I. INTRODUCTION

When an ensemble of two-level atoms interacts with a cavity field, the cavity transmission spectrum can show a double-peak structure when the coupling strength, measured as $\sqrt{N}g$, between the atoms and the cavity field is larger than the atomic decay rate $\gamma$ and cavity decay rate $\kappa$ [1] (Here, $g$ is the single atom-cavity coupling strength and $N$ the number of atoms inside the cavity mode volume.) This double-peak structure has been called "vacuum Rabi splitting" or "normal-mode splitting" in the literature. The frequency difference between the two peaks is $2\sqrt{N}g$ when both the atoms and the cavity are on resonance with the laser field. In order to overcome the Doppler effect, this normal-mode splitting phenomenon has always been studied either in an atomic beam [2–4] or in a cold atomic cloud [5]. Recently, we have shown that such a double-peak transmission spectrum can even be observed in a Doppler-broadened atomic vapor, with a relatively stronger intracavity field intensity, to saturate the atomic absorption within the Doppler profile [6].

When three-level atoms are placed inside the optical cavity (i.e., an additional coupling laser beam is used to interact with another transition, sharing a common level with the first one), three-peak transmission spectra have been observed, both in cold atoms [7] and in a hot atomic cell [8], with the narrow central transmission peak being due to electromagnetically induced transparency (EIT). The narrowness of this central peak results from the use of a two-photon Doppler-free configuration in the optical ring cavity [9].

It was shown in [8] that, at relatively low atomic densities and moderate intracavity field intensities, the three-peak transmission structure is stable, with the side-peaks moving outwards as the temperature (atomic density) or coupling laser beam power increases. At a moderate intracavity field intensity (cavity input power of 0.8 mW, corresponding to a Rabi frequency at the beam waist of $2\pi \times 11$ MHz, which is about 0.65 times the cavity linewidth.), the atomic absorption is saturated, but the dispersion is still in a “quasilinear” region, a phenomenon pointed out in [8] and analyzed theoretically (for a two-level system) in [6].

In this work, we show that if the atomic cell temperature is set higher (above 73 °C, corresponding to the atomic density of $7 \times 10^{11}$ cm$^{-3}$ or a single-pass absorption (optical depth) of 29.) with a higher intracavity field intensity (cavity input power larger than 1.5 mW), each normal-mode side peak can split into two pairs of peaks. Experimental results are presented along with a theoretical model that indicates that this phenomenon is unique to the three-level system in the large Doppler-broadening regime.

II. EXPERIMENTAL SETUP AND OBSERVATIONS

The experimental setup is shown in Fig. 1. Two commercial high-power CW diode lasers are used as the cavity input (probe) and the coupling laser beams, with linewidths of about 1 MHz. The input mirror M1 and output mirror M2 of the ring cavity have 3% and 1.4% transmissivities, respectively, and M3 is a high reflector mounted on a PZT for cavity frequency scanning and locking. The cavity length $L$ is about 37 cm. The rubidium vapor cell is $l=5$ cm long with Brewster windows, and is wrapped in μ-metal sheets for
magnetic field shielding. The coupling beam is injected into the cavity through the polarization beam splitter (PBS) and copropagates (not circulating inside the cavity) through the atomic vapor cell with the cavity field in order to eliminate the first-order Doppler effect using the two-photon Doppler-free configuration [9]. The beam waist radii are 100 and 600 μm for the probe and coupling beams, respectively. We use a third diode laser to lock the cavity frequency (not shown in Fig. 1). The empty cavity finesse is about 100. When the atomic cell is placed inside the optical cavity with laser frequency tuned far away from the atomic resonances, the cavity linewidth is increased to be about 17 MHz, corresponding to the cavity finesse of about 48. The Doppler-width of the atomic medium at the cell temperature of 67.7 °C is about 535 MHz.

We first set the experimental conditions to be the same as in [8] in order to observe the three-peak transmission spectrum with $T=67.7$ °C: the coupling beam power $P_c=7$ mW (corresponding to a Rabi frequency of about $\Omega=120$ MHz), cavity input power $P_P=0.8$ mW (at the cavity input), and the coupling laser frequency detuning $\Delta_c$ and cavity frequency detuning $\Delta_\phi$ (from the atomic resonant frequency) both equal to zero. When the temperature is increased to 73.5 °C, the side peaks get much smaller due to increased absorption. Larger cavity input power can compensate for and saturate such absorption to enhance the side peaks. Figure 2 shows the transmission spectra for different cavity input powers, for the given cell temperature. At the large cavity input power (first curve in Fig. 2 with $P_P=2.2$ mW), the transmission spectrum is the typical three-peak structure, as in [8]. However, as the cavity input power decreases, the side normal-mode peak on each side splits into two, with one moving outward, similarly as the regular normal-mode peak [8], and another (smaller one) moving inward toward the center, as shown in the second and third curves in the figure. This inward moving peak gets smaller as it moves near the central peak where the absorption is large (just outside the EIT window), and eventually disappears, as shown in the last curve of Fig. 2.

The frequencies of the split peaks are plotted as a function of the probe input power in Fig. 3, where the splitting frequency is measured from the frequency of the maximum transmission of the side-peaks to the frequency of the central peak. One can see that at both lower and higher cavity input powers, cavity transmission has the typical three-peak structure, but it shows a five-peak structure in a certain range of cavity input powers, for a given atomic density (temperature) and coupling laser power. The peak splittings are basically symmetric on both sides of the transmission spectrum.

Next, the cavity input power is fixed at $P_P=1.05$ mW and the coupling beam power at $P_c=7.4$ mW. At $T=73$ °C, the cavity transmission shows the typical three-peak structure. As the cell temperature is increased from $T=73$ °C, the cavity transmission spectrum first shows a three-peak structure, and then five peaks at $T=75.4$ °C, as shown in Fig. 4. Again the inward peaks become smaller and eventually disappear as they move closer to the middle due to larger absorption at high atomic densities (temperature). The outward peaks
again follow the typical normal-mode relation as described in [8].

Finally, we keep the cell temperature and the cavity input power fixed at \( T = 73 \, ^\circ\text{C} \) and \( P_p = 1.04 \, \text{mW} \), respectively, and change the coupling laser power. At \( P_c = 7 \, \text{mW} \), which is the value used in Figs. 3 and 4, only the typical three-peak structure appears in the cavity transmission spectrum. When the coupling beam power is increased from this point, only the three-peak structure shows, following the normal-mode relation as described in [8]. However, as \( P_c \) decreases to around 5 mW, again a five-peak structure shows up in the cavity transmission spectrum, as shown in Fig. 5. Such five-peak spectra only exist for a small range of the coupling beam powers. When the coupling beam power was set to zero (which is the two-level atom case) a five-peak cavity transmission spectrum could not be observed in the large parameter (for temperature and cavity input power) ranges available under our current experimental conditions.

### III. THEORETICAL DISCUSSION

Our observations can be qualitatively understood as follows. In the general, nonlinear regime, the intracavity intensity \( I \) is obtained as the solution of the self-consistency equation (see [3]),

\[
I = \frac{T I_{in}}{1 + r^2 e^{-\omega_0 L/2c} - 2 r e^{-\omega_0 L/2c} \cos[(\Delta + (\omega_0/2L)\chi')L/c]}
\]

where \( T \) is the transmissivity of the input mirror \( M1 \); \( I_{in} \) is the input intensity; \( r < 1 \) is a parameter describing the losses due to reflection and absorption at the various optical elements; and \( \chi = \chi' + i\chi'' \) is the medium’s complex susceptibility, which in general depends on \( I \) and \( \Delta \), the probe detuning from the cavity (and the atomic) resonance.

We recall that, as we have recently pointed out [6,8], for a Doppler-broadened medium the absorption saturates much more strongly than the dispersion as the intracavity field increases. Hence we can expect to gain some insight into the peak-splitting phenomenon starting from a consideration of the dispersive properties of the medium only, as given by \( \chi' \), the real part of the susceptibility.

Neglecting absorption in the medium altogether, Eq. (1) becomes

\[
I = \frac{T I_{in}}{1 + r^2 - 2 r \cos[(\Delta + (\omega_0/2L)\chi')L/c]}
\]

and one expects the transmission peaks to be given by the maxima of \( \cos[(\Delta + (\omega_0/2L)\chi')L/c] \) in Eq. (2), which is to say (since dispersion in our system is relatively weak, and the only relevant maximum of the cosine function is the one at zero), to correspond to values of \( \Delta \) for which the difference between \((\omega_0/2L)\chi'\) (considered as a function of \( \Delta \))
which it crosses the dashed straight line. One would expect the crossings to disappear for sufficiently small $\Delta$, due to saturation of the medium; hence, eventually, the peak splitting observed in Fig. 3.

Transmission is obtained simply by multiplying $f$ with the transmission peak on the right-hand side of Fig. 7. The plot in Eq. (3) includes Doppler-broadening and treats both the coupling and probe field to all orders in field strength. The plot is for a fixed, constant $I$; increasing $I$ generally causes $|\chi'|$ to decrease, due to saturation of the medium; hence, eventually, one would expect the crossings to disappear for sufficiently large $I$, in qualitative agreement with the disappearance of the peak splitting observed in Fig. 3.

To take into account the nonlinearity, note that Eq. (2) can be rewritten as

$$TI_{\text{in}} = I(1 + r^2 - 2r \cos[(\Delta + (\omega_0/2L)\chi')L/c]) = f(I, \Delta)$$

where the last equality defines a function $f$ of $I$ and $\Delta$. Now, the contour lines of $f$, if plotted with $\Delta$ on the horizontal axis, are in fact, except for a scaling factor, the cavity transmission curves for different values of the input intensity $I_{\text{in}}$ (see Fig. 7). This is because every point on the contour corresponds to the value of the intracavity intensity $I$ that satisfies Eq. (2) for the given $\Delta$ and fixed $I_{\text{in}}$, and the cavity transmission is obtained simply by multiplying $I$ by the transmissivity of the output mirror, $M2$, and dividing by $I_{\text{in}}$, which is constant across the contour.

As a function of $\Delta$, for constant $I$, the function $f$ has minima at the crossings identified in Fig. 6, that is, the points where $(\omega_0/2L)\chi' = -\Delta$, and if there are two crossings the value of $f$ is the same at both. Then, in a sufficiently small neighborhood of each minimum, there must be two contour lines (i.e., four lines altogether) that belong to the same contour, and that means that the contour must exhibit two transmission peaks (again, see Fig. 7). We conclude, then, that it is possible to predict when a split peak will occur by studying the crossings in Fig. 6.

Note that the nonlinearity of the problem may result in regions of dispersive bistability, such as those exhibited by the transmission peak on the right-hand side of Fig. 7. The vertical dashed line in the figure indicates the way the transmission might appear in an experiment, given the availability of a stable lower transmission branch (of course, the precise point at which the jump from one branch to the other will take place cannot really be predicted, given the unavoidable noise in an experiment). Note that the sharp left edge of the peak is, in fact, consistent with the experimental results seen in Fig. 2.

It remains to be seen how the inclusion of medium absorption changes this picture. By analogy with the above, we can define a function

$$g(I, \Delta) = I(1 + r^2 e^{-\omega_0\chi'/c} - 2r e^{-\omega_0\chi'/2c}) \times \cos[(\Delta + (\omega_0/2L)\chi')L/c]),$$

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As indicated above, in connection with Fig. 8, this explains the absorption rate, as given by \( (\omega_0/2L)\chi' \). For instance, we can see, with reference to that figure, that decreasing the atomic density naturally decreases \( \chi' \), which eventually results in a dispersion curve similar to the dotted green line, that no longer crosses the blue \(-\Delta\) line; note that this still has a (single) transmission maximum for nonzero \( \Delta \), corresponding to the point where the green and blue lines are closest. Increasing the intracavity field has an effect similar to decreasing \( N \); i.e., it causes \( \chi' \) to decrease, due to saturation of the medium.

With this background, we can use the simple picture provided by Fig. 6 to understand qualitatively the experimental results, and in particular the effect of varying the experimental parameters. For instance, we can see, with reference to that figure, that decreasing the atomic density naturally decreases \( \chi' \), which eventually results in a dispersion curve similar to the dotted green line, that no longer crosses the blue \(-\Delta\) line; note that this still has a (single) transmission maximum for nonzero \( \Delta \), corresponding to the point where the green and blue lines are closest. Increasing the intracavity field has an effect similar to decreasing \( N \); i.e., it causes \( \chi' \) to decrease, due to saturation of the medium.

The dash-dotted purple line in Fig. 6 shows the medium absorption rate, as given by \( (\omega_0/2L)\chi' \). It is clearly largest in the neighborhood of the dressed-state resonance, \( \Delta=\Omega/2 \) (the figure has been drawn for \( \Omega=90 \) MHz), where \( \chi' = 0 \). As indicated above, in connection with Fig. 8, this explains why the transmission peak corresponding to the smaller value of \( \Delta \) is always smaller, and why eventually it disappears if either the density becomes too large (Fig. 4) or the intracavity field becomes too small (Fig. 3): both of these have the effect of increasing the overall absorption as well as moving the small-\( \Delta \) peak closer to the high-absorption region. These observed features are related to the dramatic modification of the linear [9] and nonlinear [10] dispersions of the three-level atomic system, compared to the two-level one, due to the induced atomic coherence by the coupling laser beam in the EIT systems.

Figure 9(a) shows, for comparison, a similar plot for a two-level medium (no coupling field), such as the one considered in [6] (it is, in fact, essentially the same as Fig. 2 of [6] upside down). It shows that in this case it is only possible to have one (purely dispersive) side peak, since only one crossing for \( \Delta \neq 0 \) is possible, and if the curves do not cross at all then the point where they are closest is at \( \Delta = 0 \).

Figure 9(b) shows the situation for a three-level, homogeneously broadened medium (e.g., a dense gas of cold atoms), with a coupling field. The intracavity field has been assumed to be, as in the other plots, about 20 times the saturation intensity, which accounts for the large power-broadening of the dressed-state resonance at \( \Omega = \Delta/2 \). In this case, as in Fig. 6, there are two crossings, but one of them is always very close to the high-absorption point \( \Delta = \Omega/2 \) (again, the absorption is shown as the dash-dotted curve), which means that this peak should be unobservable in practice. We conclude that the phenomenon of the splitting of the side normal-mode peak is probably unique to the three-level, Doppler-broadened system.

Finally, returning to Fig. 6, we may consider the effect of changing the strength of the coupling field, \( \Omega \). This mainly shifts \( \chi' \) to the right or left, by moving the dressed-state resonance point \( \Delta = \Omega/2 \). Clearly, for sufficiently large \( \Omega \) this will cause the crossings, and hence the splitting of the side peak, to disappear, as we have observed in the experiment (Fig. 5). On the other hand, for sufficiently small \( \Omega \), the small-\( \Delta \) split peak will be brought too close to the high-absorption region, and it will become unobservable in the experiment. Hence, for this parameter also, as for the atomic density and probe field, the split peak can only be observable for a restricted range of values, as our experimental results show.

We want to point out that such splitting of the cavity + atoms normal modes only occurs for certain range of the intracavity intensity. At either too low or too high cavity input power, the splitting of the normal-mode peak would not show up for any temperature or coupling laser strength. This indicates that it is a genuine nonlinear effect, distinct from the “quasilinear” regime explored in [8]. This is also apparent from the shape of the split peaks in Fig. 2, which is consistent with the underlying bistability revealed by the calculations in Figs. 7 and 8 (compare also with the similar curves for the nonlinear two-level system in [11]). We have not been able to observe hysteresis directly because of difficulties keeping the system stable, but it is well known that nonclassical light effects, such as squeezing, can be observed in systems exhibiting optical bistability (see, e.g., [12]). It would be interesting to explore whether this is the case also in the present system.

### IV. CONCLUSIONS

In summary, we have experimentally demonstrated the splitting of the atom-cavity normal modes at certain values of intracavity field intensity, coupling beam power, and atomic density in a system of inhomogeneously-broadened three-level atoms inside an optical ring cavity. Detailed parametric dependence of such split-peak structures has been ex-
explored. The observed phenomena can be qualitatively explained by the interplay of the nonlinear dispersion and absorption in the system, by considering the Doppler broadening and cavity field nonlinearity. Such normal-mode peak splitting phenomenon seems to be a unique feature of the three-level, inhomogeneously-broadened atomic system. It would be interesting to explore the possibility of nonclassical light effects in the neighborhood of this instability. We note also that, because of the high density of the medium, the average number of photons per atom is relatively low, so we cannot in principle exclude the possibility of interesting quantum-field effects underlying our observations (see, e.g., [13]). In the experiment, we sometimes observe higher-order transverse modes, but at present we do not know whether they are due to imperfect alignment of the cavity or to the nonlinearity. We are currently working on a theoretical treatment taking into account the transverse profiles of the laser beams that we hope may help us in answering these interesting questions on the formations of transverse instabilities and transverse patterns with high density atoms inside an optical cavity.

We note that there has recently been a renewed interest in the quantized-field version of these types of systems (typically described by the Jaynes-Cummings model, or a suitable extension) in the context of so-called “circuit QED.” In particular, a recent paper has reported a nonlinear splitting of the “vacuum Rabi resonance” [14] that exhibits a superficial resemblance to our results. We do not believe, however, that the two results are directly related, in part because the splitting reported in [14] was only visible via heterodyne detection and not by direct photon counting. Nonetheless, as pointed out above, the search for possible quantum-field effects in our system remains an interesting possibility (see also [15]).

ACKNOWLEDGMENT

Funding support from the National Science Foundation is acknowledged.