Module 9

Rational Exponents
Things to Know

- A rational exponent is a fractional exponent.
  
  \[ x^{1/n} = \sqrt[n]{x} \]
  \[ x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m \]

- All properties of exponents apply to rational exponents.

  \[ b^m \cdot b^n = b^{m+n} \]
  \[ \frac{b^m}{b^n} = b^{m-n} \]
  \[ (b^m)^n = b^{mn} \]
  \[ (ab)^n = a^n b^n \]
  \[ (\frac{a}{b})^n = \frac{a^n}{b^n} \]
  \[ b^{-n} = \frac{1}{b^n} \]
Converting between radical form and exponent form

Write the following as a radical expression.

\[ \sqrt[6]{17^6} = \sqrt[6]{17^6} \quad \text{or} \quad (\sqrt[6]{17})^6 \]
Rational exponents: Basic

Simplify.

\[ \frac{3}{32}^\frac{3}{5} = \left( \sqrt[5]{32} \right)^3 \]

\[ = 2^3 \]

\[ = 8 \]

\[ 32 \sqrt[3]{3 \div 5} \]
Rational exponents: Negative exponents and fractional bases

Simplify. Write your answers without exponents.

\[ 16^{-\frac{5}{4}} = \] 

\[ \left(\frac{\sqrt[3]{16}}{4}\right)^3 = \left(2^5\right)^{-\frac{3}{2}} = \frac{1}{2^5} = \frac{1}{32} \]

\[ x^{-n} = \frac{1}{x^n} \]

\[ \left(\frac{1}{4}\right)^{\frac{3}{2}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \]

\[ 16 \sqrt[3]{5 \div 4} \text{ enter} \]

\[ \left(1 \div 4\right)^{\frac{3}{2}} \text{ enter} \]

Math 1, enter
Rational exponents: Products and quotients

Simplify the expression.

\[ y^{-\frac{1}{3}} y^{2} y^{\frac{1}{5}} = y^{-\frac{1}{3}} + \frac{1}{2} + \frac{1}{5} = -\frac{10}{30} + \frac{15}{30} + \frac{6}{30} \]

Write your answer using only positive exponents. Assume that all variables are positive real numbers.
Simplify the expression.

\[
\frac{z^{-\frac{1}{2}}}{z^{-\frac{2}{3}}} = z^{\frac{2}{3}} - (-\frac{2}{3}) = z^{-\frac{1}{2}} + \frac{2}{3} = \frac{-3}{6} + \frac{4}{6}
\]

Write your answer using only positive exponents. Assume that all variables are positive real numbers.
Rational exponents: Powers of powers

\[(x^m)^n = x^{mn}\]

Simplify the expression.

\[
\left( b^4 \cdot a^{-\frac{1}{5}} \right)^{\frac{5}{3}} = (b^4)^{\frac{5}{3}} \cdot (a^{-\frac{1}{5}})^{\frac{5}{3}} = b^{\frac{20}{3}} \cdot a^{-\frac{1}{3}}
\]

Write your answer without using negative exponents. Assume that all variables are positive real numbers.

\[
\frac{\frac{4}{5} \cdot \frac{5}{3}}{\frac{1}{3}} = \frac{\frac{20}{3}}{\frac{1}{3}} = \frac{\frac{20}{3}}{a^{-\frac{1}{3}}} = \frac{b^{\frac{20}{3}}}{a^{\frac{1}{3}}}
\]