LETTER

Surface solitons of four-wave mixing in an electromagnetically induced lattice

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Abstract

By creating lattice states with two-dimensional spatial periodic atomic coherence, we report an experimental demonstration of generating two-dimensional surface solitons of a four-wave mixing signal in an electromagnetically induced lattice composed of two electromagnetically induced gratings with different orientations in an atomic medium, each of which can support a one-dimensional surface soliton. The surface solitons can be well controlled by different experimental parameters, such as probe frequency, pump power, and beam incident angles, and can be affected by coherent induced defect states.

(Some figures may appear in colour only in the online journal)

When multiple laser beams propagate in a Kerr-type nonlinear medium, optical solitons or modulation instabilities can occur [1–4]. As the localized modes propagate at the interface between two media with different properties, surface waves could display certain intriguing features in various physical systems from the honeycomb lattice of carbon atoms [5] and periodically layered media [6] to the plasmon-polariton waves [7]. In recent years, different kinds of spatial solitons at the interfaces of different media have been studied both theoretically and experimentally [8–10]. One interesting and important type is the surface soliton formed at the interface of a periodic structure and a bulk medium [8, 9, 11–13], in which the trapping mechanism for optical surface waves is that the propagation eigenvalues fall within the photonic band-gap (PBG) of the system. For example, one-dimensional (1D) in-phase surface solitons have been demonstrated in the AlGaAs waveguide array [9] and two-dimensional (2D) discrete surface solitons have been experimentally observed [14, 15]. In general, there are many new interesting features in optical surface waves with different kinds of nonlineairities, such as nonlocal surface solitons [10] and surface light bullets [16].

In this letter, we demonstrate, for the first time, the formations of 1D and 2D surface solitons in the electromagnetically induced gratings and electromagnetically induced lattice (EIGs and EIL) in an atomic medium. Such generated surface lattice solitons show the advantages of reconfigurability and easy tunability with multiple parameters, as compared to the ones created in the solid-state systems [9, 12], due to the controllability of EIGs with induced atomic coherence in the medium [3]. We also investigate soliton dynamics under the competition between the generation of surface solitons due to the nonlinear localization and the Bragg reflection due to the created PBG.

The relevant energy levels for Na atoms (in a heat pipe) are shown in figure 1(a). The pulse laser beams
are aligned spatially as shown in figure 1(b). The levels 0, |3S1/2>, |3P3/2>, and |2>(|3D3/2, 5/2) form a ladder-type system. Two beam geometric configurations have been used in the investigations with the first one shown in figure 1(b). The pump beams E1 and E'1 with wavevectors k1 and k'1 (with nearly the same power P1, frequency ω1, connecting the transition |0> to |1> with the atomic resonant frequency Ω1) propagate in the opposite direction to the weak probe beam E3 (k3, P3, ω3) with small angles (θ1 = 0.3°) between them. These three beams are from the same near-transform-limited dye laser (10 Hz repetition rate, 5 ns pulse width, and 0.04 cm−1 linewidth). Two other pump beams E2 and E'2 (k2 and k'2, P2, ω2), connecting the transition |1> to |2> with resonant frequency Ω2) from another dye laser also propagate along the opposite direction of E3 with small angles (θ2 = 0.3°) between them. We define the frequency detuning as Δi = Ωi − ωi (i = 1, 2). A one-photon resonant degenerate four-wave mixing (FWM) process (figure 1(c)) can occur between the energy levels |0> and |1>, satisfying the phase-matching condition of kF1 = k3 + k1 − k'1 with the signal E_F1 propagating nearly opposite to the beam E1'.

Also, a two-photon nondegenerate FWM signal E_F2 (with kF2 = k3 + k2 − k'2) can be generated to propagate nearly opposite to E2.

The pumping beams E1 and E'1 (E2 and E'2) interfere with each other, as shown in figure 1(b), to induce a horizontally (vertically) aligned grating EIG1 (EIG2) in the X−Z (Y−Z) plane with a period Δ1 = πc/ω1 sin(θ1/2) (Δ2 = πc/ω2 sin(θ2/2) [3]. In figure 1(c), the incident beam E3 (launched obliquely into the gratings with angle α) contains two components, propagating in parallel and perpendicular to the interface between the induced EIG and the uniform medium, respectively. The parallel component forms a surface wave and is characterized by a propagation constant β, while the perpendicular component is Bragg reflected and characterized by a Bloch wavevector k [17, 18]. The synthesis of the surface wave and the Bragg reflection forms FWM signals, i.e., E_F1 (TM-polarized with respect to EIG1) and E_F2 (TE-polarized with respect to EIG2). The surface-wave component propagates along the z' direction in figure 1(c), while E_F1 propagates along the z axis. The transformation relations of components between these two frames are ξ' = ξ cos(θ1/2) − z sin(θ1/2) and ξ = z sin(θ1/2) + z cos(θ1/2), where i = 1 and ξ = x for EIG1 and i = 2 and ξ = y for EIG2, respectively. Since θ1 is very small, the surface-wave component is the dominant one in the FWM signals. To form a surface soliton, the powers of E_F1 and E_F2 must exceed a threshold to overcome the discrete diffraction and guarantee the emergence of surface localization [9, 14], below which both the linear and nonlinear surface modes cannot be supported.

In this first configuration, the two EIGs are in different planes, so the interaction region between them is small and they can be considered to be isolated. The experimental results presented in figures 2 and 3 are obtained under such conditions. The experimental configuration can also be changed to the second one, in which the two EIGs have considerable overlapping area to construct a 2D EIL with 2D periodic refractive index. Specifically, the strong pump beams can significantly affect the refractive index via the
Figure 2. The images of $E_{F1}$ (a-i, b-i) and $E_{F2}$ (a-ii, b-ii), when (a) $\Delta_1$ is scanned and (b) when the pump powers $P_1$ (in b-i) of the pumping fields $E_1$ and $E_1'$ and $P_2$ (in b-ii) of $E_2$ and $E_2'$ are changed respectively. (c-i) Theoretical calculations of the transverse section profile of $E_{F2}$ when $\theta_2$ is changed from small to large discretely. (c-ii) Experimental results corresponding to (c-i). (c-iii) The experimental images of $E_{F2}$, in which the first, third, and fifth correspond to (c-ii). $\Delta_2$ is set to be 0. (c-iv) The stable propagation of the $E_{F2}$ soliton.

Figure 3. (a) Experimental formation and evolution of the $E_{F2}$ surface solitons versus $\alpha$. (b) The experimental probe transmission versus $\alpha$. (c) The theoretical probe transmission spectrum with increased $\alpha$. (d) The experimental $E_{F2}$ images with $\Delta_1$ scanned within the defect-containing PBG. (e) The EIG2 transmission curve when $E_{1}'$ is transferred to induce a defect in EIG2.

strong-coupling effect [19]. At the lattice points where the antinodes of the standing wave created by fields $E_1$ and $E_1'$ come across that of the standing wave created by $E_2$ and $E_2'$, level |1⟩ will be split into three levels |+⟩, |−⟩, and |−⟩ under the first and secondary splittings induced by $E_1$ and $E_1'$ and $E_2$ and $E_2'$, respectively, as shown in figure 1(d1); so the refractive index is strongly modulated by these two field pairs [3]. In contrast, at the lattice points where the nodes of the two standing waves overlap, no strong field exists and level |1⟩ is not affected, so the index will not be modified at those points. When the antinodes of one standing wave encounter the nodes of the other standing wave, |1⟩ will be split into two levels |+⟩ and |−⟩ and the index will be modified only by a single field pair. The spatial periodic distribution of these three cases can lead to a lattice state as shown in figure 1(d2) and further into a distinct 2D periodic index modulation pattern as illustrated in figure 1(d1), in which the two periodic directions are defined as $x''$ and $y''$, respectively. If only a single pump field $E_{1}'$ ($E_{1}'$), instead of the standing wave $E_1$ and $E_1'$ ($E_2$ and $E_2'$), meets EIG2 (EIG1), a defect state will be generated in the PBG of the grating.

By selecting an appropriate Bloch wavevector falling within the 2D PBG of EIL, $E_3$ can excite the surface mode and receive an attenuated Bragg reflected component. When the laser beam propagates at the interface between the EIL and the homogeneous (non-modulated) medium,
the FWM signals can be expressed as $E^3(x', y', z) = E_0(x', y') \exp[i(k_x x' + k_y y' + \beta z)]$ for $x' > 1, y' > 1$ and $E^3(x', y', z) = E_0 \exp(i \beta z + q_{ax,y} x' + q_{ay,y} y')$ for the other regions, where $x', y' = 1$ define the two boundaries of the EIL with the homogeneous medium, and $k_x (k_y)$ are the Bloch wavevectors in the $x'$ ($y'$) directions, respectively. In the forbidden band, there exists $k_x,y = m_{ax,y} \pi/\Lambda_{1,2} + i \kappa_{ax,y}$ with $m_{ax,y}$ being the Brillouin zone index, in which the nonzero imaginary parts $\kappa_{ax,y}$ are the attenuation coefficients along the $x'$, $y'$ axes in the EIL for the Bragg reflected component. Other parameters include amplitude $E_0 = E_0(1, 1)$ and $q_{ax,y} = \sqrt{\beta^2 - (\omega_1/c)^2 n_{ax,y}^2}$, being the attenuation coefficients along the transverse axes $x', y'$ in the homogeneous medium, in which $n_{ax,y}$ are the refractive indices ($n_{ax,y} = n_1 + n_2 l_{1,2}$ with $n_1 \approx 1$). The propagation constant $\beta$ can be obtained by solving the dispersion expression $\Delta_1 = \Omega_1 \pm c \sqrt{n_x^2 - (k_x - k_2)^2 - k_2^2/(1 + \chi_{(0,0)} - \chi_{(0,1)}) + \beta^2}$ in the forbidden band, derived from the plane-wave expansion [13], in which $\chi_{m,n} (m, n = 0, 1)$ is the $(m, nh)$ Fourier coefficient of the spatial periodic susceptibility under truncation.

The propagation equations of the generated signals $E_{F1}$ and $E_{F2}$ under either 1D or 2D modulation can be expressed by

$$
\frac{\partial E_{F1}}{\partial z} = \frac{i \chi^2}{2 \kappa_{F1}} E_{F1} \left[ n_x^2 \left| E_{F1} \right|^2 + 2 \left| n_x^2 \left| E_{1} \right|^2 \left| \xi \mu(x') \right| (1 - \xi) \mu(x') \right] + n_x^2 \left| E_{1} \right|^2 \left| \xi \mu(y') \right| E_{F1},
$$

$$
\frac{\partial E_{F2}}{\partial z} = \frac{i \chi^2}{2 \kappa_{F2}} E_{F2} \left[ n_x^2 \left| E_{F2} \right|^2 + 2 \left| n_x^2 \left| E_{1} \right|^2 \left| \xi \mu(x') \right| + n_x^2 \left| E_{2} \right|^2 \left| \xi \mu(y') \right| \right] + (1 - \xi) \mu(y') \right] E_{F2}.
$$

$\chi^2$ is the Laplace operator with respect to the $x$ and $y$ axes and describes the diffraction. The $x'$ and $y'$ axes denote the orientations of the EIG1 and EIG2, respectively. In these equations, $\mu(\xi) = \cos^2(\pi \xi/\Lambda_1)$ for $\xi \geq 1$ and $\mu(\xi) = 1$ for $\xi < 1$ ($\xi = x', y'$). The periodically modulated refractive index pattern inside the EIGs ($\xi \geq 1$) and the uniform index outside ($\xi < 1$) the EIGs ($\xi = x', y'$) represent the periodically modulated refractive index pattern inside the EIGs ($\xi \geq 1$) and the uniform index outside ($\xi < 1$) the EIGs ($\xi = x', y'$) for EIG1 and $\xi = x', y'$ for EIG2), respectively. Surface solitons can be generated at the interface of these two regions with a high refractive index contrast. The factor $\xi = 0$ or 1 is introduced to describe the assembling of EIG1 and EIG2 into EIL, i.e. $\xi = 1$ for a sufficiently large overlapping area, and $\xi = 0$ when the overlapping area is small, as in figure 1(b). With $\xi = 1$ both FWM signals suffer from the modulations of the lattice with the indices, varying periodically in two directions according to expressions for $\mu(x')$ and $\mu(y')$. In equation (1), $n_x$ is the linear refractive index; $n_x^{\Lambda_1}$ and $n_x^{\Lambda_2}$ are self-Kerr and cross-Kerr coefficients of $E_{F1}$ and $E_{F2}$, respectively. These coefficients can all be calculated by solving the density-matrix equations for the atomic system [19].

Neglecting diffraction and self-phase modulation and linearizing equation (1) with respect to $x$ and $y$, we can obtain a set of solutions with phases for $E_{F1}$ and $E_{F2}$ as $\phi_{F1} = 2k_{F1} [n_x^{\Lambda_1} \left| E_{1} \right|^2 \xi \mu(x') + (1 - \xi) \mu(x')] + n_x^{\Lambda_1} \left| E_{2} \right|^2 \xi \mu(y') \xi/n_1$, and $\phi_{F2} = 2k_{F2} [n_x^{\Lambda_2} \left| E_{1} \right|^2 \xi \mu(x') + n_x^{\Lambda_2} \left| E_{2} \right|^2 \xi \mu(y') + (1 - \xi) \mu(y') \xi/n_1$, both of which are 2D (1D) periodic modulated.

The geometric configuration in figure 1(b) (with $\xi = 0$) is employed in figures 2 and 3. First, by scanning $\Delta_1$ (figure 2(a)), we can effectively control the spatial contrast of the EIG via the resonances of the split levels which can significantly affect the sign and value of the Kerr nonlinear index $n_2$ [19]. In figure 2(a), EIG1 (EIG2) is responsible for the periodic horizontal (vertical) stripes of the $E_{F1}$ ($E_{F2}$) beam. With $\Delta_1$ scanned towards $\Delta_1 = 0$ from the negative region, $n_2$ will increase and reach its maximum at $\Delta_1 \approx -7$ GHz corresponding to the resonance of one split level (figure 2(a)). Obviously, $E_{F1}$ and $E_{F2}$ beams get the clearest periodic stripes near the resonant point, at which the larger $n_2$ leads to a higher contrast for the EIGs (further bringing larger phase distortion and modulation depth). At other $\Delta_1$ values, $n_2$ is not sufficiently large and the modulation depth becomes lower, so the periodic pattern suffers from diffusion and tunneling between adjacent sites due to the diffraction in the propagation [8]. The investigation of $E_{F1}$ by scanning $\Delta_1$ in its positive region also reveals a similar result [2]. Similarly, by increasing the power $P_1 (P_2)$ of the pump fields $E_1$ and $E_2$ ($E_1$ and $E_2$) for $E_{F1}$ and $E_{F2}$ to improve the contrast of EIG1 (EIG2), the two signals evolve from blurry single spots into clear periodically modulated structures, as shown in figure 2(b). In figure 2(c-i-iii), when $\theta_2$ is increased, the $E_{F2}$ pattern gradually evolves from a nearly single spot to an asymmetrical and periodically oscillating structure. This change is due to the decrease in the period of EIG2 with an increased $\theta_2$. After all these parameters are set at their optimal values, a stable soliton is obtained at an equivalent propagating length of 57 cm [3], where the temperature is raised from 290 to 310°C, provided that the pump power is sufficiently large as shown in figure 2(c-iv).

Then, the competition between the Bragg reflected and surface-wave components is investigated by changing $\alpha$. Derived from the plane-wave expansion method [13], the transmission spectrum of EIG2 in figure 3(c) shows that a PBG in the left side of the two-photon resonance ($\Delta_1 + \Delta_2 = 0$) emerges and broadens with an increased $\alpha$. When $\alpha$ is small, the PBG in EIG2 is very narrow or even destroyed. In this case, the spots in figure 3(b) show a strong probe transmission signal (while the $E_{F2}$ beam is weak in figure 3(a)) with a clear periodic pattern due to the strong surface-wave component. Then, with the increase of $\alpha$, the transmission of the probe beam decreases, and $E_{F2}$ shows an increased intensity and fewer stripes due to the increase of the Bragg reflected component, which can be verified by the transmission spectrum in figure 3(c). When $\alpha$ is further increased, the phase matching of $E_{F2}$ will be lost, and therefore the intensity is reduced, as shown in figure 3(a). Also, a defect spike in the theoretical EIG2 PBG
(see figure 3(c)) and the corresponding sharp decrease of $E_{F2}$ due to sharply decreasing reflection can be observed in figure 3(d).

After setting the above parameters at their optimal values, the pump beams are realigned to make a sufficiently large intersection area for the two EIGs (with $\zeta = 1$), which is essential for the formation of the 2D lattice state (i.e., EIL), as illustrated in figure 1(d) (a) and (d), and 2D surface lattice solitons. In this reconstruction, EIG1 (EIG2) is anticlockwise (clockwise) rotated by an angle $\gamma$ (maximum at 45°) with respect to the Z axis, and EIG2 is slightly clockwise rotated with respect to the X axis. The images of $E_{F2}$ are recorded at nine discrete positions when EIG1 is rotated, as shown in figures 4(a-i) and (a-ii). The beam $E_{F2}$ in figure 4(a-i) only shows modulation in the $y'$-direction due to the small intersection area with EIG2 not rotated, though the rotation of EIG1 has increased the overlapping area to a certain extent. In contrast, in figure 4(a-ii), $E_{F2}$ is modulated in two directions, which becomes increasingly significant with an increased $\gamma$ value, because the intersection area has been gradually enlarged and the area with the lattice state extends. The theoretical $\gamma$-dependent EIL transmission spectrum shown as figure 4(a-iii) is obtained as $T = (T_1 + T_2)/2\sqrt{2} \cos \gamma$, with $T_{1,2}$ being the transmissions of EIG1 and EIG2. It shows that the reflection increases and PBG broadens in assembling the EIL from two independent EIGs with increasing $\gamma$, leading to the intensity enhancement for $E_{F2}$ shown in figure 4(a-ii).

Next, the evolutions of $E_{F2}$ solitons are investigated when the powers $P_1$ and $P_2$ are changed, respectively. The corresponding 2D PBG structure is shown in figure 4(b-iii). In figure 4(b-i), EIG1 is rotated to have the largest $\gamma$, while EIG2 is not rotated. In such a case, the $E_{F2}$ signal remains to be periodically modulated in the $y'$-direction. Here, increasing $P_1$ only shortens the modulation period. In figure 4(b-ii), EIG2 is rotated, so $E_{F2}$ becomes 2D-modulated. When $P_2$ increases, the periodic stripes in one direction become denser. Such tunability can be well explained by the expression $\Delta \approx \frac{n_1 \Delta k_{F2}^2}{k_F^2} \gamma^2$ derived from the phase distortion above, in which larger $P_2$ will lead to a smaller modulation period.

In conclusion, we have experimentally demonstrated the formation and dynamics of 2D surface solitons in FWM signals in the atomic medium. We have shown that the surface lattice solitons propagate along the interface between the uniform medium and the laser induced EIL, which is composed of two EIGs. Such pattern formation and propagation control can have important applications in image storage, processing, and communication.

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