Horizontal and Vertical Lines and Their Slopes

Recall the slope of a line is a numerical measure of the steepness of the line. We can find a line's slope using the slope formula or the slope-intercept form.

- If (x_1, y_1) and (x_2, y_2) are two points on the line, then the slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- An equation of the form $y = mx + b$ has slope m and y-intercept $(0, b)$.

Horizontal and vertical lines are special cases of lines that can often be confusing. Let's look at a few examples.

Horizontal Lines

A horizontal line is a line whose equation is of the form $y = k$, where k is a real number. All points that lie on the line have a y-coordinate of k.

Example: Graph the line $y = 3$ and find its slope.

Solution: Every point on this line has a y-coordinate of 3. It is a horizontal line through 3 on the y-axis.
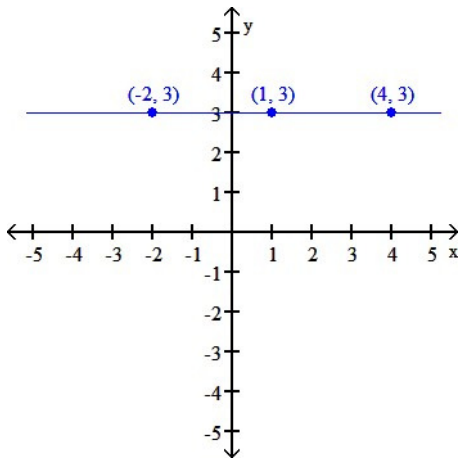
To compute the slope of this line, we can either use two points and substitute in the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$ or use the slope-intercept form.

If $(x_1, y_1) = (1,3)$ and $(x_2, y_2) = (4,3)$, we get

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{4 - 1} = \frac{0}{3} = 0$$

The equation $y = 3$ is equivalent to $y = 0x + 3$. From this form, we know the line has slope $m = 0$.
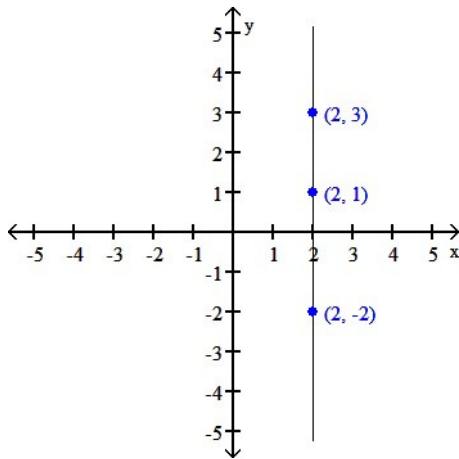

The slope of any horizontal line is zero, 0.

Example: Graph the line $x = 2$ and find its slope.

Solution: Every point on this line has a x-coordinate of 2. It is a vertical line through 2 on the x-axis.

To compute the slope of this line, we can only use two points and substitute in the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. This equation cannot be written in the form $y = mx + b$.

If $(x_1, y_1) = (2,3)$ and $(x_2, y_2) = (2,1)$, we get

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{2-2} = \frac{-2}{0}, \text{ undefined}$$

The slope of any vertical line is undefined.

Remember these two things: HO and UV which stands for

HO – horizontal, zero
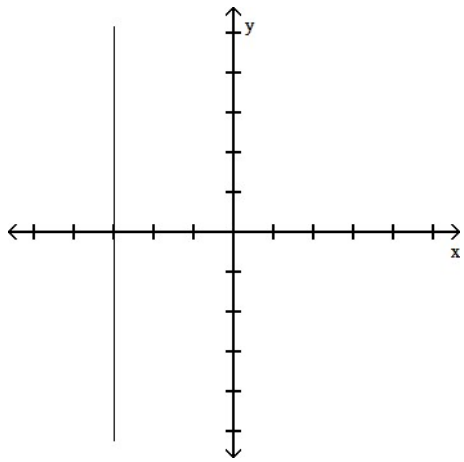
UV – undefined, vertical

Practice Exercises:

1) Answer the following questions by circling one of the underlined words that will make the statement true.

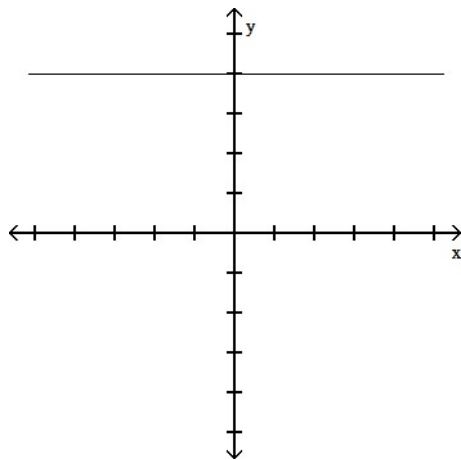a) The line $y = -2$ has <u>undefined/zero</u> slope.

b) The line $x = 4$ has <u>undefined/zero</u> slope.

2) Multiple Choice - The equation and slope of the graph below is

a) $y = -3$, slope is zero
b) $x = -3$, slope is zero
c) $y = -3$, slope is undefined
d) $x = -3$, slope is undefined

3) Multiplce Choice – The equation and the slope of the graph below is

a) $y = 4$, slope is zero
b) $x = 4$, slope is zero
c) $y = 4$, slope is undefined
d) $x = 4$, slope is undefined

Answers:
1a) zero, 1b) undefined, 2) d,   3) a