than the marginal gains from informational intermediaries. This is because at lower levels of modernization, social embeddedness and thus information are already high and the temptation to cheat does not come from the likelihood of being found out, but from the possibility of lenient punishment if found out. However, at higher levels of modernization, it is the low likelihood of being discovered that is the driving force. In other words, the stage of modernization can play a role in the relative marginal benefits of the two types of intermediaries.

When we consider modernization in a social context, it is not only the case that the non-market benefits from search clearly differ from market benefits, but the role of informational and enforcement intermediaries will differ as well. The analog to a credit bureau in providing information regarding undesirable social behavior is more difficult to imagine. Similarly, legislation and legal enforcement of such behavior is equally difficult to generate. Consequently, we contrast the consequences of economic modernization with those that are felt in
a social context when information and enforcement intermediaries have differential abilities to function in market and non-market contexts. We find that by raising the optimal level of modernization in market interactions, economic intermediaries have a negative impact on social embeddedness, which ultimately has negative effects on opportunism in non-market interactions, where analogs for economic intermediaries are absent. This is the sense in which we find that economic progress may be attendant with social regress.

If the process of modernization that enhances market (economic) performance goes hand in hand with a decline in social capital and leads to poor non-market (social) outcomes, then one must question the overall welfare consequences of optimizing with respect to the market economy while overlooking the potential impact on the non-market economy. A number of scholars recognize the impact that social capital has on market performance (see, for example, Fukayama (1995), Knack and Keefer (1997) and Dasgupta and Serageldin (1999)). The work of these scholars suggests that in addition to the potentially adverse effect of modernization on non-market interactions, there could be negative feedback effects on market interactions as well. Thus, it is important to simultaneously consider the overall impact of modernization on both market and non-market interactions.

Our paper is organized as follows. Section 2 provides a brief overview of some of the related literature in social capital. Section 3 sets up the analytical framework of the economy as a random graph. Section 4 describes the behavior of the generic prisoners dilemma game played on the graph. Section 5 considers market and non-market interactions as separate but interlinked games played simultaneously on the graph. Section 6 considers the role of external institutions in constraining opportunistic behavior and discusses the issue of overall social welfare. Section 7 concludes.

2 Related Literature

As mentioned earlier, the surge of interest in the notion of social capital is in large part due to the work of Putnam (1993, 2000). For our purposes, we focus on that part of the literature that specifically relates social complexity to a variety of social ills and the ability of institutions to replace community oversight as modernization takes place. In Bowling Alone
(2000) Putnam illustrates the relationship between increasing modernization and mistrust in many ways. He documents that people are less trusting in larger cities, and that those who trust less give less to charity and volunteer less of their time. Other examples include the finding that attorneys tend to cooperate less when their professional circle is wider and child abuse is more pervasive in neighborhoods that are less cohesive. He suggests that society is increasingly using the legal system to replace trust citing that the United States currently has three times as many lawyers and judges as that of any other profession.

A number of other scholars have made similar arguments. Alesina et. al (1999) find that ethnic diversity leads to a decrease in spending on public goods while Alesina and LaFerrara (2000) find that social capital declines as homogeneity of the group declines. Related research by Kingston (2002 a,b) examines how social structure affects parochial behavior, finding that segmented societies are able to sustain higher levels of parochialism than more integrated societies. Similarly, Platteau (1994 a,b) argues that when individuals with differing social norms interact, they are less likely to trust one another. Given that more modern economies increase the scope of individuals with whom one interacts, all of these findings are consistent with the notion that modernization can produce a general decline in social capital.

However, as an economy modernizes, the development of institutions may bolster the performance of communities. Bowles and Gintis (2002) suggest that not only can communities function where markets fail by enhancing trust and solving market failures such as public goods problems, but also that markets may solve community failures. This can occur by the creation of property rights or the social value that comes from integration and diversity which typically is enhanced through modernization. Small worlds may lead to high levels of corruption (Kingston, 2002) or the increased exclusion of individuals from small groups that form based on attributes such as race, religion or gender. It is through institutional development that many of these problems have been alleviated.

As we note in the introduction, in his survey paper, Sobel (2002) proposes that it is the interaction of social capital with institutions that matters and that in some cases institutions can replace social trust. Referencing Jacobs (1961), he recounts a story of a neighborhood in which the local deli operator held keys for the members of the community, a situation clearly reliant on trust. As Sobel notes, the deli has been replaced by the Gap, but that the
apartment building now has a doorman who performs the same service. Yamigishi (1988) shows experimentally that individuals contribute more to a public good when their ability to sanction those who do not contribute rises, suggesting that legal institutions may facilitate trust and cooperation.

In a similar vein, Ostrom (1999) also suggests the importance of institutions as farmers in developing countries negotiate arrangements for water sharing from irrigation systems. Importantly, each community creates different solutions that account for their own specific needs and in so doing recognize local conflicts and interests that outside observers cannot know. This “leads to the valuable lesson that external assistance...need not improve economic performance” (Sobel, p.148). The farmers’ successes demonstrate the role that institutions can play in creating cooperative outcomes; however, this also highlights the potential drawback of modernization and the increasing difficulty of institutions to necessarily replace local trust as modernization occurs.

Bowles and Gintis (2002) also suggest that while markets may enhance community performance in some aspects, enforcement of social values such as teamwork, pride and solidarity can be difficult. While institutions may be successful at improving community performance through the establishment of property rights, for example, it can be argued that these greatest successes of institutions are in market based circumstances. Legislating cooperation in a marriage, in the workplace or among friends is clearly a difficult prospect. Thus, as modernization increases, institutions do play a role in maintaining cooperation. However, one must question the ability of institutions to substitute for community oversight in maintaining cooperation in non-market circumstances to the extent that they can in market driven circumstances.

3 Theoretical Framework

A. The Small World Approach

The cornerstone of this approach is the idea that the process of modernization can be represented in terms of a gradually increasing likelihood of interacting with individuals outside a person’s close relational neighborhood. Thus, if the economy has very low modernization (a
traditional economy), interactions are mostly with neighbors, social embeddedness is strong and thus plays a vital role in governing transactions. However, as the economy modernizes, social embeddedness begins to weaken and consequently has less influence on economic behavior. A traditional economy is thus associated with a very low probability \( p \sim 0 \) of interacting with others outside the neighborhood. As modernization proceeds, this probability increases until finally we arrive at what could be called anonymous random interaction \( p \sim 1 \). Varying the probability of random interaction \( p \) in the economy permits the examination of the economy through intermediate stages of modernization. By grafting a prisoners’ dilemma game onto this structure, we are able to examine cooperation (and it’s collapse) in the transition from highly clustered, group-based interaction at the one extreme, to anonymous random interaction at the other.

This approach is motivated by a paper by Watts and Strogatz (1998) that uses techniques from graph theory. Starting from a regular graph where individuals correspond to the nodes and the links joining nodes are fixed, they start “rewiring” links randomly with probability \( p \). This rewiring procedure effectively converts the regular graph into a random graph. They define two statistical properties for the random graph thus formed. The clustering coefficient, which is a measure of the cliquishness of a neighborhood and the characteristic path length, which is a measure of the average number of links connecting any two people. Normally one would expect that as the probability of random rewiring increases, the clustering coefficient should fall. But their most striking discovery is that as the random rewiring probability increases, the characteristic path length does indeed fall sharply but the clustering coefficient remains at high levels over a fairly large range. In other words, there is a large interval of randomness over which the cliquishness of the graph remains high even though connectivity is high too. This phenomenon has been referred to as the “Small World” effect and graphs which display low characteristic path length and high clustering are referred to as small world graphs.

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\[ ^3 \text{The term modernization seems appropriate for this process because modernization is associated with development of infrastructure and adoption of technologies, such as roads and telecommunications, that are likely to increase random interactions in the economy.} \]

\[ ^4 \text{A social network exhibits the small-world phenomenon if, roughly speaking, any two individuals in the network are likely to be connected through a short sequence of intermediate acquaintances. This has long} \]
The clustering coefficient and characteristic path length are particularly useful statistics for our analysis. We use the clustering coefficient as a measure of the informational capabilities of the economy and the characteristic path length as a measure of the search costs. The prisoners' dilemma presents an agent with the temptation to cheat, but the clustering coefficient acts as a brake on such behavior by determining the extent to which others will know the agent’s record and punish his transgression in the future. In addition, each period an agent can search for his ‘ideal’ partner with whom gains from interaction are higher than with others. But there is a trade-off between the clustering coefficient and characteristic path length that accompanies the modernization process. When modernization is low, clustering is high, and the likelihood of cheating is low. But since search costs are high in this situation, the likelihood of finding the ideal trading partner will also be low.

In the absence of any external institutions this trade-off leads to an optimal level of modernization for the economy. This, in turn, leads to an optimal level of modernization. An interpretation of this is that an economy is likely to stay close to the degree of modernization that maximizes its relative gains from honest exchange. Deviating from this level may actually lead to the breakdown of exchange altogether.

been the subject of anecdotal observation and folklore; often we meet a stranger and discover that we have an acquaintance in common. It has since grown into a significant area of study in the social sciences, in large part through a series of striking experiments conducted by Stanley Milgram and his co-workers in the 1960’s (Milgram 1967, Corte and Milgram,1978). Recent work has suggested that the phenomenon is pervasive in networks arising in nature and technology, and a fundamental ingredient in the structural evolution of the World Wide Web (Watts and Strogatz 1998).

Milgram’s basic small-world experiment remains one of the most compelling ways to think about the problem. The goal of the experiment was to find short chains of acquaintances linking pairs of people in the United States who did not know one another. In a typical instance of the experiment, a source person in Nebraska would be given a letter to deliver to a target person in Massachusetts. The source would initially be told basic information about the target, including his address and occupation; the source would then be instructed to send the letter to someone she knew on a first-name basis in an effort to transmit the letter to the target as efficaciously as possible. Anyone subsequently receiving the letter would be given the same instructions, and the chain of communication would continue until the target was reached. Over many trials, the average number of intermediate steps in a successful chain was found to lie between five and six, a quantity that has since entered popular culture as the “six degrees of separation” principle (Guare, 1990).
B. The Economy as a Relational Graph

We consider the economy to be representable as a relational graph. Relational graphs have the defining property that the rules governing their construction do not depend upon any external metric of distance between vertices\(^5\). The distance between vertices is measured solely in terms of the graph itself, and not in terms of any externally defined space.

Specifically, the economy consists of \(n\) individuals constituting the vertices of a one-dimensional ring lattice. Distance is measured in the economy solely in terms of connections or links between the vertices. Whatever combination of factors makes people more or less likely to associate is accounted for by the distribution of those links that actually form. Hence we are not concerned with questions of spaces and metrics: only connections. Furthermore we assume that all such connections are symmetric and of equal significance: that is, given some definition of what is required in order to “know” someone (whatever it may be), either two individuals know each other or they do not.

Each individual \(i\) is directly connected to \(k\) others on the ring by undirected edges. In order to capture the idea of a group where individuals interact only with people who are socially close to them, we assume that the connections are initially with the \(k\) closest neighbors\(^6\). Each vertex is thus of degree \(k\)\(^7\). This kind of structure represents a completely ordered lattice or a regular graph.

A graph-economy is assumed to have many vertices with sparse connections, but not so sparse that the graph is in danger of becoming disconnected\(^8\). This is ensured by assuming \(n \gg k \gg \ln(n)\). The first inequality ensures that the graph is sparse while the second prevents it from becoming disconnected (Bollobas, 1985).

There are two statistics of the relational graph economy that will be of particular interest to us. The first is the characteristic path length \(L(n,k)\), that is the typical distance \(d(i,j)\) between every vertex and every other vertex. Distance here refers not to any separately

\(^5\)This is something of a fine point because the vertices of relational graphs are labelled and ordered according to some kind of geometry (such as a ring).

\(^6\)This will be relaxed later.

\(^7\)The degree of a vertex is the number of edges connected to that vertex.

\(^8\)A graph is connected if there is a path joining every pair of distinct vertices in the graph. Consider any sequence \(x_1, \ldots, x_{n+1}\) of vertices. A path \(P\) is a sequence of edges \(e_1, \ldots, e_n\) such that the endpoints of edge \(e_i\) are \(x_i\) and \(x_{i+1}\) for \(i = 1, 2 \ldots n\).
defined metric in which the graph has been embedded, but to a distinct graph metric—simply the minimum number of edges (in the edge set) that must be traversed in order to reach vertex \( j \) from vertex \( i \), or in other words the shortest path length between \( i \) and \( j \). Operationally, \( L \) is defined as the shortest path \( d(i, j) \) between two vertices, averaged over all \( \binom{n}{2} \) pair of vertices and is best computed numerically for a known graph. The idea of a neighborhood\(^9\) is useful in quantifying another statistic that will be useful, the clustering coefficient \( C(n, k) \). The clustering coefficient characterizes the extent to which vertices adjacent to any vertex \( v \) are adjacent to one another, and is defined as follows. Suppose that a vertex \( v \) has \( k_v \) neighbors; then at most \( \frac{k_v(k_v-1)}{2} \) edges can exist between them (this occurs when every neighbor of \( v \) is connected to every other neighbor of \( v \)). Let \( C_v \) denote the fraction of these allowable edges that actually exist. \( C \) is defined as the average of \( C_v \) over all \( v \). For a relational graph economy these statistics have intuitive meanings. \( L \) is the average number of acquaintances in the shortest chain connecting two people. \( C_v \) reflects the extent to which friends of \( v \) are also friends of each other, and thus \( C \) measures the cliquishness of a typical acquaintance circle. Note that \( L \) is a global property whereas \( C \) is a local property.

### C. Random Graphs

We introduced the notion of a random graph\(^10\) as a tool for thinking about the transition of the economy from a situation where individuals interact only with their close neighbors, to a situation where a market is well developed and individuals encounter and interact randomly with others spread throughout the economy. By varying the extent of randomness (which we also refer to as modernization) in interactions within the framework of a random graph, we are able to interpolate the economy from a situation of close-knit interaction (a traditional economy) to a situation of arms-length random interaction (a modern economy).

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\(^9\) Two edges are adjacent if they are both incident to the same vertex. A vertex and an edge are incident to one another if the vertex is the endpoint of an edge.

The set of vertices adjacent to the vertex \( x \) is the neighbourhood of \( x \).

\(^10\) The study of random graphs has a long history. Starting with the influential work of Paul Erdos and Alfred Renyi in the 1950s and 1960s (Erdos and Renyi 1959, 1960) random graph theory has developed into one of the mainstays of modern discrete mathematics, and has produced a prodigious number of results, many of them highly ingenious, describing statistical properties of graphs, such as distributions of component sizes, existence and size of a giant component, and typical vertex-vertex distances. See Bollobas (1985).
A random graph is simple to define. A random graph is a collection of points, or vertices, with links or edges, connecting pairs of the vertices at random. In a random graph the presence or absence of an edge between two vertices is assumed to be independent of the presence or absence of any other edge, so that each edge may be considered to be present with independent probability \( p \). A random graph is constructed by taking the \( n \) nodes or “vertices” and placing connections or “edges” between them, such that each pair of vertices \( i,j \) has a connecting edge with independent probability \( p \).

Based on Watts and Strogatz (1998), it is possible to interpolate between a regular ring lattice and a random graph without altering the number of vertices or edges in the graph. Their procedure works as follows. Take the one-dimensional ordered ring lattice in which each vertex has precisely \( k \) neighbors (\( \frac{k}{2} \) on either side) and then randomly rewire the edges with probability \( p \) using the following algorithm. Choose a vertex \( i \) and the edge that connects to its nearest neighbor \( (i+1) \) in a clockwise sense. With probability \( p \), we reconnect this edge such that \( i \) is connected to another vertex \( j \), which is chosen uniformly at random over the entire ring, with duplicate edges forbidden; otherwise we leave the edge in place. Repeat this procedure by moving clockwise around the ring, considering each vertex in turn until one lap is completed. When all the vertices have been considered once, we consider the edges that connect each vertex to its second-nearest neighbors clockwise (that is \( i+2 \)). As before, randomly rewire each of these edges with probability \( p \), and continue this process, circulating around the ring and proceeding outward to more distant neighbors after each lap, until each edge in the original lattice has been considered once. As there are \( \frac{nk}{2} \) edges in the entire graph, the rewiring process stops after \( \frac{k}{2} \) laps.

Figure 1 depicts this process for different values of \( p \). For \( p = 0 \), the original lattice is unchanged. As \( p \) increases, the graph becomes increasingly disordered until for \( p = 1 \) all edges are rewired randomly, resulting in a close approximation to a random graph. The algorithm thus allows the “tuning” of the graph between regularity (\( p = 0 \)) and disorder (\( p = 1 \)).

D. The “Small World” phenomenon

Watts and Strogatz numerically explore the properties of the characteristic path length \( L(p; n, k) \) and the clustering coefficient \( C(p; n, k) \) over the range of \( p \). The regular lattice at \( p = 0 \) is a highly clustered world where \( L \) grows linearly with \( n \). But as \( p \) grows, \( L(p) \) drops
almost immediately and falls very quickly to a value close to $L_{\text{random}}$, (when $p = 1$). $C(p)$ however, remains practically unchanged for small $p$ even though $L(p)$ drops rapidly. This existence of high clustering like a regular graph, yet small characteristic path length like a random graph is referred to as the “small-world” property of the network by analogy with the small-world phenomenon. One of Watts and Strogatz’s main contributions is the discovery of the small-world phenomenon for intermediate values of $p$ ($0 < p < 1$).

The onset of the small-world results from the immediate drop in $L(p)$ caused by the introduction of a few long-range edges. Such “short cuts” connect vertices that would otherwise be much further apart than $L_{\text{random}}$. For small $p$, each short cut has a highly nonlinear effect on $L$, contracting not just the distance between the pair of vertices that it connects, but between their immediate neighborhoods, neighborhoods of neighborhoods and so on. By contrast, an edge removed from a clustered neighborhood to make a short cut has, at most, a linear effect on $C$. Consequently, at the local level the transition from the large to the small world is almost undetectable. Figure 2 depicts the behavior of $L(p)$ and $C(p)$ as obtained by Watts and Strogatz.

We use these properties to motivate the idea of the transition of an economy from close-knit groups with a high degree of social embeddedness to the anonymous market where individuals interact randomly. The idea is that when $p$ is small the economy is like a
Figure 2: Characteristic path length and clustering coefficient for the family of randomly rewired graphs.

regular lattice, with individuals interacting only within their circle of immediate neighbors. Modernization could be modeled by the increase in $p$ and the associated interaction with individuals located further away in terms of social distance.

For our purposes, all we require are the qualitative properties of the statistics produced by the procedure\(^{11}\). As in Watts and Strogatz, we consider values normalized by $C(0)$ and $L(0)$, so that the normalized clustering coefficient $c(p)$ and characteristic path length $l(p)$ lie between zero and one. We assume the following properties of $c(p)$ and $l(p)$, as motivated by the work of Watts and Strogatz\(^{12}\).

A1 $c(p)$ and $l(p)$ are continuous and differentiable.

A2 Over the whole range, $p \in [0, 1]$, $c'(p) < 0$, $c''(p) < 0$ and $l'(p) < 0$, $l''(p) > 0$.

\(^{11}\)Exact analytical results have not yet been obtained for this procedure (See Newman, Watts and Strogatz, 2002 for a recent review of the literature). Consequently, our approach is essentially a “reduced form” use of the small world procedure.

\(^{12}\)As in Watts and Strogatz, we assume values normalized by $C(0)$ and $L(0)$, so that $c(p)$ and $l(p)$ lie between zero and one.
A3 Over a certain range, say $p \in [0, \overline{p}]$, $|c'(p)| \leq |l'(p)|$ and over $(\overline{p}, 1]$, $|c'(p)| > |l'(p)|$.

A4 Over a certain range, say $p \in [0, \overline{p}], \overline{p} < \overline{p}, |c''(p)| \leq |l''(p)|$ and over $(\overline{p}, 1]$, $|c''(p)| > |l''(p)|$.

A5 Over the whole range, $p \in [0, 1]$, $l(p) \leq c(p)$.

E. The Stage Game

The agents our relational graph economy are infinitely lived and are confronted with a symmetric two-sided prisoners’ dilemma game each period. A two-sided prisoners’ dilemma is used in order to capture the idea that each side of the transaction can cheat on the other. The payoffs are shown in Figure 3. As usual, assume $W > H > D > E$ and $W + E < 2H$.

In addition, an agent’s needs change period by period. Every period two agents are uniformly randomly selected to be a “best match” to each other in terms of their needs. If the two players whose needs and capabilities fit best interact, the payoff from honest behavior on the part of both players is $H + \theta$, instead of $H$ as it is with any other partner. $\theta$ is thus
the premium obtained from interacting honestly with the “ideal” partner, given an agent’s specific needs that period.

The fundamentals of the game are stationary and we consider stationary equilibria with the following “grim” trigger strategy.\textsuperscript{13}

(i) When faced with an opponent whose record you do not know, play Honest.

(ii) If you know your opponent’s record,
   (a) Play Honest if the opponent has an Honest record.
   
   (b) Play Cheat if the opponent has a Cheat record.

A Cheat record means that the player’s action history contains at least one “Cheat” and an Honest record means that the player’s action history contains no “Cheat.” The premium $\theta$ is not delivered when the Cheat record is found out.

F. Search and Information

Since there are $n$ agents, the probability that any player $j$ is player $i$’s ideal trading partner is $\frac{1}{n}$. In each period player $i$ does not know in advance who his ideal partner is, but can engage in search. The average cost of each search is proportional to the characteristic path length $l(p)$. Total search costs are assumed to be a quadratic function of the number of searches $q$ of the form $\frac{1}{2}q^2l(p)$\textsuperscript{14}. The search process works as follows. Agent $i$ gathers information from $q$ other individuals simultaneously. Let $Q$ be the sample of $q$ agents that $i$ gathers information from. After analyzing this information, if he finds his “ideal” partner he interacts with him. If the ideal partner is not within the sample, he interacts with a partner selected at random from the set $Q$. The probability that the ideal partner is within set $Q$ is thus $\frac{q}{n}$. The discount factor is $\delta$.

The clustering coefficient is a measure of how likely it is that information regarding an individual’s behavior is known to other agents in the economy. We thus assume that $c(p)$ is the probability that a randomly chosen individual will know your record.

\textsuperscript{13}More sophisticated equilibrium strategies are of course possible, such as two-phase carrot and stick punishments (see Fudenberg and Tirole 1991, Ch 5). But for our purpose grim trigger strategies are simpler and convey adequate intuition.

\textsuperscript{14}The quadratic form of search costs is adopted only for simplicity and the arguments are valid for a general convex cost function.
4 Analysis

In this section we analyze the sustainability of cooperative behavior in the absence of any external enforcement institutions.

Let $V_H$ and $V_C$ denote the expected present value payoff of an agent with Honest record and Cheat record, respectively. Also, let $V_{H,\theta}$ and $V_{H,0}$ stand for the present value payoffs when an agent with Honest record finds his ideal match and does not find his ideal match, respectively. $V_{C,\theta}$ and $V_{C,0}$ are similarly defined for an agent with Cheat record. Then we can write the following system of equations to characterize behavior in the game.

\[ V_{H,\theta} = \max \{ H + \theta + \delta V_H, W + \theta + \delta V_C \} \]  
\[ V_{H,0} = \max \{ H + \delta V_H, W + \delta V_C \} \]  
\[ V_H = \max_q \left( \frac{q}{n} V_{H,\theta} + (1 - \frac{q}{n}) V_{H,0} - \frac{1}{2} q^2 l(p) \right) \]  
\[ V_{C,\theta} = (1 - c(p)) \max \{ H + \theta + \delta V_C, W + \theta + \delta V_C \} + c(p) \max \{ E + \delta V_C, D + \delta V_C \} \]  
\[ V_{C,0} = (1 - c(p)) \max \{ H + \delta V_C, W + \delta V_C \} + c(p) \max \{ E + \delta V_C, D + \delta V_C \} \]  
\[ V_C = \max_q \left( \frac{q}{n} V_{C,\theta} + (1 - \frac{q}{n}) V_{C,0} - \frac{1}{2} q^2 l(p) \right) \]  

where in the curly brackets the first (second) term represents the payoff when the player chooses to play Honest (Cheat). Note that the player with Cheat record always chooses Cheat.

If a player with Honest record chooses to be honest in the current period his payoff is obtained to be

\[ V_H = \frac{1}{1 - \delta} \left[ H + \frac{q \theta}{n} - \frac{1}{2} q^2 l(p) \right] \]  

whereas if he chooses to cheat in the current period his payoff is obtained to be

\[ V_H = W + \frac{q \theta}{n} - \frac{1}{2} q^2 l(p) + \delta V_C. \]  

Since a player with Cheat record always chooses to cheat, we can obtain his payoff to be

\[ V_C = \frac{1}{1 - \delta} \left[ \frac{q \theta}{n} (1 - c(p)) + (1 - c(p)) W + c(p) D - \frac{1}{2} q^2 l(p) \right]. \]
From (7) we can find the optimal level of search \( q^* \) when Honest to be

\[
q^* = \frac{\theta/n}{l(p)}
\]  

(10)

and from (9) we find the optimal level of search when cheating to be \((1 - c(p))q^*\).

We state this intuitive result on search in the form of the following lemma.

**Lemma 1:** Search is increasing in the expected premium from the “ideal” match and decreasing in the characteristic path length of the economy.

Subtracting (8) from (7) and using (9) yields an expression for the premium from honest behavior,

\[
G = \delta c(p) \left[ W - D + \frac{\theta^2(3 - 2c(p))}{2n^2 l(p)} \right] - (W - H)
\]  

(11)

\( G = 0 \) on the boundary where an agent is indifferent between honesty and dishonesty. \( G > 0 \) implies we will be in the honest regime, when agents chose to cooperate, and \( G < 0 \) implies we will be in the dishonest regime, when agents chose to cheat. In order to examine issues relating to regime switching as the economy increases in complexity from regularity to randomness, we differentiate \( G \) with respect to \( p \). This yields,

\[
\frac{\partial G}{\partial p} = \delta \left[ c'(p) \left( W - D + \frac{\theta^2(3 - 2c(p))}{2n^2 l(p)} \right) + c(p)\theta^2 \frac{2c'(p)}{2n^2 l(p)} \left( -2c'(p) - \frac{(3 - 2c(p))l'(p)}{l(p)} \right) \right]
\]  

(12)

The first part of this expression is negative while the second part is positive. Consequently whether \( \frac{\partial G}{\partial p} \geq 0 \) depends on the relative magnitudes of the first and second parts.

In order to understand the behavior of the \( G \) function over the range of \( p \), we use A1-A4 to analyze the behavior of expressions (11) and (12).

First consider the value of \( G \) at the two extremes, \( p = 0 \) and \( p = 1 \).

At \( p = 1 \), \( l(p) \), \( c(p) \) = 0. So it is easily observed that \( G \mid_{p=1} < 0 \).

At \( p = 0 \), \( l(p) \), \( c(p) \) = 1. So, \( G \mid_{p=0} = \delta \left[ W - D + \frac{\theta^2}{2n^2} \right] - (W - H) \). We can see that this can be positive under a variety of parameter configurations. For example, this would be the case if the “ideal match” premium \( \theta/n \) were large ( \( >> W - H \) ), the discount factor \( \delta \) were high, and the punishment payoff \( D \) were small.

Since we have assumed \( l(p) \), \( c(p) \) are continuous, \( G(p) \) will also be continuous. Then by the Intermediate Value Theorem we can argue that \( G \) can go from positive to negative values as \( p \) increases.
Next, consider $\frac{\partial G}{\partial p}$ from (12). At low values of $p$ the magnitude of $c'(p)$ is low while $c(p)$ and $l'(p)$ are high. Also, because of the small world effect, $l(p)$ is low. This implies that the first term will be small while the second term will be large. Consequently (12) can be positive at low values of $p$. However, as $p$ increases, it can turn negative.

We could summarize this as suggesting a shape for the $G$ curve as depicted in Figure 4. Call the value $p$ where $G = 0$, $p^0$.

Note that this implies a value of $p$, say $p^*$, with $p^* < p^0$ where the gains from the honest regime $G$ are highest. This value could be backed into an optimal level of clustering and characteristic path length. In other words, in the absence of external enforcement mechanisms, there is an optimal level of randomness in the economy that determines the optimal size of the network. We summarize the preceding analysis in the following proposition.

**Proposition 1:** In the self-governance economy, parameter values determine an optimal level of modernization $p^*$. This level of modernization in turn determines the optimal level of social embeddedness in the economy: The optimal level of modernization $p^*$ determines the optimal degree of clustering $c(p^*)$ and the optimal characteristic path length, $l(p^*)$, of the economy.
The result could be interpreted as follows. Without external institutional mechanisms to intermediate reliable exchange, the economy is apt to confine itself to an intermediate level of interaction in order to sustain cooperation. In the absence of external institutions, an expansion in interaction with other individuals outside the relational neighborhood could prove disastrous by leading to the collapse of cooperation altogether.

5 Non-Market and Market Interactions

The analysis of the previous section provides us with an understanding of the way in which social embeddedness in the economy evolves as modernization takes place. So far we have considered a generic prisoners’ dilemma stage game to enable us to focus on cooperative behavior and its collapse as socially embeddedness disintegrates in the wake of modernization. In this section our goal is to use the preceding framework to examine the differential impact of modernization on interaction between individuals in our relational graph economy in two separate contexts, non-market (or social) and market (or economic).

We do this by considering that individuals in the economy are simultaneously engaged in two separate prisoners’ dilemma games, one of which represents market interactions while the other represents non-market interactions. Using separate prisoners’ dilemma games allows us to distinguish market interactions (we call this game 1) from non-market interactions (we call this game 2). Grafting these two games onto the relational graph enables us to consider the social embeddedness of market transactions and the social embeddedness of non-market transactions.

Since non-market (social) interactions differ in many ways from market (economic) interactions, the differences between game 1 and game 2 can be numerous. Since we are comparing social and economic interactions within a given economy, we will hold constant the basic structure of interactions; specifically, $n$, $k$ and the functions $c(p)$ and $l(p)$ must necessarily remain unchanged across models. The difference between market and non-market outcomes lies in the benefits accruing to these transactions. For purposes of comparison, we assume that the two games are identical save for the premium from the ‘ideal’ match being lower in game 2, i.e., $\theta_2 < \theta_1$; i.e., the market generates greater potential benefits from making potentially welfare improving connections. Given that our thesis is to argue that
the optimal level of modernization is likely to differ when considering market vs. non-market interactions, allowing for additional differences between economic and social transactions will only strengthen our argument on this point.

The analysis presented in section 4 then carries through here with the slight change in notation described above. The expression for the premium from honest behavior can be written for game i as,

\[ G_i = \delta c(p_i) \left[ w - d + \frac{\theta^2_i (3 - 2c(p_i))}{2n^2l(p_i)} \right] - (w - h) \]  

where \( i = 1, 2 \).

Since \( \theta_2 < \theta_1 \) it is easy to see from (13) that \( G_2 \) lies below \( G_1 \). Also, \( \frac{\partial^2 G}{\partial p \partial \theta} \bigg|_{p^*} > 0 \) [see the appendix for the proof]. It is clear, therefore, that the value of \( p_i \), say \( p^*_2 \), where the gains from the honest regime \( G_2 \) are highest in game 2 will in general be different than \( p^*_1 \) and that in this case, \( p^*_2 < p^*_1 \). In other words, the level of modernization/randomness (\( p \)) that maximizes the gains from the non-market transactions is less that that which maximizes the market transactions.

**Interlinkage**

The above analysis suggests a different optimum level of modernization in each game. However, in reality market (economic) and non-market (social) interactions are interlinked through a common value of the modernization parameter. What we mean by this is that while individuals engage in transactions in different contexts — market (economic) and non-market (social) — their interactions in both spheres are affected by a common value of \( p \) since both types of transactions take place in the same relational graph economy. We could thus refer to the two linked games together as a socio-economic system.

By comparison of the optimal level of \( p^* \) in each game, we arrive at the following proposition.

**Proposition 2:** In an interlinked socio-economic system under self governance, the economically optimal level of modernization (\( p^* \)) is excessive from a social standpoint: The level of social embeddedness dictated by economic interactions undercuts cooperation in social interactions.
This suggests that if the economic sphere drives the level of modernization in a society, then that society will arrive at a level of modernization that is detrimental to the optimal functioning of its social interactions.

6 Institutions

As we have seen in the preceding analysis, at large values of \( p \), when \( G < 0 \), the existing structure of links in no longer adequate to sustain honest economic activity. This suggests that beyond this level of modernization there is a gainful role for external institutions. As previously mentioned we consider two kinds of institutions: information intermediaries and enforcement intermediaries.

A. Information Intermediaries

Consider an intermediary who gathers and transmits information regarding cheating through the economy. In a market context we could think of such an intermediary being similar to a credit rating agency. For a given value of \( p \), such an intermediary increases the likelihood of being found out. We could say then that the probability of being found out it is not \( c(p) \) but \( c(p) + \gamma_i \), where, as before, the subscript \( i \) represents either game 1 or game 2. Bear in mind the restriction \( c(p) + \gamma_i \leq 1 \). \( \gamma_i \) is thus a parameter that stands for the function of the information intermediary. We call such an intermediary Info following Dixit (2003)\(^{15}\). The introduction of Info has the effect of shifting up the \( G \) curve as depicted in Figure 5.

The \( G \) function now takes the form

\[
G = \delta(c(p) + \gamma_i) \left[ W - D + \frac{\theta_i^2(3 - 2(c(p) + \gamma_i))}{2n^2l(p)} \right] - (W - H). \tag{14}
\]

This also has the effect of changing \( p^* \) and the maximal value \( G^* \). Agents in the economy should be willing to pay Info the difference \( S = G(\gamma_i^1, p) - G(\gamma_i^0, p) \), where \( \gamma_i^1 > \gamma_i^0 \). Note that intermediation is essential for normal economic activity beyond \( p^0 \), but is valuable even before. Consequently, whether Info enters or not depends whether the sunk cost of setting up intermediation is less than \( S \).

\(^{15}\)The terms Info and Enfo were coined by Dixit (2003) to refer to Information and Enforcement intermediaries.
Comparing Information Intermediaries in the Social and Economic Game

First let us consider the impact of raising the probability of having one’s dishonest behavior revealed in either of the two games. In other words, what is the marginal impact of $\gamma$ on $G$ in each game?

\[
\frac{\partial G}{\partial \gamma} = \delta \left[ W - D + \frac{\theta^2_i(3 - 2(c(p) + \gamma))}{2n^2l(p)} \right] - \delta(c(p) + \gamma) \left[ \frac{\theta^2_i}{2n^2l(p)} \right] \tag{15}
\]

and

\[
\frac{\partial G}{\partial \gamma \partial \theta} = \delta \left[ \frac{\theta_i(3 - 3(c(p) + \gamma))}{n^2l(p)} \right] > 0 \tag{16}
\]

Since in the social game, $\theta_2 < \theta_1$, the marginal benefit of raising the likelihood of catching someone who is engaging in undesirable behavior is greater in the market (economic) game. In other words, a given level of information will be more effective at promoting cooperative behavior in the market game than in the non-market (social) game. Moreover, it seems likely that in the non-market game, it would be more difficult to achieve a similar level of information dissemination. Intuitively, it seems evident that institutions such as credit agencies will
be more effective at eliminating information problems than an analogous institution might be in distributing information about what might viewed as undesirable social behavior. How might an institution credibly and accurately reveal that an individual might make a poor spouse, friend or neighbor? Given this, it seems reasonable that not only will the marginal benefit of social institutions designed to raise information be lower than in analogous economic institutions, but their ability to achieve the desired increase in information is less as well. As a result, we expect \( \gamma_1 > \gamma_2 \).

**B. Enforcement Intermediaries**

These kind of intermediaries inflict punishment on cheaters. We refer to such an intermediary as Enfo. If found out, the cheating payoff goes down because of Enfo. We could think of this as a decrease in the mutual cheating payoff \( D_i \) where \( i = 1, 2 \). This also has the effect of shifting up the \( G \) curve as with Info.

We can compare between Info and Enfo.

\[
\frac{\partial G}{\partial D} = -\delta(c(p) + \gamma_i) < 0
\]

From the above discussion, we expect \( \gamma_1 > \gamma_2 \) which implies that lowering the returns to cheating is more effective at achieving cooperation in the economic game. Again, this compromises the ability for a given level of enforcement to be effective in the social game. Further, as with social information, in addition to being less effective, social enforcement is likely to be less achievable as well. Intuitively, legal institutions designed to punish fraudulent economic behavior seem more plausible than analogous institutions designed to punish uncooperative social behavior such as cheating on one’s spouse or being generally difficult as a friend or neighbor. How possible is it to even lower \( D \) in this circumstance? Alternatively, the costs of lowering \( D \) are conceivably quite large in the social context while the marginal benefits as shown in equation (17) are likely to be lower than those in an economic context. As with informational enforcement, the combination of these factors would indicate that \( D_1 < D_2 \).

Finally, this lower level of enforcement will have a further feedback effect on the marginal
benefits of information. Referring back to equation (17) above, we see that

$$\frac{\partial^2 G}{\partial \gamma \partial D} = -\delta < 0. \quad (18)$$

In other words, the less a society is able to punish undesirable social behavior (indicated by higher values of $D$), the less effective information dissemination will be also. In other words, because undesirable social behavior is not as punishable as undesirable economic behavior, information that reveals such behavior is also less valuable.

We could summarize the preceding discussion in the form of the following proposition.

**Proposition 3:** The benefits of both information and enforcement are greater in market interactions than in non-market interactions. Simultaneously, the costs are greater in the non-market interactions.

As a result, it seems evident that both sets of institutions will be less likely to be present and to the extent that they exist, they will be less effective at promoting honest behavior in the social context. Thus, the ability for such institutions to replace social embeddedness as society modernizes is diminished. Combined with the results stated in proposition 2, it is clear that the optimal level of modernization warranted by economic efficiency exceeds that which would be warranted by social efficiency. To the extent that such third-party institutions allow the economy to modernize effectively, we would expect the presence of these economic institutions to harm the social fabric. We thus arrive at the disconcerting conclusion that institutions that are good for the economic sphere are dysfunctional for the social sphere. We summarize this reasoning in the form of the following proposition.

**Proposition 4:** Economic intermediation has a detrimental impact on cooperation in social transactions by increasing the level of complexity in the socio-economic system. Improvements in institutional effectiveness have a dysfunctional impact on the social sphere by weakening incentives for social cooperation.

We could say that our analysis formalizes arguments made by Putnam (2000). Putnam makes the argument that over the last forty years the U.S. has experienced a sharp decline in almost all forms of social cooperation. This period has also been a period of unprecedented
economic growth and development, mobility and urbanization. Dramatic advances in the legal and financial infrastructure of the economy have also taken place. These phenomena could be collectively referred to as being correlated to what we have labeled modernization. Our framework thus provides us with an understanding of why economic progress might go hand in hand with social regress.

C. Social Welfare

Given the tension between modernization intended to achieve economic efficiency and the potential breakdown of social cooperation, this suggests that a social welfare function that includes both spheres is warranted. We propose that a socially optimal level of modernization would maximize a weighted average of both spheres. Assuming that a society’s citizens receive utility from a society that functions well both economically and socially, we could write a general welfare function as follows:

\[
W = \alpha U(E) + \beta U(S)
\]  

(19)

where \(U(E)\) and \(U(S)\) represent the utility that a typical citizen receives from economic and social performance. Of course, \(\alpha\) and \(\beta\) represent the relative weights that individuals in a given socio-economic system place on each sphere. It is obvious that if the optimal level of modernization in the market sphere exceeds that of the non-market sphere, then the optimization of (19) will generate a solution that lies between those values.

To the extent that across countries the relative weights attached to these ideals differs, we might expect to see countries making different choices regarding modernization. For instance, one might argue that the differences we observe between European countries and the United States can be interpreted in this model as a result of different preferences between economic performance and the social function of society. In fact, a number of recent papers (Alesina et. al. (2001), Gordon (2002)) have noted that there are systematic differences between the U.S. and Europe in their preferences, and consequent choices between economic modernization and social embeddedness. Both of these papers argue that while economic modernization is higher in the U.S., social embeddedness is higher in Europe.

Finally, a number of scholars, including Fukayama (1995), Knack and Keefer (1997) and contributors to the Dasgupta and Serageldin (1999) volume among others, suggest that there
exists a feedback from the non-market world to the market world. Specifically, trust can be effective in enhancing economic growth, and as a result, a decline in social capital can impede economic performance. In other words, in the context of the social welfare function in equation (19), $E$ should be dependent upon $S$. If the degree of this feedback is extensive, then even a society that predominantly values economic performance may over-modernize if it ignores the impact of modernization on social performance.

7 Conclusion

In this paper, we study the impact of modernization on both market and non-market interactions. Using the small-world based framework, we address the notion that social benefits from an increasingly modern society may not be as large as the economic benefits that derive from decreased search costs and enhanced efficiency. In addition, because increasing modernization impedes community governance of behavior, it can lead to dishonest behavior, a cost which must be weighed against the benefits.

To mitigate this trade-off, societies have developed institutions that can to some degree replace community governance. However, the ability of institutions to promote honest behavior in non-market circumstances is likely to be less than in market circumstances. While legal institutions have the ability to punish those found to engage in dishonest transactions, there is no corresponding punishment for behaving dishonestly toward one’s spouse, friends or neighbors. In this paper, we show that if the social benefits of modernization are less than the economic benefits, and if the ability to enforce honesty through institutions is also inferior for social interactions, then the optimal level of modernization from an economic perspective exceeds that from a social perspective. This tension can explain the growing concern over the recent decline in social capital. Given different preferences across societies, it may also explain differences in modernization strategies we observe across cultures.

Finally, given a growing body of evidence that social capital has a direct impact on economic performance, it is a mistake to ignore the well being of the social fabric. If a well functioning society from a non-market perspective contributes to improved market performance, then allowing economic modernization to occur to the detriment of the social fabric may create an inadvertent decline in market performance as well. Thus, even a society
whose preferences are purely economic in nature needs to consider the detrimental impact that excessive modernization from a social perspective might inflict upon the economy.
Appendix

1. Sign of $\frac{\partial^2 G}{\partial p \partial \theta} |_{p^*}$.

The gains from honest exchange are maximized at the value of $p$ (call this value $p^*$) where equation (12) is equal to zero. That is,

$$\frac{\partial G}{\partial p} = \delta \left[ c'(p) \left( W - D + \frac{\theta^2 (3 - 2c(p))}{2n^2 l(p)} \right) + \frac{c(p)\theta^2}{2n^2 l(p)} \left( -2c'(p) - \frac{(3 - 2c(p))l'(p)}{l(p)} \right) \right] = 0$$

This condition can be written as,

$$c'(p) (W - D) + \frac{\theta^2}{2n^2 l(p)} \left( c'(p)(3 - 2c(p)) - 2c'(p)c(p) - \frac{(3 - 2c(p))l'(p)c(p)}{l(p)} \right) = 0 \quad (20)$$

Evaluated at the optimal value $p^*$, equation (20) is of the form $F(\theta) = A + B\theta^2$, where $A = c'(p^*) (W - D)$ and $B = \frac{1}{2n^2 l(p^*)} \left( c'(p^*)(3 - 2c(p^*)) - 2c'(p^*)c(p^*) - \frac{(3 - 2c(p^*))l'(p^*)c(p^*)}{l(p^*)} \right)$.

Since $A < 0$ as $c'(p^*) < 0$, it must be that $B > 0$ to satisfy the first order condition, $F(\theta) = 0$.

Consequently, a small change in the value of $\theta$ holding $p = p^*$ implies $\frac{\partial F}{\partial \theta} = B > 0$. Thus, $\frac{\partial^2 G}{\partial p \partial \theta} |_{p^*} > 0$. 

28
References


