Valley Hall Effect

Suguru Yoshida

sqy5298@psu.edu

Materials Research Institute & Department of Physics,
Pennsylvania State University
Quick review of the last two lectures

Anomalous Hall and Spin Hall effects
Spin-dependent Hall effects: Anomalous and Spin Hall effects

AHE: Anomalous Hall Effect
SHE: Spin Hall Effect
ISHE: Inverse Spin Hall Effect

Spin-dependent scattering is the key for both AHE and SHE

Main mechanisms of spin-dependent scattering

✓ a) Intrinsic deflection

Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

\[
\frac{d\langle r \rangle}{dt} = \frac{\partial E}{\hbar \partial k} + \frac{e}{\hbar} E \times b_n
\]

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry’s phase curvature.

✓ b) Side jump

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.

✓ c) Skew scattering

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.

Nagaosa et al., Rev. Mod. Phys. 82, 1539 (2010).

\[ b_n \text{ (sometimes denoted as } \Omega_n \text{) is Berry curvature} \]
What is Berry curvature?

Berry curvature $\Omega_n$ is the curl of Berry connection, $a_n$

$$\Omega_n(k) = \nabla_k \times a_n(k)$$

$$a_n(k) = - \langle u_{n,k}(r) | i \nabla_k | u_{n,k}(r) \rangle$$

where $u_{nk}$ is the cell-periodic part of Bloch wave function.

Semi-classical equation that describes the motion of electron wave packet under electric field $E$ is

$$\hbar \nu = \nabla_k E_n(k) - eE \times \Omega_n(k)$$

anomalous velocity

Berry curvature = magnetic field in the momentum space
Symmetry constraints on Berry curvature

\[ \hbar \mathbf{v} = \nabla_k E_n(k) - e \mathbf{E} \times \Omega_n(k) \]

- Under time reversal
  \[ \Omega_n(k) \Rightarrow -\Omega_n(-k) \]

- Under spatial inversion
  \[ \Omega_n(k) \Rightarrow \Omega_n(-k) \]

Berry curvature vanishes if the system has both the time reversal and inversion symmetry

Lifting of either or both symmetries \( \Rightarrow \) anomalous transport phenomena
Valley Hall effect
What is valley?

Valley is a pocket of electrons or holes in $k$-space

In Si, there are six hole pockets (valleys) in the first BZ

Visualized by VESTA
https://warwick.ac.uk/fac/sci/physics/current/postgraduate/regs/mpagswarwick/ex5/bandstructure/
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https://warwick.ac.uk/fac/sci/physics/current/postgraduate/regs/mpagswarwick/ex5/bandstructure/
Valley in graphene

Valleys are identical but inequivalent

$\Rightarrow$ Valley is a new degree of freedom in addition to charge and spin
Valleys are identical but inequivalent

$K$ and $K'$ points are inequivalent

$\Rightarrow$ Valley is a new degree of freedom in addition to charge and spin
Valleys are identical but inequivalent

$\Rightarrow$ Valley is a new degree of freedom in addition to charge and spin
How pseudospin manifests itself?

Physical quantities have odd parity under time reversal
⇒ good candidates to distinguish valley pseudospin states.

pseudospin-dependent Berry curvature

pseudospin-dependent orbital magnetic moment


<table>
<thead>
<tr>
<th>Table 1</th>
<th>Internal degree of freedom of Bloch electrons in 2D hexagonal crystals and the associated physical phenomena.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spin</td>
</tr>
<tr>
<td>Magnetic moment</td>
<td>✓</td>
</tr>
<tr>
<td>Hall effect</td>
<td>✓</td>
</tr>
<tr>
<td>Optical selection rule</td>
<td>✓</td>
</tr>
<tr>
<td>Electrical polarization</td>
<td>✓</td>
</tr>
</tbody>
</table>

Valley Hall effect is closely analogous to the spin Hall effect, where the spin-polarized electrons replaced by valley-polarized carriers.

\[ \hbar \nu = \nabla_k E_n(k) - eE \times \Omega_n(k) \]

Valley Hall


Spin Hall

Inversion symmetry breaking is needed

- Time reversal symmetry
  \[ \Omega_n(k) = -\Omega_n(-k) \]

- Spatial Inversion symmetry
  \[ \Omega_n(k) = \Omega_n(-k) \]

\[ \Omega_n(k) = \Omega_n(-k) = 0 \]

\[ \Omega_n(k) = \Omega_n(-k) \neq 0 \]
Inversion symmetry breaking is needed

Epitaxial graphene on h-BN

Transition metal dichalcogenides (TMDs)

\[ \Omega_n(k) = -\Omega_n(-k) \]

\[ \Omega_n(k) \neq \Omega_n(-k) \neq 0 \]
Demonstration of valley Hall effects

Detection of pure valley current by non-local transport measurement

1. Apply current $I_c$, which will be converted into in-place electric field $E$

2. $E$ is converted into the valley current via the valley Hall effect.

3. Valley current reaches the right side while being attenuated by scattering.

4. Valley current is converted back into the voltage via inverse valley Hall effect

By measuring nonlocal resistance $R_{NL} = \frac{V_{BC}}{I_c}$, valley Hall effect can be detected

Demonstration of valley Hall effects

Spin-valley locking state in TMDs

TMDs = graphene + broken inversion symmetry + strong SOC

- Under time reversal
  \[ E_{+s} (k) = E_{-s} (-k) \]

- Under spatial inversion
  \[ E_{+s} (k) = E_{+s} (-k) \]

Spin-degeneration is recovered
Spin-valley locking state in TMDs

TMDs = graphene + broken inversion symmetry + strong SOC

- Under time reversal
  \[ E_{+s}(k) = E_{-s}(-k) \]

- Under spatial inversion
  \[ E_{+s}(k) = E_{+s}(-k) \]
  \[ E_{+s}(k) \neq E_{-s}(k) \]
Spin-valley locking state in TMDs

Especially for K and K' points;

\[ E_{+s}(K) = E_{-s}(K') \]

Spin orientation of each splitting band is opposite between K and K' valleys
⇒ Spin-valley locked state

Broken inversion symmetry and SOC leads to coupled spin and valley physics

Xiao et al., PRL 108, 196802 (2012)
• Due to the broken inversion symmetry, electrons in distinct valleys experience effective magnetic fields (i.e., Berry curvature) with equal magnitude but opposite signs.

⇒ Valley Hall effect

• Valley Hall effect is analogous to the spin Hall effect, where the deflection depends on the valley index rather than spin.

Still trying to understand;

• Why are valleys accompanied with Berry curvature?
• What is orbital magnetic moment?
• Deriving optical selection rule