Simple Inference for Constrained Optima*

Hiroaki Kaido† Francesca Molinari‡ Jörg Stoye§

October 30, 2018

We present a simple method for inference on the optimal value

$$\max p'\theta \quad \text{s.t.} \quad f(\theta) = 0, g(\theta) \leq 0, \tag{1}$$

where the vector p is known and well-behaved estimators of the (vector valued) constraint functions f and g are available; in particular these could be moment (in)equalities. A leading application is projection of partially identified vectors, notably inference on their components. We call the method "simple" because a critical value is computed at most once per direction of inference, i.e. at most twice for a two-sided test. This is in contrast to very generally valid methods like Bugni, Canay, and Shi (2017), Kaido, Molinari, and Stoye (2017), or also projection of confidence regions defined in Andrews and Soares (2010), all of which recompute critical values many times. The price is that assumptions became stronger. Compared to other approaches with similar computational cost, however, assumptions are still rather weak.

Similarly to Kaido, Molinari, and Stoye (2017), the basic idea is to relax sample analogs of studentized constraints by an amount $\hat{c}_n$ calibrated so that the problem's optimal value is covered (uniformly asymptotically) with pre-assigned probability. This can be computationally demanding if –as is the case in the most general settings– the limit of $\hat{c}_n$ depends on the parameter value θ at which it is evaluated, so that $\hat{c}_n$ must be computed many times.

We show that, under arguably reasonable assumptions, it suffices to compute $\hat{c}_n$ once. The main theoretical insights are as follows:

- Suppose a sample analog of the arg max in (1) is consistent in the relatively weak sense of asymptotically being inside its population counterpart, though not necessarily exploring it. Then it suffices to compute $\hat{c}_n$ at (an arbitrary element of) this sample analog. In particular, neither the population nor the sample arg max need to be singletons.

*We thank Ivan Canay for conversations that brought this paper into focus. Financial support from NSF grants SES-1824344 (Kaido) and SES-1824375 (Molinari, Stoye) is gratefully acknowledged.

†Department of Economics, Boston University, hkaido@bu.edu.

‡Department of Economics, Cornell University, fm72@cornell.edu.

§Department of Economics, Cornell University, stoye@cornell.edu.

- We provide low-level conditions that are sufficient for this simplification to apply. The broadest condition mimics m-estimator consistency requirements and is easily verified in many examples. We also provide more structured conditions that restrict the geometry of problem (1) near its solution(s).

Even our strongest assumptions do not require the feasible set in (1) to be convex, the solution to the problem to be unique, or this solution to be characterized by unique Lagrange multipliers or by a Linear Independence Constraint Qualification; all of these assumptions appear in the related literature. In particular, the approach can be compared to a comparably simple inference method from Pakes, Porter, Ho, and Ishii (2011).[1] That method (i) has essentially the same computational requirement, (ii) is currently justified under assumptions exceeding the strongest conditions above, and (iii) does not studentize sample moments, thereby potentially sacrificing statistical power. We provide examples in which the difference in power is steep.

In auxiliary contributions, we show that a similar simplification is available for Bugni, Canay, and Shi (2017) and clarify a strengthening of assumptions in Pakes, Porter, Ho, and Ishii (2011) that is needed to formally justify that inference method. We briefly discuss trade-offs between the three simple methods thus justified.

References

Andrews, D. W. K., and G. Soares (2010): "Inference for Parameters Defined by Moment Inequalities Using Generalized Moment Selection," *Econometrica*, 78(1), 119–157.

Bugni, F. A., I. A. Canay, and X. Shi (2017): "Inference for subvectors and other functions of partially identified parameters in moment inequality models," *Quantitative Economics*, 8(1), 1–38.

Kaido, H., F. Molinari, and J. Stoye (2017): "Confidence Inference for Projections of Partially Identified Parameters," CeMMAP working paper 49/17.

Pakes, A., J. Porter, K. Ho, and J. Ishii (2011): "Moment Inequalities and Their Application," Discussion Paper, Harvard University.

——— (2015): "Moment Inequalities and Their Application," *Econometrica*, 83, 315ï¿œ334.

[1]We refer to this unpublished version of Pakes, Porter, Ho, and Ishii (2015) because the latter does not contain the inference part.