Interacting photon pulses in a Rydberg medium

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Understanding the dynamic evolution of interacting photon pulses in a Rydberg atomic ensemble is a prerequisite for realizing quantum devices with such a system. We present an approach that efficiently simulates the dynamical processes, using a set of local functions we construct to reflect the profiles of narrowband pulses. For two counter-propagating photon pulses, our approach predicts the distinct phenomena, from the widely known Rydberg blockade to the previously less noticed significant absorption in the anomalous dispersion regime, that can occur by respectively setting the pulse frequency to the appropriate values. Our numerical simulations also demonstrate how spatially extending photon pulses become deformed under realistic nonuniform interaction over their distributions. © 2016 Optical Society of America

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1. INTRODUCTION

Since its earlier experimental observations [1–3], the phenomenon of electromagnetically induced transparency (EIT) in ensembles of cold Rydberg atoms has attracted extensive research. Different from the ordinary EIT [4], there exists van der Waals (vdW) or dipole–dipole interaction between Rydberg atomic excitations to modify the absorption and dissipation of a light field propagating in the medium (see, e.g., [5–18]). More recent experiments [19–24] have demonstrated numerous interesting features related to such long-range interactions, which make Rydberg EIT medium a promising candidate for implementing quantum information processing devices such as photon–photon gates [25–32] and photon switches (photon transistors) [33–37]. These applications involve individually prepared and mutually interacting photon pulses.

So far there has been no study about the real-time evolution of photon pulses based on the full dynamics and incorporating the general interaction between pulses as well as their realistic dissipation in the medium. A majority of the previous works about Rydberg EIT apply the steady-state propagating picture for continuous-wave (CW) light, according to which the time derivatives of the induced atomic excitations are assumed to be zero (see, e.g., the phenomenological superatom model in [7]). A process of finite-sized pulses in the same medium can be much more complicated, since the atomic excitations induced by the pulses varying with time are surely time-dependent, too. Regarding the accompanying physical effects, the Rydberg blockade [38,39] was the theme of the previous research.

This well-known phenomenon takes place when atoms in the medium are under strong interaction to prevent their excitation to Rydberg levels. The processes of interacting photon pulses have also been approximated with a similar blockade model [28], i.e., the medium becomes a two-level system when photons meet inside a blockade radius and is under the EIT condition when they are away from each other. It is important to understand photon–photon processes with a more realistic picture.

Here we present an approach based on the complete dynamical equations of the quantum field involved to study the processes of interacting photon pulses in a Rydberg medium. A main difference from the blockade potential model [28] and the quantum spin model [40], which describe the interaction mediated by Rydberg excitations as the potentials of either constant or discretely distributing values, is that we consider ever-changing interaction throughout pulse evolution. In reality, photon pulses evolve continuously in space and time without a clear-cut boundary like a Rydberg blockade radius and, as we will show below, a variable interaction between pulses can lead to totally different phenomenon (significant pulse absorption) from the commonly concerned Rydberg blockade. More realistically, our approach goes beyond the assumption of uniform dissipation over pulses’ spatial distributions [30] to capture the effects of inhomogeneous interaction magnitude.

The rest of the paper is organized as follows. In Section 2 we develop a theoretical approach, by using a type of local functions, to the dynamical processes of interacting photon pulses in a Rydberg EIT medium, starting from the exact dynamical
equations of the quantum fields involved. The model used for illustration and the numerical simulation procedure are explained in detail. The simulation results described in Section 3 concern mainly the dynamical evolutions of counter propagating photon pulses. We illustrate the evolution processes of such single-photon pulse pairs, which manifest speedup propagation in the Rydberg blockade regime or significant absorption in the anomalous dispersion regime, according to the sign of their detuning. The evolution of polarization fields of the decaying atomic level and the change of pulse profiles under the realistic inhomogeneous interaction are also demonstrated with examples. Finally, Section 4 contains the conclusions.

2. DYNAMICS OF INTERACTING PHOTONS

A. General Two-Photon Process

We start with two arbitrary weak light fields \( \hat{E}_l(x, t) \) \((l = 1, 2)\) propagating in ensembles of the atomic level scheme in Fig. 1(a). Together with the pump beams or control fields with the Rabi frequency \( \Omega_{l}(t) \), they induce atomic excitation distributions and the polarization fields \( \hat{P}_l(x, t) = \sqrt{N} \hat{\sigma}^l_{a}\hat{a}(x, t) \) and spin-wave fields \( \hat{S}_l(x, t) = \sqrt{N} \hat{\sigma}^l_{a}\hat{a}(x, t) \). The flip operators \( \hat{\sigma}^l_{a}\hat{a}(x, t) \), representing the transitions between the levels of atoms in an ensemble of high density \( N \), are treated as continuous fields. When the two light fields propagate in parallel along the \( z \) axis, the dynamical equations of the involved quantum fields read \((\hbar = 1)\)

\[
\begin{align*}
\partial_t \hat{E}_l(x, t) + c \partial_x \hat{E}_l(x, t) = & \, i g \sqrt{N} \hat{P}_l(x, t), \\
\partial_t \hat{P}_l(x, t) = & \, -\gamma - i \Delta_{\gamma} \hat{P}_l(x, t) + i \Delta_{\gamma} \hat{E}_l(x, t) \\
& + i g \sqrt{N} \hat{E}_l(x, t) - \sqrt{2} \hat{S}_l(x, t), \\
\partial_t \hat{S}_l(x, t) = & \, -\gamma' + i \Delta' \hat{S}_l(x, t) + i \Delta' \hat{P}_l(x, t) - \sqrt{2} \hat{E}_l(x, t) \\
& - i \int \mathrm{d}x' \hat{S}_l(x - x', t) \hat{S}_l(x', t) \hat{S}_l(x, t), \\
& + i \int \mathrm{d}x' \hat{S}_l(x - x', t) \hat{S}_l(x', t) \hat{S}_l(x, t), \\
\end{align*}
\]  

(1)

(2)

(3)

\[\begin{align*}
\text{Fig. 1.} \quad \text{(a) Atomic level scheme. Here } \Delta_{\gamma} = \omega_{\gamma} - \omega_{\gamma} \text{ and } \\
\Delta' = \omega_{\gamma} - \omega_{\gamma'}, \text{ as the differences between a level gap and a field central frequency. (b) Magnitude of the time-dependent interaction potential at a location on one of the pulses, as the result of interacting with the other photon pulse passing by it. The dashed one represents the potential in a blockade model. (c) Geometry of pulse propagation in two parallel waveguides. They either propagate face to face or go together, and can also be stopped inside the ensembles.}
\end{align*}\]

in a frame rotating at the central frequency \( \omega_{\gamma} \) of the pulse fields \( l = 1 \) and 2. Here we consider the slowly varying fields \( \hat{E}_l(x, t) \) with time derivatives much smaller than their multiplication by \( i\omega_{\gamma} \), and they couple to the atoms with a constant defined as \( g = \mu_{g}\omega_{\gamma}/\epsilon_0 \) (\( \mu_{g}\) is the electric dipole matrix element and \( \epsilon_0 \) the vacuum permittivity). The two-photon detuning \( \Delta = \Delta_{\gamma} + \Delta_{\gamma'} \) in Eq. (3) vanishes under the EIT condition \( \Delta_{\gamma} = -\Delta_{\gamma'} \) [see the definitions of these detunings in Fig. 1(a)] and, via a nonlocal potential \( \Delta(x - x', t) \), a spin-wave field \( \hat{S}_l(x, t) \) experiences interaction with the other one, \( \hat{S}_{\gamma'}(x, t) \), as well as with itself. The quantum noise operators \( \hat{S}_{\gamma'}(x, t) \) are introduced to preserve the commutation relation for the quantum field operators, \( \hat{D}_l(x, t) = \hat{P}_l(x, t), \hat{S}_l(x, t) \) and \( \hat{E}_l(x, t) \), in the presence of energy level decay at the rates \( \gamma, \gamma' \).

Next we restrict the above process to what happens to two single-photon pulses, whose quantum state takes the form

\[
\begin{align*}
|1, 1\rangle = \int \mathrm{d}x_1 f_1(x_1) \hat{E}^\dagger_1(x_1) \int \mathrm{d}x_2 f_2(x_2) \hat{E}^\dagger_2(x_2)|0\rangle,
\end{align*}
\]

(4)

before entering the medium, where the normalized functions \( f_1(x_1), f_2(x_2) \) are their snapshots at \( t = 0 \) in free space, and \( |0\rangle \) is the vacuum state for the whole system plus reservoirs. The dynamical equations [Eqs. (1)–(3)] can be obtained from the evolution operator \( U(t) = T \exp[-i \int_0^t \mathrm{d}t' H(t')/\hbar] \) of a total Hamiltonian \( H(t) \), including a stochastic part that accounts for the energy level decay (see Supplement 1 for details). Note that \( U(t) \) is not an ordinary unitary operator. Under the action \( U(t) \) the two-photon state in Eq. (4) will evolve to a general form as follows:

\[
\begin{align*}
U(t)|1, 1\rangle = \int \mathrm{d}x \int \mathrm{d}x' E(x, x', t) \hat{E}^\dagger_1(x) \hat{E}^\dagger_2(x')|0\rangle \\
+ \int \mathrm{d}x \int \mathrm{d}x' (EP(x, x', t) \hat{P}_2^\dagger(x') + (E\rightarrow P)|0\rangle \\
+ \int \mathrm{d}x' PP(x, x', t) \hat{P}_1^\dagger(x')|0\rangle \\
+ \int \mathrm{d}x' ES(x, x', t) \hat{E}^\dagger_1(x) \hat{S}_2^\dagger(x') + (E\rightarrow S)|0\rangle \\
+ \int \mathrm{d}x' SS(x, x', t) \hat{S}_1^\dagger(x) \hat{S}_2^\dagger(x') + (S\rightarrow S)|0\rangle \\
+ \int \mathrm{d}x' (PS(x, x', t) \hat{P}_1^\dagger(x') \hat{S}_2^\dagger(x') + (P\rightarrow S)|0\rangle \\
+ \text{noise components.}
\end{align*}
\]

(5)

Meanwhile, the quantum state of only one of the pulses will become

\[
\begin{align*}
U(t)|1\rangle = \int \mathrm{d}x E(x, t) \hat{E}^\dagger_1(x)|0\rangle \\
+ \int \mathrm{d}x P(x, t) \hat{P}_1^\dagger(x)|0\rangle \\
+ \text{noise components}
\end{align*}
\]

(6)

in the absence of the other, also exhibiting possible loss with the converted noise components.

B. Dynamical Equations for Photon Pulses

Any two-photon process can be described by the nine evolving two-particle functions \( E(x, x', t), P(x, x', t), \ldots \) in Eq. (5). Although the Schrödinger equations governing their dynamical evolutions (see, e.g., the supplementary material of [19]) are linear
compared with the coupled nonlinear Heisenberg–Langevin equations [Eqs. (1)–(3)] of the field operators, it is not so straightforward to solve the dynamical equations from an initial profile \( \mathcal{E}(x, x', t, \phi) = f_0(x) j(x') \) in Eq. (4). A main difficulty in numerically solving this initial value problem of the evolved twoparticle functions \( OO(x, x', t) = \langle 0 | \hat{O}_1(x, t) \hat{O}_2(x', t) | 11 \rangle \) \((\hat{O} = \hat{E}, \hat{P} \text{ and } \hat{S}) \) is that the size of a pulse will undergo tremendous change in the course of becoming a slowly propagating wavepacket in an EIT medium; see, e.g., [36] for a discussion on the similar numerics for other functions. Many previous simulations have started from the compressed pulses already inside the EIT medium, and cannot reflect their entrance process, which is especially important to two pulses going together into the medium. So far, in different previous works, the two-particle functions have been calculated only with various simplifications, such as neglecting the photon losses [25, 26], using the simplified dynamical equations from adiabatically eliminating the decaying level [28], working with their steady states (setting the time derivatives of the two-particle functions to be zero in the dynamical equations) [19], and adopting the analytical continuation from the results, neglecting photon loss at high detuning \( \Delta \) [41, 42].

Different from all other works, we will apply the functions defined as

\[
O_1(x, t) = \langle 0, 1 | \hat{O}_1(x, t) | 1, 1 \rangle,
\]

\[
O_2(x, t) = \langle 1, 0 | \hat{O}_2(x, t) | 1, 1 \rangle,
\]

where \( \hat{O}_1(x, t) = \hat{E}_1(x, t), \hat{P}_1(x, t) \) and \( \hat{S}_1(x, t) \) \((l = 1, 2) \), to deal with the dynamical problem. For one of the pulses, the spin-wave function of this type takes the exact form

\[
\begin{align*}
\hat{S}_1(x, t) &= \langle 0, 1 | U(t) | \hat{S}_1(x, U) | U(t) t \rangle | 1, 1 \rangle \\
&= \int dx' \langle \hat{S}_1(x', t) \hat{S}_1(x', t) \rangle S(x, x', t) \\
&\quad + |j_0(x', t) \rangle \langle j_0(x', t) \rangle S(x, x', t) \\
&\quad + \langle |j_0(x', t) \rangle \rangle \langle j_0(x', t) \hat{S}_1(x', t) \rangle S(x, x', t) \\
&\quad + \langle |j_0(x', t) \rangle \rangle \langle j_0(x', t) \hat{S}_1(x', t) \rangle S(x, x', t).
\end{align*}
\]

This is an inner product of the general two-body state equation \[Eq. (5)\] with the corresponding single-particle state equation \[Eq. (6)\] of the other pulse freely evolving in the absence of the one with which we are concerned [only three terms in Eq. (5) appear because the operator \( \hat{S}_1(x) \) kills all others]. The existence of the two-particle functions in Eq. (5) and single-particle functions in Eq. (6) renders such functions well defined in any situation, and they truly reflect the profiles of evolved pulses, except for their correlations.

To find the dynamical equations for the defined functions, we multiply the vector \([1, 1]\) to the right side of each term in Eqs. (1)–(3), and \([0, 1]\) for \(l = 1\) and \([1, 0]\) for \(l = 2\) to the left side of each term, resulting in the following set of equations:

\[
\begin{align*}
\partial_t E_1(x, t) + c_1 \partial_x E_1(x, t) &= ig \sqrt{N} P_1(x, t), \\
\partial_t P_1(x, t) &= -\gamma + \Delta_0 \hat{E}_1(x, t) + B \hat{E}_1(x, t) + i\sqrt{N} \hat{E}_1(x, t), \\
\partial_t S_1(x, t) &= -\gamma' + iV_1^\text{eff}(x, t) S_1(x, t) + i\Delta_0 \hat{P}_1(x, t).
\end{align*}
\]

The derivations of the first two equations are straightforward, but the meaning of the third should be explained. For the term on the second line in Eq. (3), the above procedure leads to

\[
\begin{align*}
&\langle 0, 1 | \int dx' \Delta(x - x') \hat{S}_1(x', t) \hat{S}_2(x', t) \hat{S}_1(x, t) | 1, 1 \rangle \\
&= \int dx' \Delta(x - x')/\langle 0, 1 | U(t) | \hat{S}_1(x', t) \rangle \int dk_1 dk_2 |k_1, k_2 \rangle \langle k_1, k_2 | \hat{S}_1(x', t) \hat{S}_1(x, t) | 1, 1 \rangle \\
&\times \hat{S}_2(x', t) \hat{S}_1(t) U(t) | 1, 1 \rangle \\
&= \int dx' \Delta(x - x')/\langle 0, 1 | \hat{S}_1(x', t) \rangle | 0, 0 \rangle \langle 0, 0 | \hat{S}_2(x', t) \hat{S}_1(t) | 1, 1 \rangle \\
&= \int dx' \Delta(x - x') \langle \hat{S}_1(x', t) | S_1^0(x', t) | \rangle^4 SS(x, x', t),
\end{align*}
\]

where the operator \( \hat{S}_2(x', t) \hat{S}_1(x) \) projects the component \( SS(x, x', t) | 0, 0 \rangle \) out of the evolved state \( U(t) | 1, 1 \rangle \), while another operator \( \hat{S}_2(x') \) projects out \( S_2^0(x', t) | 0, 0 \rangle \) from \( U(t) | 0, 1 \rangle \), so that only the vacuum component in the inserted complete set of wavevectors will be left after taking the inner products with them. This is equivalent to adding a complex valued two-photon detuning as the potential

\[
V_1^\text{eff}(x, t) = \int dx' \langle S_1^0(x', t) | \rangle^4 \Delta(x - x') SS(x, x', t) / S_1(x, t)
\]

given system parameters that are capable of realizing slow light to have the first term as an integral of the pure spin-wave components dominating in Eq. (8). This time-dependent effective potential for two counterpropagating pulses obviously looks like the one shown in Fig. 1(b), with its magnitude (the absolute value) going continuously to a peak value when the pulses are separated by the shortest distance. Meanwhile, performing the same procedure to the self-interaction term on the third line of Eq. (3) gives an exactly zero two-photon detuning, implying no self-interaction for single photons.

In terms of our defined functions \( S_1(x, t) \) in Eq. (8), the two-spin-wave function \( \langle 00 | \hat{S}_1(x, t) \hat{S}_2(x', t) | 11 \rangle \) can be approximated as \( SS(x, x', t) \approx S_1(x, t) S_2(x', t) \) \((l = 1 \text{ or } 2) \) if the pulses have sufficiently narrow bandwidths (see Supplement 1). Substituting this approximate form into Eq. (13) gives

\[
V_1^\text{eff}(x, t) = \int dx' \Delta(x - x') | S_1^0(x', t) | 2.
\]

This is the only major approximation we use, and the condition for the validity of the above separable form of \( SS(x, x', t) \) (see Supplement 1) indicates that it works for arbitrary interaction between pulses as long as they are narrowband ones compatible with the EIT medium.

The independent evolution of the functions \( S_1^0(x, t) = \langle 0, 0 | \hat{S}_1(x, t) | 1, 0 \rangle \) and \( S_2^0(x, t) = \langle 0, 0 | \hat{S}_2(x, t) | 0, 1 \rangle \) used in Eqs. (13) and (14) follow the equations

\[
\begin{align*}
\partial_t E_1(x, t) + c_1 \partial_x E_1(x, t) &= ig \sqrt{N} P_1(x, t), \\
\partial_t P_1(x, t) &= -\gamma + i\Delta_0 \hat{E}_1(x, t) + B \hat{E}_1(x, t) + i\sqrt{N} \hat{E}_1(x, t), \\
\partial_t S_1^0(x, t) &= -\gamma' + iV_1^\text{eff}(x, t) S_1^0(x, t) + i\Delta_0 \hat{P}_1(x, t).
\end{align*}
\]

which are found in a similar way to Eqs. (9)–(11). These functions evolve in the absence of pulse interaction, but are still subject to the dissipation due to a limited EIT width and other...
The uniqueness of our approach is to simultaneously solve two sets of differential equations, Eqs. (9)–(11) and (15)–(17), for finding the evolution of the functions defined in Eq. (7). The advantage of this approach in numerical calculations will be discussed below. These local functions provide a substitute of the two-particle functions for studying the evolution of interacting pulses, at the price of dispensing with their correlations, such as the entanglement discussed in [26,30,43,44]. Moreover, instead of the simple point–point potential \( \Delta(x - x') \) appearing in the equations about the two-particle functions with more spatial variables, an effective potential \( V_{eff}(x, t) \) determined from the distributions of pulses should be used, since there are fewer spatial derivatives to be integrated out in solving our equations.

With respect to the single-photon pair states with which we are currently concerned, the expectation values of the quantum field operators involved, \( \hat{O}_l(x, t) = \hat{E}_l(x, t), \hat{P}_l(x, t) \) and \( \hat{S}_l(x, t) \) \((l = 1, 2)\), always vanish. The functions in Eq. (7), however, give the amplitudes of the quantum fields of narrowband pulses through their average occupation numbers \( |O_l(x, t)|^2 = (1, 1)\hat{O}_l^\dagger \hat{O}_l(x, t), (1, 1) \) [following Eq. (S16) in Supplement 1], the electromagnetic component among which is measurable in principle. For narrowband interacting pulses with their entanglement neglected, their second-order correlation functions, for example, \( g_2(\tau) = \langle \hat{E}_l(z = L, z' = L - \nu_g \tau) \rangle \) (the unnormalized form of a function used in [19]), where \( L \) is the medium size and \( \nu_g \) the pulse group velocity, can be approximated as \( g_2(\tau) \approx \langle \hat{E}_l(z = L) \hat{E}_l(z' = L - \nu_g \tau) \rangle \). These relations directly connect the functions defined in Eq. (7) with measurable quantities.

C. Model for Illustration

The purpose of the current work is to apply the above general theory to photon pulses in Rydberg EIT media, where the nonlocal interaction potential is usually a vdW one: \( \Delta(x - x') = C_6/|x - x'|^6 \). Apart from what we have described above, there may exist some other factors relevant to the processes in a realistic Rydberg atomic ensemble. One of them is the nonuniform density \( N(x) \) of the atoms, which is decided by how they are trapped. The extra decoherence from collision between atoms [34,45] and due to anisotropic interaction of the D levels of Rydberg atoms [46] can exist. Moreover, the transverse profiles of the pulses will change due to diffraction, which can be depicted with an additional kinetic term in the dynamical equations [26,43].

The most important question about a two-photon process in a Rydberg medium is how the propagation of one pulse can be affected under interaction with the other. Another essential point is how the pulse profiles should change if the interaction and dissipation over them are not, in reality, uniform. To answer these questions, we adopt the setup illustrated in Fig. 1(c). In this parallel waveguide setup, two photon pulses either travel along the same direction (copropagation) or enter the opposite tips of two pencil-shaped ensembles (counterpropagation). Given a time-dependent control field \( \Omega_l(t) \), they can also be stopped inside the atomic ensembles, in which the evenly distributed atoms are assumed to be motionless. The prominent longitudinal extensions of the pulses make the setup suitable for illustrating the effects from the inhomogeneous pulse interaction between the different parts. Beyond illustrative purposes, the technical advances toward the realization of the setup have been reported in [47,48]. The calculations with this model setup are also close to those for a process inside a single atomic ensemble, where the diffraction of the Gaussian beams can be neglected in certain domains (see Fig. S1(b) in Supplement 1 or the setup proposed in [29]).

Without loss of generality the profiles of the pulses at the entries to the ensembles \((z = 0 \text{ or } L)\) are supposed to be \( \Omega_p(\lambda, t) = \Omega_p^M e^{i\omega_p(\lambda, t)^2/2\tau_p} j_0(2\kappa_{01}/\sigma d) \), where \( \Omega_p^M \) is the maximum of the photons' Rabi frequency \( \Omega_p = g\hat{E}_l \) and \( \tau_p \) and \( \tau_d \) are the time scales that indicate the peak arrival and pulse duration, respectively. We consider a single transverse mode \( j_0(2\kappa_{01}/\sigma d) \), the Bessel function of order zero with its first zero point \( \kappa_{01} \), which is more general transverse profiles can be used in the integral of Eq. (14) to find the effective potential \( V_{eff}(x, t) \) over an ensemble that is separated from the other one by an adjustable distance. The field profiles on the ensemble axis \( \beta = 0 \) will be illustrated as the representation of those in the whole space. A dynamical process of the interacting three-dimensional pulses in the setup is thus reduced to a problem of finding the relevant functions \( \hat{E}_l(z, t) \) over an ensemble that is separated from the other one by an adjustable distance.

D. Numerics in Brief

Now we come to the practical numerical calculations with the model. Using Eqs. (10) and (11), one can expand the right-hand side of Eq. (9) as

\[
ig \sqrt{N} \hat{P}_l(x, t) = \frac{g^2 N}{|\Omega_l(t)|^2} \frac{\partial}{\partial x} \hat{E}_l(x, t) - \frac{g^2 N \hat{E}_l(x, t)}{|\Omega_l(t)|^2} \frac{\partial}{\partial x} \frac{1}{\Omega_l(t)} \left( \frac{\partial}{\partial x} + \gamma + i\Delta_p \right) \cdot \frac{\partial}{\partial x} \hat{E}_l(x, t) \Omega_l(t)^{-1} \cdot \ldots (18)
\]

Under the condition \( g^2 N/|\Omega_l|^2 \gg 1 \), which is capable of realizing slow light, the time derivative on the left-hand side of Eq. (9) will be absorbed into the leading term of the above, reducing the equation to one with only one spatial derivative. This rearranged form of Eq. (9) with only one spatial derivative is discretized for finding the spatial distribution of \( \hat{E}_l(z_j, t_j) \) (over the lattice of \( z_j \) for \( 1 \leq j \leq N_z \)) at a specified moment \( t_j \leq N_t \), given the distribution of the polarization field profile \( \hat{P}_l(z_j, t_j) \) obtained with discretized Eqs. (10) and (11).

We simply apply a fourth-order Runge–Kutta method in the iterative procedure toward the field profiles \( \hat{E}_l, \hat{P}_l \), and \( \hat{S}_l \) over the \( N_x \times N_t \) space–time grid. The distribution of the constantly updated potential \( V_{eff}^l(z_j) \) at a specific moment, which is used determine the evolution to the next moment, is found with another set of field profiles \( \hat{E}_l^p, \hat{P}_l^p \), and \( \hat{S}_l^p \) from a similar iteration procedure with discretized Eqs. (15)–(17). The group velocities \( v_{p1}(t) \) and \( v_{p2}(t) \) of the evolving pulses can be directly read from their simulated real-time trajectories, unless the wavepackets lose their distinct contours due to a significant group velocity dispersion that can also be well simulated [see Figs. 2(c) and 2(d)].

At each temporal point \( t_j \) on the side \( z_j \) (the ensemble entry) of the space–time grid, we successively input the quantity
Fig. 2. (a), (c) Dynamical evolutions of the photon pulses and (b), (d) the induced spin wave in the unit μm^{3/2}. The 100 μm long ensembles contain ^87^Rb atoms with the relevant levels |g⟩ = 5S_{1/2}, |e⟩ = 5P_{1/2}, and |f⟩ = 10S_{1/2}, which give C_0 = -2.3 \times 10^9 \text{GHz}\cdot\text{μm}^6, γ = 2π \times 6.1 \text{ MHz}, and γ – γ' = 1.8 kHz. The control field Ω_c(t) = 2π \cdot 1.5 \tanh(80 - t)/τ, MHz with τ_c = 1 μs. With the index l = 1, 2 neglected for symmetric pulse propagation, the plots are obtained from the numerics, with iteration step size 0.002 μs along the time axis and 0.02 μm in the longitudinal direction z, assuming a negligible change in the transverse profile of the pulses. We set Ω_c = Ω_c' = γ in (a), (b) and Δ_0 = Δ_0' = 0 in (c), (d). The profile of the input photon pulses on the boundary is Ω_P(t) = 0.01e^{-|t-10^7|/5} MHz with τ_0 = 5.0 μs. The parameters for the ensembles used are N = 2 \times 10^{13} \text{ cm}^{-3}, d = 6 μm, and d = 2 μm. The insets show the corresponding imaginary (solid) and real (dashed) parts of the normalized central frequency susceptibility to the integers, using the two-photon detuning as the horizontal axis.

Ω_{0i}(x_i, t_k) = gE_i(x_i, t_k) from a given pulse’s temporal profile, which lead to the distributions of Ω_{pig}(x_i, t_k) (i ≥ 1) and P_{ig}(x_i, t_k), S_i(x_i, t_k) (i ≥ 1) over the further points x_i through the iteration procedure. This is to solve differential Eqs. (9)–(11) and (15)–(17) as boundary value problems, and can clearly simulate the pulses’ entry into the medium. In contrast, it is not straightforward to deal with the evolving nonlocal functions in Eq. (5) as a boundary value problem, and their numerical calculations as an initial value problem must consider the pulse distributions outside the medium, which overwhelm the size of the EIT medium itself and thus make the simulation less efficient.

3. SIMULATION RESULTS

For simplicity, only symmetric propagation of two identical pulses will be discussed in what follows. The generalization to the situations of pulses with different group velocities and different shapes is straightforward by using the different boundary conditions for each ensemble. The factors mentioned in the beginning paragraph of Section 2.C can also be included by the extensions of the numerical algorithm so as to apply the approach to more realistic situations.

A. Counterpropagating Photon Pulses

The effects of a gradually increasing interaction between pulses can be best seen from two counterpropagating photon pulses. Figure 2 illustrates the dynamical evolutions of two photons passing by each other under their attractive interaction. As shown in Fig. 2(a), the red-detuned (Δ_0 > 0) photon pulses accelerate due to the mutual interaction. However, in Fig. 2(c), the pulses will be almost totally absorbed on the way to approaching each other, if their detuning changes the sign. Such difference can be explained with the susceptibility χ(ω_0), defined as P(ω_0) = √N|χ(ω_0)L_{10}(ω_0)|, for the central frequency component under a constant interaction potential V; see the insets of Fig. 2. In the former situation, the negative potential “pulls” the central frequency component initially under the EIT condition away from the absorption peak to the regime of an effective two-level system, but it “pushes” the corresponding frequency component in the latter toward the peak of absorption in the anomalous dispersion regime. When they get closer, the red-detuned photon pulses entering the two-level regime will quickly escape from the medium due to much increased group velocity. However, the same medium becomes opaque to the blue-detuned (Δ_0 < 0) pulses because of the significantly enhanced absorption. It is evidenced by these different scenarios that, when two photon pulses approach each other in a Rydberg EIT medium, there can exist richer phenomena than the well-known Rydberg blockade that leads to a medium of an effective two-level system.

The accuracy of the numerical simulations manifests with two features in the illustrated spin-wave evolution. In Fig. 2(b), which illustrates Rydberg blockade scenario, the spin-wave reappears around the exit of the medium, upon the restoration of the three-level system after the pulses separate. We particularly adopt a gradually switching-off control field Ω_c(t), which can stop the pulses in the medium. One sees a slight tendency of storing the spin wave in Fig. 2(d) (a series of parallel platforms of remnant spin waves after the control field is turned off), indicating that the medium is still a three-level one under interaction between the negatively detuned pulses.

A potential application of the scenarios is to implement a photon switch or photon transistor [33–37] with a slightly modified scheme of first storing one pulse in the medium. The stored pulse can easily block the blue-detuned ones coming into the medium and let the red-detuned ones go through. Such processes do not rely on the specific forms of interaction. For example, if the stored spin wave could be focused on only one point x_0, the corresponding point–point potential Δ(x – x_0) can still give rise to the effects, since the approaching photon pulses nonetheless experience a gradually increasing interaction to modify their absorption and dispersion in the same ways. These effects can even be qualitatively captured by replacing the dynamical Eq. (11) with a corresponding Gross–Pitaevskii equation about the mean fields (see Supplement 1). The interaction potential in the Gross–Pitaevskii equation considerably differs from the exact potential in Eq. (13) and the approximate potential in Eq. (14) by quantity, but also has a time-dependent pattern similar to that shown in Fig. 1(b). Such flexibility with interaction makes the observation of the predicted phenomena more feasible.

The EIT width for the photon pulses used narrows down with their increased detuning Δ_0, impacting on their evolution under mutual interaction. The scenarios in Fig. 2 happen when the interaction magnitude reaches the order of 10^{6} γ, and will take place under even lower interaction potentials, as shown in Fig. 3 about a higher detuning. To a pair of highly detuned pulses getting closer from where the mutual interaction is negligible, a perturbative interaction can easily alter their absorption and dispersion so that they are more likely to speed up or to be heavily
damped, as in Fig. 2. To fit into the narrow EIT width, the pulses with a high \( \Delta_p \) should have sufficiently long duration \( \tau_p \), and such widely spreading photon pulses in the medium induce much lower interaction potentials than the corresponding point–point \( \text{vdW} \) potentials obtained by shrinking them to single points. The evolution of pulses with larger sizes will also become more complicated due to the inhomogeneity of interaction (see Section 3.D). All of these factors imply that forming the bound states of photons as recently proposed by employing the regime of high detuning \([41,42]\) is experimentally difficult.

**B. Further Discussion on Detuning Sign**

The different dynamical evolutions of the negatively and positively detuned pulses shown in Fig. 2 is one of our major predictions. A more interesting feature is that such difference can exist only under a gradually increasing rather than a suddenly increasing interaction potential, i.e., a potential like the solid curve instead of the dashed one in Fig. 1(b). To see this fact more clearly, we simplify the dynamical processes in Fig. 2 with a model of pulses propagating under constant external potential. Then the equation about the spin waves \([\text{Eq. } (11)]\) will reduce to the exact one,

\[
\dot{\mathbf{x}} \cdot S(x, t) = -i \mathbf{v}^0 \cdot S(x, t) + \mathbf{\delta}_0 P(x, t),
\]

with a constant \( \mathbf{v}^0 \).

For a red-detuned pulse, the successively increased external potential magnitude turns its slow-light propagation under the EIT condition \([\text{Fig. } 3(a1)]\) into much faster propagation in the two-level regime \([\text{Fig. } 3(d1)]\). As shown in Fig. 3(e1), the gradient of the dispersion curves at the detuning point \( \Delta_p = 10 \gamma \) lowers with the increased potential magnitude, indicating that the corresponding group velocity will go up to that of the effective two-level system. If the same series of potential is applied to a blue-detuned pulse, its evolution can significantly differ. Figure 3(b2) shows a complete absorption in contrast to the accelerated propagation in Fig. 3(b1). The dispersion curve for the central frequency of the pulse in Fig. 3(b2) has a negative gradient at \( \Delta_p = -10 \gamma \) \([\text{see Fig. } 3(e2)]\), implying a heavy absorption accompanying the negative group velocity. Given the detuning \( |\Delta_p| = 10 \gamma \) and the chosen EIT width in Fig. 3, rather low potential (of the order of \( 10^{-2} \gamma \)) is sufficient to see the difference.

The existing huge difference between the blue- and red-detuned pulses cannot be predicted for two counterpropagation pulses if their mutual interaction throughout the time is approximated by a blockade potential, shown by the dashed line in Fig. 1(b). Interpreted with constant interaction potential as in Fig. 3, an abrupt increase of the potential magnitude will directly turn the evolution in Figs. 3(a1) and 3(a2) into the almost identical ones in Figs. 3(d1) and 3(d2). Under the highest interaction potential, the dispersion curves of both blue and red detuning, which are illustrated in Figs. 3(e2) and 3(e1), respectively, stick together with that of the corresponding two-level system, simply having a Rydberg blockade. Therefore, multiphoton CW beams, which create high and stable interaction within themselves, exhibit only blockade behavior that can be analyzed in a steady-state framework \([7]\). In contrast, we consider the completely dynamical processes of single-photon pulses, and the phenomena illustrated are very different.

**C. Evolution of Polarization Fields**

Like the spin waves, the polarization fields \( \mathbf{P}(x, t) = \sqrt{N} \mathbf{P}_0(x, t) \) are also evolving in the processes with which we are concerned. We present two numerical simulations of their evolution in Fig. 4. The first one, in Fig. 4(a), shows the ordinary EIT of one pulse going through the medium. The "gap" between two symmetric parts is the EIT window in which the dissipation from the induced polarization field is small. The polarization field inside the "gap" nonetheless changes with time (generally \( \partial \mathbf{P}(x, t)/\partial t \neq 0 \)) as seen from the cross section views in Fig. 4(c). In the other example, which involves two counterpropagating pulses, as shown in Figs. 4(b) and 4(d), a peak value of the polarization fields emerge where the pulses are close to each other, leading to heavier damping due to their interaction. Here we consider a resonant pulse with \( \Delta_p = 0 \). For pulses with nonzero \( \Delta_p \neq 0 \), their polarization field profiles \( \mathbf{P}(x, t) \) become asymmetric and vary

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**Fig. 3.** (a1)–(d1) Evolutions of the positively detuned photon pulses with \( \Delta_p = 10 \gamma \) under various constant interaction potential. (a2)–(d2) Evolutions of the negatively detuned pulses with \( \Delta_p = -10 \gamma \). The profile of the pulses at the entry is \( \Omega_k(t) = 0.01 e^{-|t|^{0.5}} \) MHz with \( \tau_p = 20 \mu s \). The Rabi frequency of the constant control field is \( 1.5 \times 2\pi \) MHz. (e1) and (e2) show the dispersion curves (the normalized real part of the susceptibility) for the central frequency component of the red- and blue-detuned pulses, respectively. The dashed line is the dispersion curve for the corresponding two-level system.
with time more drastically, and it is also true for pulses with rather narrow bandwidths, like those used in Fig. 3.

The above results indicate that the approximation of adiabatically eliminating the degrees of freedom of the decaying level $|e\rangle$, i.e., setting $\partial \hat{P}(x, t) / \partial t = 0$ in Eq. (2), is not so suitable to pulses, although it works well for slow processes (compared with the time scale $1/\gamma$) in atomic systems driven by CW light. The polarization field induced by a varying pulse is certainly time dependent, even with a considerable decay rate $\gamma$. A direct simulation using adiabatic elimination also shows the disappearance of the distinct evolution for pulses with opposite-sign detunings (see Supplement 1 for an example). The complete dynamics involving the whole set of quantum fields $\hat{E}(x, t)$, $\hat{P}(x, t)$, and $\hat{S}(x, t)$ is therefore necessary to photon pulses.

D. Inhomogeneity of Pulse Interaction

Intuitively, atoms residing at different locations a pulse covers "feel" different long-range interaction from the other pulse due to their relative positions. The corresponding interaction potential $V^{\text{eff}}(x, t)$ in Eqs. (13) or (14) is equivalent to a two-photon detuning that violates the EIT condition $\Delta_p + \Delta_e = 0$ at the location $x$ and the moment $t$. Its nonuniformity leads to a space-time-dependent dissipation of the pulses. In the processes with which we are concerned, narrowband pulses are considered for reducing losses in an EIT medium and achieving good quantitative simulations with our approach as described in Section 2.B. Their large sizes in the medium make such inhomogeneity of interaction more obvious.

Here we illustrate the effects of inhomogeneous interaction with two examples in Fig. 5. The first group of plots in Figs. 5(a) and 5(b) is about two counterpropagating pulses. Since the pulses have considerable longitudinal extensions, the interaction between their front sides is much stronger than that between their back sides. We also apply a variable control field to stop the pulses. A large portion of their fronts can be absorbed under interaction, resulting in asymmetric shapes after they are stopped. The low decay rate of the Rydberg levels maintains the deformed spin-wave profiles inside the medium after the control field is off. The other example, in Figs. 5(c) and 5(d), illustrates the dynamical evolution of two copropagating pulses until they are stopped together. A rather narrow pulse bandwidth corresponding to a large pulse size is used, so that the pulse dissipation is almost due to their interaction. Because of a longer interaction time for the copropagating pulses, only a small portion of the initially induced spin wave will remain in the end.

4. CONCLUSION

While promising possible applications in quantum information processing technology, interacting single-photon pulses provide a clean channel to study the many-body physics of light in a Rydberg medium, since they are without the self-interaction that makes their evolution more complicated. However, understanding of the completely dynamical processes is still rather challenging, as there exists simultaneous evolution of different types of quantum fields, which determines the nonlocal interaction between and the dissipations of the pulses. To deal with the fully dynamical problem, we provide an approach based on the local functions defined in Eq. (7), which can well describe the pulse profiles but dispenses with their correlations. Highly efficient numerical simulations of the complicated dynamical processes can be realized with this method, to capture realistic photon losses in a Rydberg EIT medium. It is also possible to extend the approach to situations of more than two photons, as recently discussed in [49,50].

An important application of the approach is to the processes of counterpropagating photon pulses. Our simulations give the complete dynamical pictures of how the pulses propagate under their mutual interaction in a Rydberg EIT medium, showing that,