Chapter 7 Frequently Asked Questions

Example 1

Find the area to the left of z = -1.52 and to the right of 2.52.

Use the Calculator !! Remember that you will not have tables on the final exam!

To the left means from negative infinity to -1.52 (we obviously can't type infinity into the calculator, so we use a very very very small number, like -10^99)

normalcdf(-10^99, -1.52) = 0.0643 (rounded to four places)

To the right means from 2.52 to infinity (we obviously can't type infinity into the calculator, so we use a very very very large number, like 10^99)

normalcdf(2.52, 10^99) = 0.0059 (rounded to four places)

```
normalcdf(-10^99
,-1.52)
      .0642555095
normalcdf(2.52,1
0^99)
      .0058677603
```

0.0643 + 0.0059 = 0.0702 (which is what they got in MSL)

Example 2

Find the z-score such that the area under the curve to the left is 0.72

invNorm gives you the z-score for the area to the left

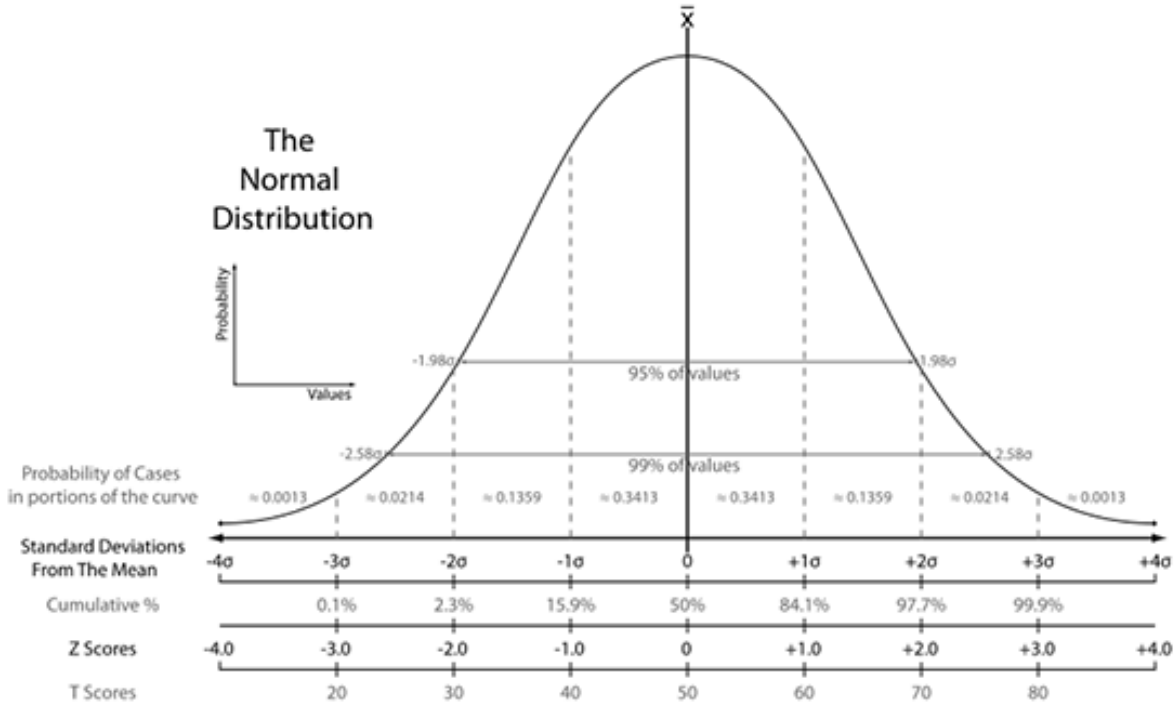
So, invNorm(.72) = 0.58 (rounded to two places)

```
invNorm(.72)
      .582841502
```

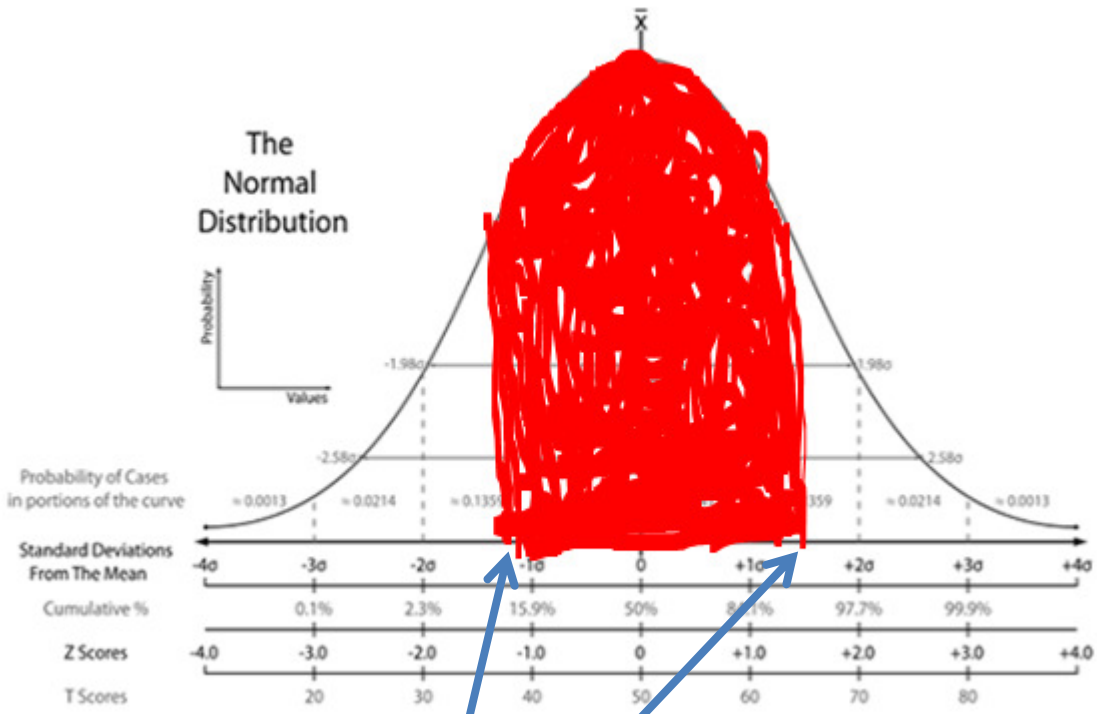
Example 3

Find the z-scores that separate the middle 79% of the distribution from the area in the tails of the standard normal distribution.

Here is a generic normal distribution:



Here is a normal dist. With 79% shaded between the tails.



So we are looking for these two z-scores:

If 79% of the total area (the total area is 100%) then how much is to the LEFT of that z-score on the left? (We want to know what's on the LEFT because invNorm tells us the z-score if we know the area on the left.)

100% - 79% = 21% but that 21% is divided equally between the two tails.

21% / 2 = 10.5%

So find invNorm(.105) = -1.25 (when rounded to two places)

```
invNorm(.105)
     -1.253565439
```

Since these z-scores have the same area to the right and left (there is also 10.5% to the right of the other z-score) we know that it will be the mirror image on the right side. So our other z-score is +1.25 (there's no need to actually calculate it).

However, just to prove that to you, let's figure out what the area to the left of that z-score would be.

79% (shaded) + 10.5% (in left tail) = 89.5% to the left

invNorm(.895) = 1.25

so the z-scores are -1.25, 1.25

Example 4

Assume the random variable X is normally distributed with a mean of 50 and a standard deviation 7.

Compute the probability P(34 < X < 63).

```
normalcdf(34,63,
50,7)
     .9572191889
```

MSL will accept this answer - - 0.9572 but on some of the homework it won't match (See http://facstaff.gpc.edu/~tlamar2/calculatorvsMSL.html for more explanation.)

Example 5

The number of chocolate chips in an 18-oz bag of chocolate chip cookies is approximately normally distributed with a mean of 1252 and standard deviation 129 chips. (Round all answers to four decimal places.)

a) What is the probability that a randomly selected bag contains between 1100 and 1500 chips?

```
normalcdf(1100,1
500,1252,129)
      .8533877602
```

normalcdf(1100,1500,1252,129) = 0.8534

b) What is the probability that a randomly selected bag contains fewer than 1025 chips?

```
normalcdf(-10^99
,1025,1252,129)
      .0392301522
```

normalcdf(-10^99,1025, 1252,129) = 0.0392

c) What is the probability that a randomly selected bag contains more than 1200 chips?

```
normalcdf(1200,1
0^99,1252,129)
      .6565629116
```

normalcdf(1200,10^99, 1252,129) = 0.6566

However, because of the rounding issues (see http://facstaff.gpc.edu/~tlamar2/calculatorvsMSL.html for more explanation)
So if you convert 1200 to a z-score as demonstrated in the above link, you could do this and get their answer:

```
normalcdf(-.4,10
^99)
      .6554216971
```

If you ever have an answer on this quiz from the calculator that was just a few ten-thousandths off the answer in MSL, email me and you'll get full credit back. Always be sure to tell me the question number you think you have correct.

d) What is the percentile rank of a bag that contains 1475 chips?

The percentile is the area to the left of the z-score. Since we want the area to the left we want the 1475 value and everything below it.

```
normalcdf(-10^99
,1475,1252,129)
      .9580670413
```

normalcdf(-10^99,1475, 1252, 129) = 0.96 (rounded to two places)

0.96 is 96% so it's the 96th percentile.

Example 6

The lengths of a particular animal's pregnancies are approx. normally distributed with a mean of 279 and standard deviation 20 days.

a) What proportion of pregnancies last more than 284 days?

More than 284 days – so the area to the right of 284

```
normalcdf(284,10
^99,279,20)
      .4012937256
```

normalcdf(284,10^99,279,20) = 0.4013

b) What proportion of pregnancies last between 244 and 289 days?

```
normalcdf(244,28
9,279,20)
      .6514033543
```

normalcdf (244,289,279,20) = 0.6514

c) What is the probability that a randomly selected pregnancy lasts nor more than 269 days?

No more than 269 days means LESS THAN 269 days – area to the left of 269

```
normalcdf(-10^99
,269,279,20)
      .3085375322
```

normalcdf (-10^99,269,279,20) = 0.3085

d) A "very preterm" baby is one whose gestation period is less than 234 days. Are very preterm babies unusual?

"unusual" is if the probability is less than 0.05. So we would want to find out if the area to the left is less than 0.05. (see page 235, section 5.1)

```
normalcdf(-10^99
,234,279,20)
     .0122244334
```

normalcdf(-10^99,234,279,20) = 0.0122

Since 0.0122 is 1.2% which is less than 5%, yes this is unusual.

Example 7

Rods are manufactured with a mean length of 24 cm. Because of variability in the manufacturing process, the lengths of the rods are approx.. normally distributed with a standard deviation of 0.08 cm.

a) What proportion of rods has a length less than 23.9 cm?
So we want the area to the left of 23.9.

```
normalcdf(-10^99
,23.9,24,.08)
     .105649839
```

normalcdf(-10^99, 23.9, 24, 0.08) = 0.0156

b) Any rods shorter than 23.81 cm or longer than 24.19 cm are discarded. What proportion of rods are discarded?
Shorter than 23.81 is the area to the left of 23.81

```
normalcdf(-10^99
,23.81,24,.08)
     .0087744625
```

normalcdf(-10^99, 23.81, 24, 0.08) = 0.0088

```
normalcdf(24.19,
10^99,24,.08)
     .0087744625
```

normalcdf(24.19,10^99, 24, 0.08) = 0.0088

Since it's both of these areas combined (both those shorter than 23.81 AND longer than 24.19)
0.0088 + 0.0088 = 0.0176

c) Using part b, if 5000 rods are manufactured, how many would we expect to discard?

If we discard 0.0176 of them and 5,000 are manufactured, then we expect to discard

5,000* 0.0176 = 88 rods

d) If an order comes in for 10,000 rods, how many should the manager expect to make if the order states that ALL rods must be between 23.9 and 24.1 cm?

```
normalcdf(23.9,2
4.1,24,0.08)
      .7887003221
```

normalcdf(23.9, 24.1, 24, 0.08) = 0.7887

This means that only 0.7887 of the rods will be between these two lengths. We want 10,000 of them to be between those two lengths.

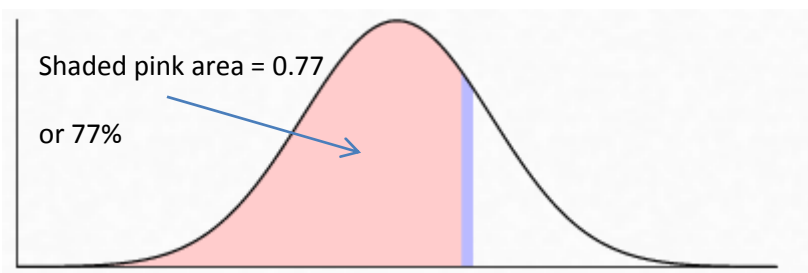
10,000 = 0.7887*(total number of rods made)

10,000 / 0.7887 = 12,679 rods need to be made

Example 8

Assume the random variable X is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 7$. Find the 77th percentile.

The 77th percentile means that 77% of the data is less than the random variable C. So the AREA under the normal curve to the LEFT of this value, X, is equal to 0.77.



Use your calculator to find the Z-score associated with an area of 0.77 to the left of your random variable.

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7↓χ²pdf(
```

```
invNorm(0.77)
      .7388468537
```

So, the z-score associated with the 77th percentile is 0.74 (when rounded to two places – since the tables in the book always use two decimal places for z-scores, this should be good enough).

To obtain the normal value use the fact that $X = \mu + Z\sigma$.

X = 50 + (0.74)(7)

= 55.18

Example 9

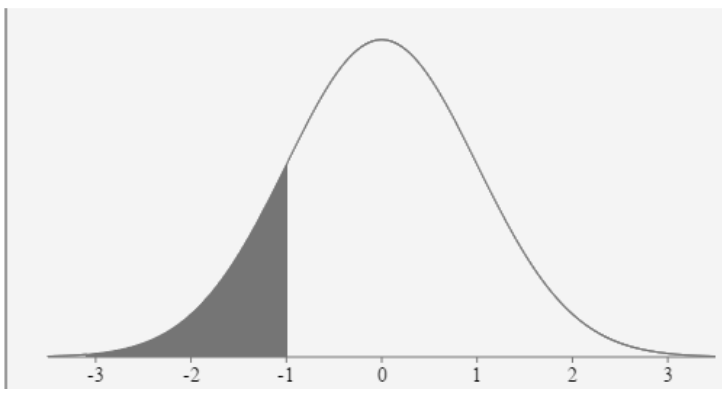
The mean incubation time of fertilized eggs is 23 days. Suppose the incubation times are approximately normally distributed with a standard deviation of 1 day.
(a) Determine the 16th percentile for incubation times.
(b) Determine the incubation times that make up the middle 39%.

(a) The 16th percentile for incubation times is 22 days.
(Round to the nearest whole number as needed.)

(b) The incubation times that make up the middle 39% are 22 to 24 days.
(Round to the nearest whole number as needed. Use ascending order.)

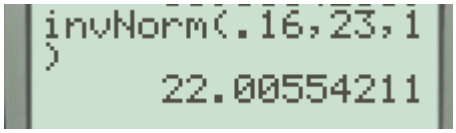
a) We need to calculate the value of a normal random variable to correspond to a certain proportion or probability. In this case, that is 0.16.

This is a normal curve with 0.16 shaded from the left. This shows the "16th percentile".



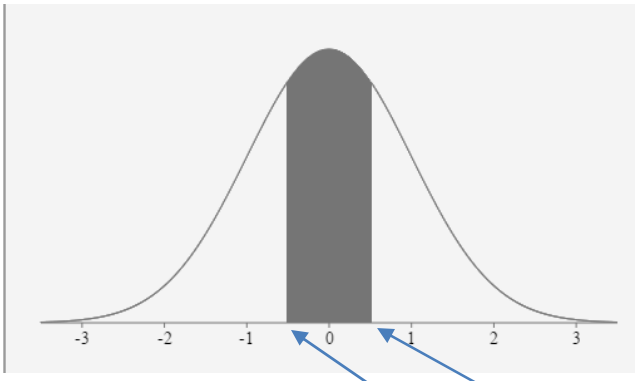
In this graph we see "z-scores" for a mean of zero and

We need to find the value that corresponds to an area of 0.16 from the left.
We can ask the calculator to give us this value by telling it the area to the left of this value, then the mean and then the standard deviation.
invNorm(%,μ, σ)



22 days (rounded to nearest whole number)

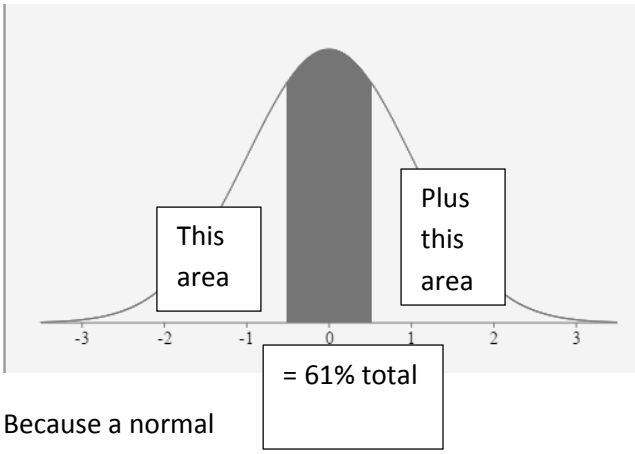
b) This is a normal curve with the middle 39% highlighted.



Because we have to tell the calculator the area to the LEFT of the value we want to know, we need to find out what is to the left of this value and this value

Since we know this 39% is exactly in the middle of 100% of area, we can find out what percentage lies to the left of both of these values.

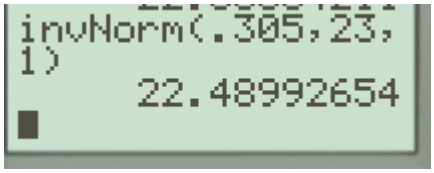
First, the one on the far left. 100% - 39% = 61%. So we have 0.61 in the two non-highlighted areas of this curve.



Because a normal curve is symmetric, each part has an area of 0.61 / 2 = 0.305.

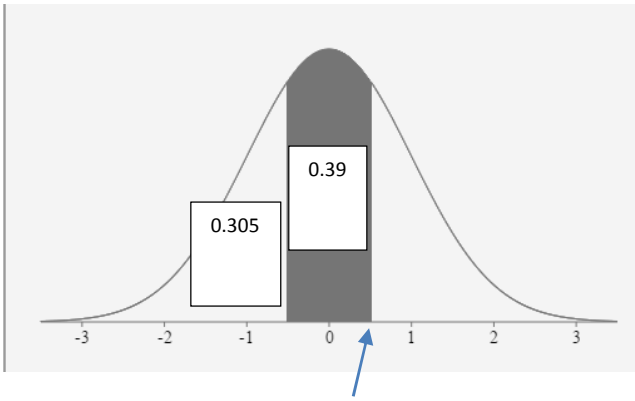
Since the left has an area of 0.305 we now know the "area to the left" that we need to give the calculator.

So again, we enter the percent to the left, then the mean, then the standard deviation.

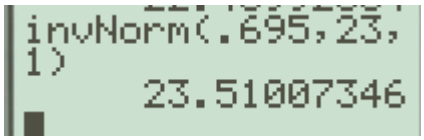


22 days (rounded to nearest whole number)

Now, what's the area to the left for the other value we need?



The area to the left of this value is 0.305 + 0.39 = 0.695



24 days (rounded to nearest whole number)