Module 5

Functions
Midpoint Formula
Things to Know

- Function notation: $y = f(x)$
- To evaluate a function means to replace the variable $x$ with the given value and simplify.
- For instance $f(-3)$ means the $y$-value that corresponds to $x = -3$. We find it by replacing $x$ with -3 in the given function.
The midpoint of a line segment is the point halfway between two points. If \((x_1, y_1)\) and \((x_2, y_2)\) are two points, then the midpoint is

\[
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]
Evaluating functions: Problem type 1

The functions $f$ and $g$ are defined as follows: $f(x) = 3x + 2$ and $g(x) = -2x^3 - 3$. Find $f\left(\frac{1}{2}\right)$ and $g(-4)$.

\[
f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right) + 2 = \frac{3}{2} + 2 = \frac{3}{2} + \frac{4}{2} = \frac{7}{2}
\]

\[
g(-4) = -2(-4)^3 - 3 = -2(-64) - 3 = 128 - 3 = 125
\]
Finding inputs and outputs of a function from its graph

The graph of a function $f$ is shown below. Find $f(-1)$ and find a value of $x$ for which $f(x) = 5$.

$\begin{align*}
\text{x = input} & \quad \text{y = output} \\
f(-1) = 2 & \quad (-1, 2) \\
f(x) = 5 & \quad \left(-\frac{3}{2}, 5\right) \\
x = -2, 1 & \quad \left(\frac{1}{2}, 5\right)
\end{align*}$
Variable expressions as inputs of functions

The function $f$ is defined as $f(x) = x - 4$. Find $f(x + 1)$.

$$
\begin{align*}
  f(x) &= x - 4 \\
  f(x+1) &= (x+1) - 4 \\
  &= x - 3
\end{align*}
$$
Evaluating functions: Problem type 2

The functions \( f, g, \) and \( h \) are defined as follows.

\[
\begin{align*}
 f(x) &= 3 + \sqrt{x + 20} \\
 g(x) &= \left| -\frac{1}{4}x - 15 \right| \\
 h(x) &= \frac{4 + x^2}{4 + x}
\end{align*}
\]

Find \( f(-4), g(12), \) and \( h(4). \)
Midpoint of a line segment in the plane

The midpoint of $AB$ is $M = (-1, 3)$. One endpoint is $A = (-5, 7)$. Find the coordinates of the other endpoint $B$.

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(-1, 3) = \left( -\frac{5 + x_2}{2}, \frac{7 + y_2}{2} \right)$$

$$-\frac{5 + x_2}{2} = -1 \quad \frac{7 + y_2}{2} = 3$$

$$-5 + x_2 = -2 \quad 7 + y_2 = 6$$

$$x_2 = 3 \quad y_2 = -1$$