

A History of Undergraduate Research in Mathematics

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Introduction

Research in mathematics by undergraduates is now commonplace. Summer undergraduate research programs abound. Many institutions fund undergraduate student research. Senior theses routinely include original results. There are numerous conferences where the focus is on presentations by undergraduates. There are mathematics journals that specialize in publishing papers with undergraduate authors. Research experience is expected for admission to leading graduate programs. The annual Joint Mathematics Meetings (JMM) of the American Mathematical Society (AMS) and the Mathematical Association of American (MAA) and the annual summer MAA MathFest are attended by large numbers of undergraduates. These are recent developments. In this article we identify the key events that have led to the current widespread acceptance of the importance of opportunities for undergraduates to engage in research in mathematics.

URP programs

The first national effort to promote research by undergraduates began in 1959 when the National Science Foundation funded Undergraduate Research Participation (URP) programs. In 1961 the NSF promoted the URP program by sponsoring a five-day conference at Carleton College at which 75 mathematics professors representing 70 colleges and universities from across the United States gathered “to discuss certain fundamental questions regarding undergraduate research in mathematics [].” The questions were:

1. Is research desirable at the undergraduate level?
2. Is there a role for research in the basic undergraduate curriculum?
3. What are the aims of undergraduate research?
4. What are the criteria for undergraduate research?
5. Why is it that so few of the colleges and universities have undergraduate programs in mathematical research?

Following is the content of a letter I received from Jerry Alexanderson in 2011 commenting on his recollections of the era and the Carleton conference.

From my own experience an introduction of undergraduates to research began seriously in the late 50s. There was lots of opposition to the idea among faculty. They thought that it made sense in the other sciences—test tubes needed to be washed, white rats had to be tended to, . . . there were lots of things that undergraduates could do to help in research. But it was widely viewed as unworkable in mathematics. Undergraduates just didn't know enough to do research. So it was about that time that I came to Santa Clara (1958) and Abe Hillman . . . was very keen on the idea of undergraduate research and he pulled me into the effort locally. There were some good supporters in the mathematical community: Arnold Ross, R. L. Wilder, former president of both the AMS and MAA, and Ken May at Carleton. But it was a hard idea to sell.

The great step forward was the organization of a week-long conference on undergraduate research in mathematics by Seymour Schuster and Ken May, held at Carleton College in the summer of 1961. That same year NSF started funding "undergraduate research projects in mathematics" and we got a grant for that first year and continued with them, for summers and academic years, for 20 years. But in my view, it all started at Carleton. And the projects certainly did not consist of the mathematical equivalent of test-tube washing. One of my students (now on the faculty at the University of Edinburgh) was the first to break into the *AMS Proceedings* with a nice paper. Other projects led to publications in the *Pacific Journal of Mathematics*, *Linear and Multilinear Algebra*, *Journal of Mathematical Analysis and Its Applications*, *Semigroup Forum*, *Proceedings of the Edinburgh Society*, *The American Mathematical Monthly*, and on and on. Of course at some point funding was discontinued at NSF, but then it was reinstated with the introduction of the REUs.

Although the URP program had noticeable success, when it ended—in 1981 when the Reagan administration greatly reduced the NSF education budget—research by undergraduates in mathematics was still uncommon. The attitude of the math community regarding research by undergraduates is well illustrated by the following statement made to the National Science Board by Lynn Steen, President of the Mathematical Association of America, in November 1985 [].

Research in mathematics is not like research in the laboratory sciences. Whereas undergraduate research can thrive in most chemistry, biology, or physics research laboratories, research in mathematics is so far removed from the undergraduate curriculum that little if any immediate benefit to the undergraduate program ever trickles down from standard NSF research grants. Publication patterns provide vivid proof: hardly ever does one see papers in mathematics jointly authored with students, either graduate or undergraduate.

For the historical record, I have included in the appendix a complete list of the NSF-funded URPs between 1964–1966 as well as some other URP programs I was able to identify.

REU programs

Although the killing off of the NSF URP program in 1981 was a temporary setback, the first group of REU programs, in 1987, began a new era when the URP program was resurrected under the NSF Research Division. This came at the recommendation of the National Science Board, the policy-making arm of the NSF.

That year eight mathematics sites were funded. The 1987 NSF Division of Mathematical Sciences (DMS) budget for REU sites and NSF grant supplements was \$380,000. The annual math REU budget for 1988–1990 was \$500,000. In 1991 it rose to \$750,000. In 1987 and 1988 REUs ran four to six weeks with four to twelve participants. By 1991 seventeen of the eighteen REU sites ran from eight to ten weeks with six to ten participants. These numbers have been the norm ever since.

“Paradigm shift” is a term that is overused but I think it is appropriate to describe the change in attitude that has occurred in the mathematics community regarding undergraduate research since the beginning of the REU program.

A dramatic way to see this change in mindset in the math community regarding research by undergraduates is to compare the expectations stated in the NSF announcement of the REU program in 1987 with the role that undergraduates played at the Joint Mathematics Meetings in 2012.

Here is the announcement in 1987 in the *Notices of the American Mathematical Society* of the first REU program sponsored by the NSF [1].

To clarify the range of activities eligible for support under this program, the DMS has formulated the following examples.

- Direct involvement of a student in a research project operating in an experimental mode, e.g., generating data or working out examples in order to develop conjectures.
- Independent study activities where the student is expected to carry out literature searches that indicate the development over time of the area under study, possibly working through the details in seminal papers. Depth and difficulty of the material could be adjusted to meet the student’s background.

Today most people who run REUs would consider these activities as the starting point, not the end product of an REU.

In contrast to low expectations for the role of research by undergraduates in the 1987 NSF announcement, in 2012 948 undergraduates attended the Joint Mathematics Meetings (15.4% of the registered mathematicians) with 152 giving talks. The undergraduate poster session had 310 entries representing the research of 525 students. (See Table 1.)

Here is an excerpt from the REU announcement that appeared in the *AMS Notices* [2] in 1988 that addressed the problem that the REU program was meant to rectify.

The decline in recent years in the number of mathematics doctorates has been a continuing source of concern within the mathematical community. Many believe that the root of the problem lies at the undergraduate level, where students are rarely exposed to the excitement of mathematical research when they are in the process of choosing a field of study. In the laboratory sciences, students can participate in research in a variety of ways, and even simply observing the laboratory environment

can demonstrate how research is conducted in such fields. But in mathematics, it is more difficult to convey the nature of the research process, and the lack of technical background can be a barrier.

A major goal of the program is to involve students who come from institutions where research programs are limited.

Diversity of REU programs

One of the noteworthy strengths of REU programs is their great diversity in structure and target audiences. Although most REUs have students do group work in one or two areas under the supervision of a few faculty advisers, some institutions match students one on one with a faculty adviser. Examples include Indiana University, the University of Michigan, Kansas State, the University of Tennessee, and a number of schools that had NSF VIGRE grants. Less common is for an REU to be run by a single faculty member. This is the model used at the University of Minnesota Duluth by the author, by Anant Godbole at Michigan Technological University and East Tennessee State, and by Charles Johnson at the College of William and Mary. For long-running REUs involving multiple faculty members it is common for the faculty and the program leadership to change over the years.

Some REUs are designed for students who are from institutions that offer limited opportunities for research while others are aimed at students from leading PhD-granting institutions who have already done successful research. Some programs—particularly those in locations that have a large pool of students from underrepresented groups and possessing faculty from those groups—have had outstanding success at attracting those students. Other programs have had little success in this regard.

There have been three large-scale REUs. Williams College has about twenty participants per year, Iowa State University has had thirteen to twenty-seven students, DIMACS and Rutgers University administer four REU programs each year that run concurrently with up to thirty students in total. Since it began in 1990, more than 450 undergraduates have participated in the National Security Agency's Director's Summer Program.

Beginning with three sites in 2004, the MAA National REU Program (NREUP) has had funding from the NSF and NSA to support faculty and local minority undergraduates to engage in research at their own campuses during the summer. In 2012 twelve NREUP sites were funded, each with a minimum of four students.

Since 1997 Worcester Polytechnic Institute has operated an REU that focuses on industrial mathematics and statistics. The program partners with industries to identify problems of both industrial and mathematical significance. Students, faculty and industrial liaisons work together on these problems.

By 2012 twelve REUs had run for 20 years or more. They are listed below with the names of the initial directors, years of operation, and when available, the number of participants, PhDs produced, and papers published in refereed journals.

Long-running REUs

- Oregon State University, Robert Robson, 1987–2012, 252 participants, 57 PhDs.
- University of Minnesota Duluth, Joseph Gallian, 1987–2012, 191 participants, 106 PhDs, 189 publications, 8 AMS Fellows.

- Rose-Hulman Institute of Technology, Gary Sherman, 1988–2012, 182 participants, 39 PhDs from 1995 on, 23 publications.
- University of Washington, James Morrow and Charles Curtis, 1988–2012.
- Mt. Holyoke College, Donal O’Shea, 1988–2012 (missed 2003).
- Williams College, Frank Morgan, 1989–2012, 500 participants, 20 PhDs, 200 publications (all numbers are estimates).
- College of William and Mary, Charles Johnson, 1990–2012.
- NSA Director’s Summer Program, 1990–2012.
- Indiana University, Daniel Maki, 1991–2012 (missed a few years).
- Michigan Technological University, Anant Godbole, 1991–1999 moved to East Tennessee State University 2000–2012 (missed 2005 and 2009).
- Hope College, David Carothers, 1991–2012 (missed 1997 and 2007), 118 participants, 30 PhDs, 22 publications.
- DIMACS Rutgers University, 1992–2012.
- Louisiana State University, Neal Stofzfus and William Hoffman, 1993–2012.

Highlights in Undergraduate Research in the REU Era

In this section we document the growth in interest in fostering research in mathematics by undergraduates in the first quarter-century of the REU era by providing some of the important events along the way.

1987

- The first NSF math REU site awards were granted to Harvey Mudd College, the University of Colorado, Oklahoma State University, the University of Minnesota Duluth, Oregon State University, the University of Tennessee, Rice University and the University of Utah. In addition to REU sites, individuals with standard NSF research grants could request a supplement to support a few undergraduate students [1].
- The MAA sponsors an “experimental” contributed paper session for undergraduates at the summer meeting that has four speakers. Thereafter, the session becomes an annual event.
- Pi Mu Epsilon sponsors a contributed paper session for undergraduates at the MAA summer meeting that attracts thirty speakers.

1988

- The MAA forms a subcommittee on Research by Undergraduates. This committee is responsible for many of the advances that appear on this list.
- NSF funds fourteen REUs; among them are Worcester Polytechnic Institute and Indiana University.

1990

- The NSA Director’s Summer Program is established. It provides a twelve-week REU-like program for top-level undergraduates.

- The Council of Undergraduate Research (CUR) establishes a division for mathematics and computer science.
- The Association for Women in Mathematics Alice Schafer Prize is established. The award was not created to recognize outstanding research but over the years research has become the decisive factor in the selection of the winner.

1991

- The MAA committee on Research by Undergraduates sponsors a panel on “Models for Undergraduate Research” at the joint meetings.
- The first MAA poster session is held at the Joint Math Meetings. There were twelve posters.
- The AMS and the MAA jointly sponsor a three-part special session for research papers by undergraduates that features twenty-two talks representing the work of fifty-four students.

1994

- The MAA sponsors the first minicourse on undergraduate research at the Joint Meetings.
- The MAA and CUR sponsor the second poster session on undergrad research with nineteen students participating. The poster session now becomes an annual event.
- The AMS sponsors a four-part special session for research by undergraduates at the Joint Meetings with thirty-eight talks representing the work of sixty-eight students.

1995

- Aparna Higgins gives the first Project NExT course on undergraduate research. This course becomes an annual offering and that has thirty or so participants each year.
- The MAA, AMS and SIAM jointly establish the Morgan Prize for Outstanding Research by an Undergraduate.

1996

- The *Notices of the American Mathematical Society* begins identifying talks by undergraduates at the Joint Meetings in the program announcement. There was not a special session for research by undergraduates but six undergraduates spoke at contributed paper sessions at the Joint Meetings.

1997

- The MAA sponsors a minicourse on undergraduate research at the Joint Meetings.
- Fifteen papers are presented in a contributed paper session for faculty on establishing and maintaining undergraduate research programs in mathematics.

1999

- The number of students involved in the poster session at the JMM is a record sixty-eight.

- The National Security Agency and the AMS sponsor a three-day conference on summer research programs in mathematics for undergraduates. The proceedings are published by the AMS [].
- The MAA minicourse on getting undergraduates involved in research at the Joint Meetings becomes an annual event. Registration varies between 35–50 per year.

2000

- One hundred and forty students are involved in the poster session at Washington D.C. In many subsequent years the number is limited by the size of the room as demand exceeds the space available. The poster session annually becomes one of the best attended events at the Joint Meetings.
- The AMS sponsors a four-part special session on research by undergraduate students with thirty-eight talks representing seventy students at the joint meetings.

2001

- The MAA includes in its Mission Statement: “We support research, scholarship, and its exposition at all appropriate levels and venues, including research by undergraduates.”

2002

- A special session for research by undergraduates becomes standard at all subsequent annual Joint Mathematics Meetings.

2003

- The number of undergraduate posters at the Joint Meetings reaches 200 for the first time.

2004

- The MAA initiates a national REU with emphasis on providing opportunities for under-represented groups. With support from the NSA and the NSF, the program continues in subsequent years.

2006

- Of 377 undergraduates attending the annual Joint Meetings, forty-four give talks.
- Michael Dorff from BYU receives \$1.28 million from the NSF to provide twelve to fifteen mini-grants to fund faculty each working with two to four undergraduate students during the academic year on research.
- The MAA sponsors its twentieth annual contributed paper session at MathFest in Knoxville. Sixty students give talks in six sessions.
- The NSA and the AMS sponsor a three-day conference on promoting undergraduate research in mathematics. The proceedings are published by the AMS [].

2008

- The Joint Mathematics Meetings has 527 undergraduates in attendance (11.1% of the mathematicians registered).

Year	Undergraduates at the Joint Meetings			At MathFest
	Number Registered	Talks Given	Posters Presented	Number Registered
1991	-	-	12	-
1992	-	-	-	-
1993	71	-	-	-
1994	153	-	19	-
1995	125	-	13	-
1996	141	6	32	-
1997	109	23	13	-
1998	176	22	36	-
1999	236	23	68	-
2000	275	35	140	96
2001	276	15	148	122
2002	300	38	185	149
2003	377	50	200	123
2004	292	38	-	184
2005	361	51	120	166
2006	377	43	129	158
2007	476	64	140	255
2008	527	62	150	269
2009	650	94	210	247
2010	683	95	241	336
2011	759	137	265	279
2012	948	152	310	235

Table 1. Undergraduate Participation at National Meetings

2012

- The NSF funds sixty-four math REUs.
- The number of undergraduates at the JMM exceeds the number of graduate students in any previous year.
- The NSA, NSF and the MAA sponsor a three-day conference on trends in undergraduate research in mathematics.

Reasons for growth in undergraduate research

Although there are many reasons for the dramatic rise in the number of undergraduates doing research in mathematics, here are the ones I feel are the most significant.

- **NSF and NSA funding**

By a far margin, the generous support from the NSF and the NSA for summer REUs and REU-like programs has been the most important impetus for nearly all other efforts.

- **The Council on Undergraduate Research**
The lobbying efforts by the Council on Undergraduate Research were largely responsible for the NSF creating the REU programs.
- **MAA minicourses**
Between 1994 and 2012 enrollment in the sixteen minicourses at the Joint Meetings on how to involve undergraduates in research was approximately 600. Most of these were offered jointly by Aparna Higgins and the author.
- **MAA poster session**
The MAA poster session at the Joint Meetings has become a showcase event for undergraduate students to exhibit their work.
- **Project NExT**
Over 500 new faculty have taken Aparna Higgins's Project NExT workshop on getting undergraduates involved in research.
- **MAA undergraduate conferences**
Between 2003 and 2012 the MAA regional conferences, funded by the NSF, featured over 5500 talks by undergraduates and over 17,000 participants.
- **Pi Mu Epsilon**
For many years Pi Mu Epsilon has sponsored regional conferences and paper sessions at MathFest dedicated to undergraduate research.
- **MAA paper session at MathFest**
The MAA has sponsored a student paper session at MathFest for twenty-six years.
- **NSA Director's Summer Program**
The NSA Director's Summer Program is one of the largest research programs for undergraduates in existence. Moreover, it is one of the few that provides a research opportunity for students in the summer prior to their entering graduate school.
- **VIGRE programs**
Several major PhD-granting institutions had REU-like programs as part of their VIGRE program.
- **Pipeline effect**
REUs have been around long enough that REU alumni are now providing research opportunities for their own students.
- **Deans are demanding it**
More and more deans are demanding that faculty in all disciplines provide research opportunities to undergraduates. This is even the case at many schools where faculty have very high teaching loads. Not long ago it was commonplace for math departments to be exempt from this pressure but this is becoming less the case.

With Lynn Steen's statement made in 1985 given earlier in this article in mind, I wish to close this essay with my own made at the 2006 AMS-NSA Conference on Promoting Undergraduate Research in Mathematics.

“Under the right circumstances, undergraduates CAN participate in mathematics research.”

Bibliography

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- [5] L. A. Steen, Restoring scholarship to collegiate mathematics, *MAA FOCUS* 6:1 (1986) 1–2, 7.
- [6] *Proceedings of the Conference on Summer Undergraduate Mathematics Research Program*, Edited by J. A. Gallian, American Mathematical Society, Providence, RI, 2000, available at www.ams.org/employment/REUproceedings.pdf.
- [7] *Proceedings of the Conference on Promoting Undergraduate Research in Mathematics*, Edited by J. A. Gallian, American Mathematical Society, Providence, RI, 2007, available at www.ams.org/programs/edu-support/undergrad-research.

APPENDIX I

URP Sites

NSF URPs 1964--1966

Harvey Mudd College, John Greever
 University of Santa Clara, A. P. Hillman
 Wesleyan University, Robert Singleton, James Cronin
 Florida Presbyterian College, Forrest Dristy
 Emory University, John Neuberger
 Illinois Institute of Technology, Lennert Pearson
 University of Illinois, Urbana-Champaign, Hiram Paley
 Mount Saint Scholastica College [Kansas], Sr. H. Sullivan
 University of Kansas, William Scott
 Bowdoin College, David Christie
 University of Maryland, College Park, John Brace
 College of the Holy Cross, Patrick Shanahan
 Michigan State University, C. P. Wells
 University of Michigan, Ann Arbor, Nicholas Kazarinoff
 Jackson State College, Benjamin McLemore
 University of New Hampshire, Evans Munroe
 Princeton University, Ross Finney
 Rutgers University, Hyman Zimmerberg
 Stevens Institute of Technology, Henry Polowy

New Mexico State University, John Giever
Brooklyn College of SUNY, Meyer Jordan
City College New York of SUNY, Gerald Freilich
Fordham University, Henry DeBaggis
New York University, Albert A. Blank
Polytechnic of Brooklyn, Robert N. Dheedene
Rosary Hill College, Sr. Marion Beiter
SUNY of Buffalo, Frank Olson
Syracuse University, Paul Gilbert
North Carolina College, Durham, Marjorie Brown
Case Institute of Technology, Fred C. Leone
Kenyon College, Daniel T. Finkbeiner
University of Oklahoma, Gene Levy
Reed College, Thomas P. Dennehy
University of Oregon, Andrew F. Moursund
Bucknell University, Harvey Arnold
Lehigh University, Everett Pitcher
University of Pennsylvania, George Schweigert
University of the South, Stephen Puckette
Vanderbilt University, Donald Coleman
Washington State University, Donald Bushaw

Other URP programs I was able to track down include: Indiana University, George Springer (late 1960s); Wesleyan University (1961); Clemson University, Bill Hare and John Kenelly, (1967, 1968, 1970); Duke University, Bill Hare and Joe Kitchen (early 1960s); Caltech, Gary Lordon (1971, 1972); Reed College; St. Olaf College, Lynn Steen, Arthur Seebach and Loren Larson; University of Wisconsin at Madison, Don Crowe; Macalester College, Wayne Roberts (1970s); University of Minnesota Duluth, Joe Gallian (1977, 1979–1981).

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