CONTROLLED SPATIAL BEAM SPLITER USING FOUR-WAVE-MIXING IMAGES

Yanpeng Zhang,1,* Cuicui Zuo,1 Huabin Zheng,1 Changbiao Li,1 Zhiqiang Nie,1 Jianping Song,1 Hong Chang,2 and Min Xiao3,†

1 Key Laboratory for Physical Electronics and Devices of the Ministry of Education and Shaanxi Key Laboratory of Information Photonics Technique, Xi’an Jiaotong University, Xi’an 710049, China
2 National Time Service Center, Chinese Academy of Sciences, Lintong 710600, China
3 Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA

(Received 28 July 2009; published 25 November 2009)

We report our experimental observations of spatial shift and splitting of four-wave mixing (FWM) signal beams induced by additional dressing laser beams. These effects are caused by the enhanced cross-Kerr nonlinearity due to atomic coherence in a two-level atomic system. The spatial separation and number of the split FWM beams can both be controlled by the intensity of the dressing beam, the Kerr nonlinearity, and atomic density. Theoretical results agree quite well with the observations. Studies of such controlled beam splitting can be very useful in understanding spatial soliton formation and interactions, and in applications for spatial signal processing.

DOI: 10.1103/PhysRevA.80.055804 PACS number: 42.50.Gy, 42.65.Jx, 42.65.Tg

Spatially shifting and splitting one weak laser beam by another stronger beam in Kerr nonlinear optical media were predicted and experimentally demonstrated in early 90s [1,2]. These interesting beam coupling effects are governed by the cross-phase modulation (XPM) between the two laser beams in the Kerr nonlinear medium [1–5]. Also, degenerate and nondegenerate four-wave mixing (FWM) processes in two-level atomic systems have been investigated previously [6–9]. Here, we experimentally demonstrate that by arranging the pump and coupling laser beams in a specially designed spatial configuration (to satisfy phase-matching for the FWM processes), the generated FWM signals from the degenerate and nondegenerate FWM processes can be spatially split easily. Both the spatial separation and the number of the split beams of the FWM signals can be well controlled by the additional dressing laser beams via XPM. The enhanced self- and cross-Kerr nonlinearities due to induced atomic coherence in the system [5] are essential in generating the efficient FWM processes, and in the spatial splitting of the FWM signal beams. Full theoretical simulations are carried out and used to provide good matches to the observed phenomena.

Studies of the spatial beam shift and splitting can be very useful in understanding the formation and interactions of spatial solitons [10], gap solitons [11,12], vortex solitons [13], as well as their dynamics [12], in the Kerr nonlinear systems. Also, such spatial beam controls can be very useful for signal processing applications, such as spatial beam splitter [14], routing [15], and switching [16].

Let us consider the two-level atomic system, as shown in Fig. 1(a). Five laser beams (with diameters of 0.2 mm) are applied to the atomic system with the spatial configuration given in Fig. 1(b). E1 (k1, and the Rabi frequency G1) and E′1 (k′1, and G′1) are the pump beams propagating in one direction with a small angle (0.3°) between them. E3 (k3, G3), is the probe beam propagating in the opposite direction with a small angle (0.05°) from beam E1. These three beams (E1, E′1, and E3) have the same frequency ω1 (from the same laser), and generate an efficient degenerate FWM signal E′′1 = k1 + k′1 + k3 in the direction shown at the lower right corner of Fig. 1(b). Another pair of beams, E2 (k2, G2), and E′2 (k′2, G′2), are the coupling beams with E2 propagating in the same direction as E1 and E′2 having a small angle (0.3°) from E2. E2, E′2 and E3 have the same frequency ω2 (from the same laser) and they interact with the probe beam E3 to generate an efficient nondegenerate FWM signal E′′2 = k2 − k′2 + k3 in the direction of the upper left corner in Fig. 1(b) due to the given phase-matching condition.

The experiment was done with Na vapor in a 18 cm long heat pipe. The ground state of the two-level system (|0⟩) is the 3S1/2,1S1/2 energy level and the excited state (|1⟩) is the 3P3/2 level. Both lasers (with frequencies ω1 and ω2, respectively) are near-transform-limited dye lasers with 10 Hz repetition rate, 3.5 ns pulse width, pulse-by-pulse stability of 3%, and energy per pulse of 0.1 mJ. One laser is split to produce beams E′1, E′′1, and E′′′1 with frequency ω1, and another laser is used for beams E2 and E′′2. These laser beams are carefully aligned in the spatial configuration as shown in Fig. 1(b). In order to optimize the beam shift and splitting effects, E′′2 beam is made to be the strongest, approximately five times larger than E′1 beam, 100 times larger than the beam E3, 103 times larger than the beam E2, and 104 times larger than the weak probe beam E3, as well as the two generated FWM signal beams (E′′1 and E′′′2). These weak beams are recorded by a charge-coupled device and a fast gated integrator (gate width of 50 ns).

As shown in the insets of Figs. 2(a) and 2(b) (lower panels), the probe and FWM signal (E′′1) beams are displaced and split as the frequency of the probe beam (frequency detuning is defined as ωp = ω1 − ω2, where ω2 is the atomic transition frequency) is scanned through resonance. The displacements of the probe and FWM beams follow the shape of nonlinear dispersion [5,15]. Here, we concentrate only on the beam splitting effect. Figure 2(a) gives the splitting dis-
In the atomic system is a defocusing medium and the beam profile shifts in the down-right direction due to the repulsion of the weak probe beam $E_1$. For $\Delta_1 > 0$, the atomic system becomes in the $x$ (horizontal) direction and gets closer to the $E'_1$ beam, which can split the probe beam in the $y$ direction. In the $\Delta_1 > 0$ region, the probe beam shifts to the down-right direction due to the repulsion ($n_2 > 0$) of the strong $E'_2$ beam. This makes it get closer to the $E'_2$ beam, which can split the probe beam in the $x$ direction.

![Diagram](image)

**FIG. 2.** (Color online) (a) Measured probe beam splitting versus $\Delta_1$ (square) and the fitted $n_2$ curve (solid) with $G'_1=20.6$ GHz at 250 °C. Inset: spots of the probe beam versus $\Delta_1$. (b) Measured $E_{F1}$ beam splitting versus $\Delta_1$ (square) and the fitted $n_2$ curve (solid) with $G'_1=20.6$ GHz at 265 °C. Inset: $E_{F1}$ beam profiles at $\Delta_1=-30$ GHz (triangle), $\Delta_1=-22$ GHz (square), $\Delta_1=-17$ GHz (circle) in top right corner; and spots of the $E_{F1}$ beam versus $\Delta_1$ in bottom. (c) $E_{F1}$ beam profiles at 230 °C (triangle), 240 °C (square), 260 °C (circle), and 280 °C (reverse triangle), respectively, with $G'_1=20.6$ GHz at $\Delta_1=-10$ GHz. Inset: $E_{F1}$ beam splitting versus atomic density $N$. (d) $E_{F1}$ beam splitting versus $G'_1$ with $\Delta_1=-10$ GHz at 265 °C. The other parameters are $\Delta_2=0$, $G_2=0$, $G_1=1.5$ GHz, and $G'_2=10.8$ GHz.

**FIG. 3.** (Color online) (a) $E_{F2}$ beam profiles versus $\Delta_1$ at 250 °C. $E_{F2}$ beam profiles at (b) $\Delta_1=-3$ GHz and (c) $\Delta_1=3$ GHz at 230, 240, 250, and 260 °C. The other parameters are $G_1=G'_1=0$, $G_2=1.9$ GHz, and $G'_2=20.8$ GHz. The solid lines are the experimental results and the dotted lines are the calculated $E_{F2}$ beam profiles.

$\Delta_1 < 0$ the beam splitting occurs in the $y$ (vertical) direction, but becomes in the $x$ (horizontal) direction when $\Delta_1 > 0$ in Figs. 2(a) and 2(b). This phenomenon can be explained by the relative positions between the weak beams and the strong dressing (controlling) beams shown in Fig. 1(b). For the weak probe beam $E_3$ [Fig. 2(a)], at the $\Delta_1 < 0$ region, the beam shift in the $y$ direction results from the attraction ($n_2 > 0$) of the strong $E'_2$ beam. This makes it get closer to the $E'_2$ beam, which also splits the probe beam in the $y$ direction. In the $\Delta_1 > 0$ region, the probe beam shifts to the down-right direction due to the repulsion ($n_2 < 0$) of the strong $E'_1$ beam (which is slightly misaligned to the left side) and gets closer to the $E'_1$ beam, which can split the probe beam in the $x$ direction. Also, for the weak FWM beam $E_{F1}$ [Fig. 2(b)], in the $\Delta_1 < 0$ region, it shifts above the $E'_1$ beam in the $y$ direction induced by the $E'_2$ beam, and such $E'_1$ beam can split $E_{F1}$ in the $y$ direction. While in the $\Delta_1 > 0$ region, the FWM $E_{F1}$ beam shifts in the down-right direction due to the repulsion ($n_2 < 0$) of the $E'_2$ beam and, therefore, splits in the $x$ direction induced by the $E'_1$ beam.

**FIG. 4.** Figure 4 depicts the beam profiles of the FWM beam $E_{F2}$ as functions of various parameters. Figure 4(a) presents the FWM beam $E_{F2}$ profiles for different probe frequency detunings. As one can see that the beam breaks up into two beams at certain value of $\Delta_1$. For $\Delta_1 < 0$, the atomic system is a focusing medium due to self-Kerr nonlinearity. For fixed parameters ($\Delta_1=-3$ GHz, $\Delta_2=0$, $G_3=0.1$ GHz, $G_1=G'_1=0$), as the temperature of the atomic medium (atomic density $N$) increases, the FWM beam changes from one into four pieces as shown in Fig. 4(b). Similarly, in the $\Delta_1 > 0$ region, the atomic system is a defocusing medium and the beam profile becomes wider. Under the similar experimental parameters (except $\Delta_1=3$ GHz), the single FWM beam also breaks up into four as the temperature gets higher [Fig. 4(c)]. The insets in Fig. 4 show the spatial beam images.
Here, $G_1=0 \& G_2=1.5$ GHz (ii), $G_1=20.6$ GHz and $G_2=1.5$ GHz (iii), and $G_1=G_2=20.6$ GHz (iv). The solid lines are the experimental results and the dotted lines are the calculated $E_{F2}$ beam profiles. Inset of (b): spatial shift of the $E_{F2}$ beam versus $\Delta_1$ with $E'_1$ dressing (square), $E'_2$ dressing (reverse triangle), $E''_1$ dressing (triangle), respectively. The scattered points are the measured results and the solid lines are theoretical $n_2$ curves. The other parameters are $G_1=G_2=1.5$ GHz.

Figure 4 presents the effects due to the doubly-dressing fields. The images in Fig. 4(a) show (i) the FWM $E_{F2}$ beam at different probe detunings; (ii) the FWM $E_{F2}$ beam shifts with $E'_1$ dressing; and (iii) splitting into three or four parts with the $E'_2$ dressing. So, with both the $E'_1$ and $E'_2$ dressing fields on, the FWM $E_{F2}$ beam not only shifts, but also splits, as shown in (iv) of Fig. 4(a). These phenomena are evident in Fig. 4(b), which presents the FWM beam profiles of Fig. 4(a) at $\Delta_1=25$ GHz. With stronger $E'_2$ dressing beam, the $E_{F2}$ beam changes from two into five split peaks. The inset of Fig. 4(b) shows that the shift of $E_{F2}$ beam is mainly caused by $E'_1$, since $E'_1$ is at the down-right corner of the $E_{F2}$ beam while $E'_2$ almost overlaps with $E_{F2}$. Thus, $E_{F2}$ is shifted by $E'_1$ and split by $E'_2$.

To understand the observed beam splitting and spatial shift of the probe and FWM beams, we need to consider various self-phase modulation (SPM) and XPM processes. The spatial beam breaking is mainly due to the overlap between the weak probe and/or FWM beams and the strong coupling or pump beams [1]. Due to XPM, the nonlinear phase can have more than one minimum when the cross-Kerr index $n_2$ increases, which generates several intensity minima in the profiles of the FWM beams. The propagation equations for the probe and FWM beams with only the most relevant coupling or pump beams for beam splitting are

$$\frac{\partial E_3}{\partial z} - \frac{i\nabla^2 E_3}{2k_3} = \frac{i k_3}{n_1} n_2 [n_2 S_1 |E_3|^2 + 2n_2^* |E'_1|^2] E_3,$$

(1a)

$$\frac{\partial E_{F1}}{\partial z} - \frac{i\nabla^2 E_{F1}}{2k_{F1}} = \frac{i k_{F1}}{n_1} n_2 [n_2 S_1 |E_{F1}|^2 + 2n_2^3 |E''_1|^2] E_{F1}, \tag{1b}$$

$$\frac{\partial E_{F2}}{\partial z} - \frac{i\nabla^2 E_{F2}}{2k_{F2}} = \frac{i k_{F2}}{n_1} n_2 [n_2 S_2 |E_{F2}|^2 + 2n_2^4 |E'_2|^2] E_{F2}. \tag{1c}$$

Here, $z$ is the longitudinal coordinate; $k_3=k_{F1}=k_{F2}=$ $\omega n_1/c$; $n_1$ is the linear refractive index; $n_2^{S_1-S_3}$ are the self-Kerr coefficients of $E_{3, F1,2}$; $n_2^{X_{1-4}}$ are the cross-Kerr coefficients of $E_{3, F1,2}$ induced by $E_{1,2}$ and $E'_{1,2}$. Using Gaussian profiles for the input fields, Eqs. (1) are solved by using the commonly employed split-step method. Note that the linear and FWM coupling terms

$$\rho^{(3)}_{F1} = -iG_3G_1G''_1 \exp(ik_{F1} \vec{r}) F_1^{\dagger} [F''_1 - F_1^\dagger] (F''_1 + F_1^\dagger),$$

$$\rho^{(3)}_{F2} = -iG_3G_2G''_2 \exp(ik_{F2} \vec{r}) F_2^{\dagger} [F''_2 - F_2^\dagger] (F''_2 + F_2^\dagger)$$

are neglected [15], where $F_i$ factors are the parameters related to the dressing field, the frequency detuning, and the atomic coherence rate.

The Kerr nonlinear coefficient is negative for a self-defocusing medium and positive for a self-focusing one, which is given by $n_2=\Re \rho^{(3)}_{F1}/(\epsilon_0 d_{31} n_0)$. One can solve the coupled density-matrix equations to obtain all $\rho^{(3)}$, i.e., $\rho^{(3)} = -iG_3G''_1 [d_3 d_1 + G''_1/T_0 + G''_1/G_1] n_2^{X_3}$ (induced by the strong $E'_1$ field), $\rho^{(3)}_{E_2} = -iG_3G''_2 [d_3 d_2 + G''_2/T_0 + G''_2/G_1] n_2^{X_3}$ (induced by $E'_2$) and $\rho^{(3)}_{E_2} = -iG_3G''_1 [d_3 d_2 + G''_2/T_0 + G''_2/G_1] n_2^{X_3}$ (induced by both $E'_1$ and $E'_2$ fields) with $d_1=\Gamma_{10}+\Delta_1$, $d_2=\Gamma_1+i(\Delta_1-\Delta_2)$, $d_3=d_1+G''_1 + G''_2/d_1$, $d_4=\Gamma_1-\Delta_2$. Here, $G_1,F_{1,2}$ are the Rabi frequencies of $F_{F1,F2}$ and $\Delta_1$ is the detuning of the fields $E_{F1,3}$ and $E'_1 (E_2$ and $E'_2$). In addition, these three weak beams can be spatially shifted by the other coupling or pump beams that do not have total overlaps with them [15].

The solid curves in Figs. 2(a) and 2(b) are the calculated cross-Kerr nonlinear coefficients, which show good fits to the measured data. So, the measurements of spatial splitting can be used to determine the cross-Kerr nonlinear index. With fixed experimental parameters (such as atomic density, frequency detunings, spot sizes, and atomic decay rates), the measured beam profiles are fitted to the calculated results (from the propagation equations) with adjustable signal amplitudes and constant background, which show excellent agreements, as shown in Figs. 3 and 4. Such direct comparisons indicate the validity of Eqs. (1) in describing the spatial splitting of the FWM beams by other laser beams. The nonlinear phase for the split beams is given by $\varphi_{NL}(z, \vec{r}) = 2k_{F1,2} n_2^{X_{1-4}} e^{-\vec{\xi}^2/2} / n_0$ and the transverse wave vector is $\partial_{\vec{\xi}} = \partial \varphi_{NL}/\partial \vec{\xi}$. So the amounts of the splitting for the probe and FWM beams are proportional to the dressing beam intensities $I_{2,1}$, the nonlinear dispersion $n_2$, and the propagation distance $z$ (or equivalently atomic density).

The ability to control spatial position and beam profile of one laser beam (or the FWM signal beam) by another laser beam can be very useful in understanding nonlinear dynamics between multiple laser beams in nonlinear media. In the self-defocused nonlinear medium ($\Delta_1 > 0$), the XPM due to the presence of another laser beam can generate a focusing effect [1,2], which can be used to generate spatial solitons and other interesting nonlinear effects in the media.

In summary, we have experimentally demonstrated controllable beam splitting for the degenerate and nondegenerate FWM signal beams in a two-level atomic system. The separation and number of the split beams can be well controlled by the intensities and frequencies of the laser beams, as well
as atomic density. Theoretical calculations indicate that the separation of the split FWM beams can be fitted well with the cross-Kerr nonlinear indices, and the split FWM beam profiles can also be fitted well by the propagation equations based on partial and complete overlapping beams. The current study opens the door to manipulate the SPM and XPM processes in the Kerr nonlinear media for spatial shaping and all-optical control of optical beam profile for soliton formation and soliton communications.

This work was supported by NSFC (Grants No. 10974151 and No. 60678005), PANEDD (Grant No. 200339), RFDP (Grant No. 20050698017), FYTEFYTIEC (Grant No. 101061), and NCET (Grant No. 08-0431).