Things to Know

- A quadratic equation is an equation of the form \( ax^2 + bx + c = 0 \).
- Methods to solve quadratic equations include
  - Factoring (Zero Products Property)
  - Square Root Property
  - Completing the Square
  - Quadratic Formula
The Square Root Property states that if \( x^2 = k \), then \( x = \sqrt{k} \) or \( x = -\sqrt{k} \); that is, \( x = \pm \sqrt{k} \).

Completing the square means to make a perfect square trinomial from a binomial of the form \( x^2 + bx \). The value \( c \) must equal \( \left( \frac{b}{2} \right)^2 \).

The Quadratic Formula states that if \( ax^2 + bx + c = 0 \), then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).
Solving a quadratic equation using the square root property: Problem type 1

Solve $x^2 = -96$, where $x$ is a real number.
Simplify your answer as much as possible.

If there is more than one solution, separate them with commas.

$x^2 = -96$
$x = \pm \sqrt{-96}$
$x = \pm \sqrt{-16 \cdot 6}$
$x = \pm 4i \sqrt{6}$

No real solutions
Complex solutions

$x = 4i \sqrt{6}, -4i \sqrt{6}$
Solving a quadratic equation using the square root property: Problem type 2

Solve \((y+9)^2 - 27 = 0\), where \(y\) is a real number.
Simplify your answer as much as possible.

\[
(y+9)^2 = 27 \\
y+9 = \pm \sqrt{27} \\
y+9 = \pm 3\sqrt{3} \\
y = -9 \pm 3\sqrt{3}
\]

\[-9 + 3\sqrt{3}, -9 - 3\sqrt{3}\]
Completing the square

Fill in the blank to make the expression a perfect square.

\[ x^2 + 2x + [\_\_] \]

\[ x^2 + 2x + \left( \frac{2}{2} \right)^2 \]

\[ x^2 + 2x + 1 \]

Check: \((x + 1)(x + 1) = (x + 1)^2\)
Solving a quadratic equation by completing the square

Solve the quadratic equation by completing the square.

\[ x^2 + 4x - 6 = 0 \]

First, choose the appropriate form and fill in the blanks with the correct numbers.

Then, solve the equation. If there is more than one solution, separate them with commas.

<table>
<thead>
<tr>
<th>Form:</th>
<th>( (x + \square)^2 = 10 )</th>
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<tr>
<td>( (x + \square) )</td>
<td>( (x + 2)^2 = 10 )</td>
</tr>
<tr>
<td>( (x - \square)^2 )</td>
<td>( 10 )</td>
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Solution:

\[ x = -2 + \sqrt{10}, -2 - \sqrt{10} \]

\[ x^2 + 4x - 6 = 0 \]
\[ (\frac{4}{2})^2 = 4 \]

\[ x^2 + 4x + 4 = 6 + 4 \]
\[ (x + 2)^2 = 10 \]
\[ (x + 2) = \pm \sqrt{10} \]
\[ x = -2 \pm \sqrt{10} \]
Applying the quadratic formula: Exact answers

Use the quadratic formula to solve for \( x \).

\[ \alpha x^2 + \beta x + \gamma = 0 \]

\[ x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \]

\[ 3x^2 - 2x - 6 = 0 \]

\[ a = 3 \]
\[ b = -2 \]
\[ c = -6 \]

\[ x = \frac{(-2) \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)} \]

\[ x = \frac{2 \pm \sqrt{4 + 72}}{6} = \frac{2 \pm \sqrt{76}}{6} \]

\[ \frac{1 + \sqrt{19}}{3}, \frac{1 - \sqrt{19}}{3} \]
Solving a quadratic equation with complex roots

Find all complex solutions of $3x^2 + 2x + 5 = 0$.

$a = 3$
$b = 2$
$c = 5$

\[ X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ X = \frac{-2 \pm \sqrt{2^2 - 4(3)(5)}}{2(3)} \]

\[ X = -2 \pm \frac{\sqrt{4 - 60}}{6} = -2 \pm \frac{\sqrt{-56}}{6} \]

Solutions: $\frac{-1 + i\sqrt{14}}{3}, \frac{-1 - i\sqrt{14}}{3}$
Discriminant of a quadratic equation

Compute the value of the discriminant and give the number of real solutions of the quadratic equation.

\[ 3x^2 - 2x + 4 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ b^2 - 4ac = \text{discriminant} \]

\[ b^2 - 4ac > 0 \Rightarrow 2\text{ real sol.} \]
\[ b^2 - 4ac = 0 \Rightarrow 1\text{ real sol.} \]
\[ b^2 - 4ac < 0 \Rightarrow 0\text{ real sol.} \]

\[ a = 3, \ b = -2, \ c = 4 \]

\[ b^2 - 4ac = (-2)^2 - 4(3)(4) = 4 - 48 = -44 \]

no real sol.
Solving equations that can be written in quadratic form: Problem type 1

Solve.

\( x^4 - 6x^2 + 5 = 0 \)

If there is more than one solution, separate them with commas.

\[(x^2 - 3)(x^2 - 2) = 0\]

\( x^2 - 3 = 0 \) or \( x^2 - 2 = 0 \)

\( x^2 = 3 \)

\( x = \pm \sqrt{3} \)

\( x^2 = 2 \)

\( x = \pm \sqrt{2} \)

\( \sqrt{3}, -\sqrt{3}, \sqrt{2}, -\sqrt{2} \)