The effect of child support on labor supply: An estimate of the Frisch elasticity*

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JOB MARKET PAPER

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November 6, 2019

In many countries, child support is set as a function of income and child support liabilities end when the children involved reach emancipation age. Exploiting these two features, this paper estimates the intertemporal elasticity of labor supply (Frisch elasticity) in an event study design using individual-level panel data from four countries. Empirically, I find that the observed child support rate that fathers face drops to nearly zero upon emancipation of the children; correspondingly, these fathers increase their work hours and earnings at this time. Based on these results, I estimate Frisch elasticities of 0.7–0.8 on the intensive margin and 0.1–0.2 on the extensive margin.

*PinchuanOng2014@u.northwestern.edu. I am deeply indebted to Matthew Notowidigdo, Seema Jayachandran, and Diane Schanzenbach for their insightful guidance and advice throughout the process of writing this paper. I thank Matthias Doepke, Ivan Canay, Robert Miller, Richard Blundell, Alexandre Mas, Basit Zafar, Douglas Webber, Shing-Yi Wang, and participants of the Northwestern Applied Microeconomics Lunch group for valuable feedback and suggestions at various points of this project. This paper would not have been possible without all the data collection efforts that went into the PSID, NLSY, BHPS, UKHLS, HILDA, SHP, and SIPP; I thank all parties involved for these important datasets, and for making the data easily available. All mistakes are my own.

1 Introduction

Labor economists have been interested in credible estimates of the Frisch elasticity—the intertemporal elasticity of labor supply holding constant the marginal utility of wealth—for three decades and counting. There are at least three reasons for interest in this parameter. The first is based on a decomposition perspective: labor supply responses to wage changes reflect wealth effects as well as an intertemporal shifting of labor supply to take advantage of temporarily higher wages, and the Frisch elasticity governs the latter. Second, the Frisch elasticity is an upper bound on the compensated (Hicksian) elasticity, a parameter necessary to compute the efficiency cost of government intervention in static settings.[1] Third, the Frisch elasticity is directly used in macroeconomic and dynamic public finance models [Golosov et al., 2011, Reichling and Whalen, 2012], and hence these estimates are important for understanding the economic and welfare impact of fiscal policies.

Despite its importance, few experimental or quasi-experimental estimates of the Frisch elasticity exist. Appropriate settings for its estimation are hard to find. The most common method of estimation requires that agents know the future price of labor that they will face in advance, or can predict it based on an instrument.[2] If using an instrument, we require that the econometrician observes this instrument. In addition to modeling what the agents know, we also require exogeneity: whatever drives the variation in price cannot just be shifting the taste for labor over time.

This paper proposes one setting in which the price of labor changes predictably in advance so that the above concerns are mitigated. All developed countries have child support systems in place that oblige noncustodial parents—I focus on child support transfers from fathers to mothers in this paper—to make child support payments in cases of divorce or nonmarital births.[3] These systems frequently have two features that are particularly favorable towards estimating a Frisch elasticity. First, in many cases, child support is expressed as a function of the father's income. Similar to a tax, it reduces the disposable income available for consumption, hence acting as a price on labor supply. Second, child support systems exhibit

[1]The Frisch elasticity compensates for changes in the marginal utility of wealth (equivalently the marginal utility of consumption in standard models) while the Hicksian elasticity compensates for changes in utility. For concave utility functions, it can be shown that the former requires a larger compensation than the latter, and hence the Frisch elasticity is an upper bound on the Hicksian elasticity.

[2]Predictability plays a key role in handling income effects when estimating the Frisch elasticity. Intuitively, a future wage increase leads to a negative income effect in the future, but if this increase were anticipated in advance, the agent works less even before the change happens. This smoothing of the income effect over time implies that it can be handled with simple fixed effects.

[3]Countries have incentives to put in place formal systems of child support, for otherwise, the burden of caring for the children involved falls on the state budget. The Organisation for Economic Co-operation and Development (OECD) describes the system in place among most member countries [OECD, 2014]; members not on the list are in Beaumont and Mason [2014, Table 8].

a drop in the child support rate (the amount of child support paid as a percentage of income) at a known point in time: when the last child involved becomes old enough to be legally independent. This is known as the age of emancipation of the children in some jurisdictions, a term which I adopt in this paper. The emancipation age is usually publicly known, and the drop upon emancipation is large, both factors which reduce concerns about inattention that might bias the estimated elasticity downwards [Chetty, 2012].

These facts motivate estimating the Frisch elasticity by using the emancipation of the youngest child eligible for support as an instrumental variable (IV) for the child support rate. To operationalize this, I follow the strand of the literature started by MaCurdy [1981], which makes sufficient assumptions on an individual's optimization problem to derive a simple estimating equation. To increase statistical power, I apply the same strategy to five longitudinal datasets spanning four countries with institutional settings that have the above properties, and combine the estimates. In the sample I examine, as the last child eligible for support becomes emancipated, fathers decrease the child support that they pay as a fraction of income (the support rate) to almost zero. In response to this, fathers increase their work hours by 3 percent. The formally estimated Frisch elasticity ranges from 0.7 to 0.8 on the intensive margin and 0.1 (statistically insignificant) to 0.2 (statistically significant at the 5 percent level) on the extensive margin.

There are two main threats to the validity of interpreting the IV estimates as Frisch elasticity estimates. First, we might not be satisfied that taste shocks are sufficiently accounted for. Here, we might think that taste for labor changes with the ages of children not living with the father. To address this, I show that labor supply does not respond to emancipation of children ineligible for support. Second, the above results assume that fathers only care about the consumption of their own families, and ignore the effects of receiving support for the support-receiving children and mothers. Without good data that links fathers to the consumption of the children and mothers, this is difficult to address empirically. Instead, I perform simulations that show the size of the elasticity in each case. Under reasonable assumptions on the fathers' preferences for the children's or mothers' consumption, the true Frisch elasticity is bounded between 0.6 and 1.2.

While the primary focus of this paper is on estimating the Frisch elasticity, we can also use the child support setting to study the labor supply of mothers. In this case, emancipation of the child leads to an anticipated change in unearned income for the mother. Because the child lives with the mother, the strategy above must be modified to avoid picking up consumption effects as the child leaves home. In the Survey of Income and Program Participation (SIPP), I find that the amount of child support received drops rapidly on emancipation of the youngest eligible child, but that children do not leave home immediately. This motivates a fuzzy regression discontinuity (RD) design strategy centered on emancipation. RD estimates of the labor supply response of mothers to child support payments are not significantly different from zero. Because the change in unearned income is anticipated, the lack of an income effect often found in other settings [e.g. Imbens et al., 2001, Kimball and Shapiro, 2008, Bengtsson, 2012, Cesarini et al., 2017] is consistent with mothers smoothing the drop in income across emancipation.

This paper contributes to two strands of the literature, the first being a large literature estimating Frisch elasticities [MaCurdy, 1981, Altonji, 1986, Altuğ and Miller, 1990, Pistaferri, 2003, Ziliak and Kniesner, 2005, Blundell et al., 2016, among others]. As mentioned, the main difficulty thus far has been in finding a credible setting or set of instruments. The usual instruments used are age and education, which have been criticized as being possible shifters for the taste for labor, and in any case it is unclear that individuals use the age-wage profile to forecast wages. In contrast, I control for these variables in my main specification. Pistaferri [2003] avoids the second issue discussed by using expectations data, but is open in admitting that the data might not allow him to control sufficiently for taste shifters. On the other extreme are estimates of the Frisch elasticity based on experimental or very controlled settings, but these estimates are based on very selected populations and shorter time horizons [Oettinger, 1999, Fehr and Goette, 2007]. Conceptually, the closest settings to that used in this paper are the "tax holiday natural experiments", which exploit tax reforms in Iceland [Bianchi et al., 2001, Sigurdsson, 2018] and Switzerland [Martinez et al., 2018] that result in one year of income being tax-free. By constructing appropriate control and treatment groups, and after restricting the periods of analysis so that tax changes are anticipated, the authors estimate a Frisch elasticity. Besides using a different source of price variation—child support instead of taxes—this paper differentiates from the tax holiday experiments by using long-run variation instead of focusing on one particular year or groups of years.

This paper also contributes to our understanding of the implications of child support systems. A substantial number of people interact with these systems; in the four countries I examine, about 14 percent of families with children are single parent families, affecting 20 percent of children younger than 15 [OECD, 2014]. Neither are payments small; support-paying non-custodial parents in the US pay 13 percent of family income in child support on average, a figure comparable to their effective tax rate of 8 percent.[4] Given the above and that the population involved tends to be economically vulnerable, many researchers have looked at implications of child support for the child. However, few have examined the labor supply response of the parents. There are three main difficulties: it is difficult to find a

[4]Estimates based on the Supplemental Poverty Measure module of the 2010 to 2016 Current Population Survey (CPS).

source of exogenous variation, most causal strategies require linking the father's data to the child, and most datasets are not powered towards looking at divorce and child support issues. To my knowledge, there are four studies examining the labor outcomes of the father. Earlier papers find little correlational effect on labor supply [Freeman and Waldfogel, 1998, Holzer et al., 2005]. Cancian et al. [2013] do not look at child support payments directly; instead, they exploit quasi-experimental variation in unmarried mothers' childbirth fees—these get charged to the fathers for Medicaid cases—and find that child support debt lowers fathers' labor supply. Rossin-Slater and Wüst [2018] construct a simulated instrument in Danish data and find no response in fathers' labor supply.[5] Lastly, on the mother's side, both Cuesta and Cancian [2015] and Rossin-Slater and Wüst [2018] find that child support receipt has no significant effect on the mothers' labor supply.

The paper proceeds as follows. Section 2 describes the relevant elements of child support policies, and the datasets used in estimating the Frisch elasticity for fathers. Section 3 presents the model and the estimating equation used. Section 4 presents estimates of the Frisch elasticity, as well as other related issues when considering fathers. Section 5 describes the data, strategy, interpretation, and estimates of the response of mothers to receiving child support. Section 6 concludes.

2 Institutional setting and data

2.1 Institutional setting

A key feature of my empirical strategy is that it is broadly applicable in many datasets: in principle, we get a causal elasticity estimate as long as we have enough information on marriage and fertility. This is important in overcoming data limitations; most datasets are small once we restrict to support-eligible parents. In this paper, I use data from several countries, selected based on panel data availability and the presence of key institutional features. To my knowledge, panel datasets that include clear questions on child support payments by the *noncustodial parent* exist in the US, UK, Australia, Switzerland, and Canada. Of these, access to the Canadian dataset is restricted, and hence was not included for this study.[6] In

[5] Heterogeneity across countries is one possible reason for the difference in results obtained. Another reason is that their estimated elasticity reflects substitution and income effects which act in opposite directions, and hence their response is likely to be smaller in magnitude.

[6] Three other countries, Germany, Russia and South Korea, have panel datasets that include questions on transfers to children. However, the wording of the question is more suggestive of transfers to current children in the family or children from the current marriage living elsewhere rather than from a previous marriage. The institutional settings are also not suitable for estimating the Frisch elasticity, with support amounts determined less by fixed rules and more by discretion [OECD, 2014].

this section, I provide a summary of important features of countries involved in this paper; details of each country's laws are provided in the online institutional details appendix.

Across all countries, there are two ways in which child support cases begin. In the majority of cases, child support starts when a couple with children divorce. Here, child support is usually determined as part of the divorce proceedings. In other words, when the judge determines who gets which assets, she also determines who gets custody of the child, and how much should be made in child support. The second type of child support cases are those in which the parents were never married. In these cases, the custodial parent has the additional step of determining involvement of the other parent. Child support payments are then determined by the courts or institution with authority based on formulas similar to divorce cases. The distinction between divorce and nonmarital cases does not pose a problem in my setting, and I combine both groups in my analysis.

In order for child support to be a price on the fathers' labor supply, payments need to be determined as a function of their income, possibly among other factors. In the countries that I consider, support amounts are determined by publicly-announced formulas that use the fathers' income, which helps ensure that the fathers know what to expect. In the UK and Australia, these formulas are rigid rules that the authorities must follow; in the US and Switzerland, judges are allowed to deviate from these guidelines, but deviation requires explanation (instituted as a federal requirement in the US). There is some heterogeneity in the guidelines used by different jurisdictions. 42 US states as of 2013, Australia, and some cantons of Switzerland consider both father and mother income, while others set child support amount as a percentage of the father's income. In the latter system, the interpretation of child support as a rate is clear. In the former system, a child-rearing expenditure amount that increases with both parents' income is first computed, and then the father pays his share of both parents' income multiplied by this expenditure. Here, the interpretation of child support as a rate is approximate, with the approximation being perfect if child-rearing expenditure were linear in the parents' income with an intercept of zero.

After the authorities make a support determination, it is presented to the parents as a dollar amount to be paid, to be updated when circumstances change in later years. This updating is also crucial for interpretation of support as a price; without, fathers should be treating the payment amounts as lump sum transfers.[7] In Australia and in the UK since 2012, updating is annual and automatic using tax income data. In the US, updating is only done when either the father, the mother, or a child support agency reviews the case.

[7] Non-contemporaneous updating implies that the effect of increasing labor supply on support amount comes with a lag. In Section 3.1, I model this lag and show that the price on the fathers' labor supply is attenuated by a discount factor. Intuitively, the benefit of delaying the "tax" by each additional year is an additional year's worth of interest obtained by saving or investing this amount.

Whether fathers expect reviews is untestable given the data available, but several factors help to keep the issue of review salient. First, the mothers obviously have a strong incentive to report increases in the fathers' income. Second, for all cases that go through the formal legal system (87.9 percent according to Grall, 2018), federal law requires parents to be notified of their right to review every three years, and the review is automatic in some cases (e.g. if the mother is receiving Temporary Assistance for Needy Families). In the UK before 2012, similar considerations exist, with the addition that the law requires the father to report changes in his income to the child support agency. The Swiss process for child support is similar to the US, except that (to my knowledge) there does not exist a rule that requires regular review of cases.

The end date of child support is mainly determined by the emancipation age, a feature that is publicly known and easily understood by the fathers. This supports the assumption that fathers anticipate future changes in the child support rate, which is what allows us to interpret the estimated labor supply elasticity as a Frisch elasticity. The emancipation age varies across jurisdictions, ranging from 16 in the UK to 21 in a few US states. Payments are usually allowed to continue past emancipation age if the child is still in high school, and a few jurisdictions allow payments through college; this likely weakens the first stage of the IV regression, especially if we focus on the years around emancipation.

2.2 Data

I use five panel datasets covering 4 countries to estimate the labor supply responses of fathers to child support obligations. The five datasets are the Panel Study of Income Dynamics (PSID) in the US; the National Longitudinal Survey of Youth 1979 (NLSY) in the US; the British Household Panel Survey combined with its successor the UK Household Longitudinal Study (BHPS+); the Household, Income and Labour Dynamics in Australia (HILDA) dataset; and the Swiss Household Panel (SHP). For consistency in treatment, I harmonize the method of construction of all variables across all five datasets as much as possible. Details of the data and cleaning procedures are provided in the online data appendix.

My identification strategy requires information on which children are eligible for child support, and their ages. I construct these variables based on the timings of marriages and child births for each father. To improve identification of eligibility, I exclude the child if she lives mostly with the father during the years in which she is eligible, or if she has died (if information is available). Marital histories are provided as derived variables in the PSID, NLSY, and HILDA, and fertility histories are provided as derived variables in the former two; in all other cases, I construct the relevant histories based on retrospective questionnaires (usually asked once per person), supplemented with marital status and family relationship information from each wave.[8]

The child support rate is constructed by dividing child support amount paid by the total individual income of the father. The degree to which the former specifically targets child support varies; data for the PSID before 1985 comes from a question on the amount of monetary support for any persons not living in the household, while data from 1985 onwards explicitly asked for child support. This does not pose additional problems for identification in an IV framework; the former implies that we should observe larger support amounts after the youngest eligible child hits emancipation age, but should not affect the drop at emancipation age itself. Total income includes earnings from work and income from other sources if recorded in the data, since this is generally the income base for computations of child support. The resulting child support rate computed has large outliers; to reduce their influence, I winsorize the rate at the first and ninety-ninth percentiles separately for each dataset.

In principle, a child support rate exists even if the father is not earning income. As such, I impute the child support rate if it is missing. For each individual, I impute using the previous observed value for observations before emancipation of the youngest eligible child, half of the previous observed value in the year of emancipation, and the median for observations after emancipation. To avoid periods of education and retirement, I do not extrapolate beyond the first and last observed child support rate of the father. Imputation serves two purposes: it increases the sample size slightly (by about 1 percent), and it allows estimation of an extensive margin elasticity to the child support rate.

I use all male observations between the ages of 26 and 59 for which information on marriage start and end dates are available, subject to a few dataset-specific restrictions and the following restrictions applied across all datasets.[9] First, I use only fathers who have at least one eligible child below emancipation age in at least one wave; this is motivated by the strategy of following these fathers and observing their reactions to emancipation of the children. Second, I drop fathers who have ever had to pay child support for more than 4 children. Third, I exclude years before the fathers are supposed to pay child support; these are years before divorce or nonmarital births. Fourth, for simplicity, I exclude years before and including the last observed positive change in the number of eligible children; this can occur because of multiple divorces, and the restriction ensures that there is only one year in which emancipation age is reached. Fifth, I exclude observations for which the youngest

[8]Retrospective questionnaire information may be missing for some individuals. In this case, histories are based only on in-panel information.

[9]Dataset-specific restrictions are described in the subsection for each dataset in the online data appendix.

eligible child is younger than 5; empirically, I observe that the average child support rate is lower at those ages, which likely reflects noise in the data that affects child age construction.

Appendix Table A1 shows summary statistics for the fathers I examine and all men in the datasets. Fathers in the sample I use are less educated, work less, and earn less early in life. They also marry and have children earlier, which is unsurprising given how the sample is constructed. These are the relevant differences if we wanted to compare with studies of the Frisch elasticities based on similar datasets. A few of the datasets oversample economic disadvantaged groups; differences increase once we incorporate sampling weights.

3 Model and estimation of the Frisch elasticity

3.1 The Frisch elasticity estimated through child support

In this section, I present a model that motivates the Frisch elasticity interpretation of the labor supply response of fathers to child support. Intuitively, the estimate is a Frisch elasticity because the father knows that at a known point in the future, the price of labor will change. A decrease in the price of labor makes the father work less through an income effect, but if he could forecast this change perfectly, he would also work less even before the change occurs. Any income effect is then cancelled out in a comparison across two periods. What makes the child support regime appealing for estimating a Frisch elasticity is the magnitude and the salience of the labor price drop when the youngest eligible child hits emancipation age.

The model I use is adapted from MaCurdy [1981] and related papers. At time period 0, father i solves

$$\max_{\{c_{it}, a_{i,t+1}, h_{it}\}_{t=0,1,\ldots,\text{all states}}} \hat{E}\left[\sum_{t=0}^{T} \beta^t \left[u\left(c_{it}\right) - \frac{e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}}}{1+\frac{1}{\gamma}} h_{it}^{1+\frac{1}{\gamma}}\right] |\Phi_{i0}\right] \tag{1}$$

$$\text{s.t. } c_{it} + \frac{1}{1+r} a_{i,t+1} = a_{it} + w_{it}h_{it} - S_{it}, \quad \forall t, \text{ states}, \tag{2}$$

$$S_{i,t+1} = s_{it} w_{it} h_{it}, \quad \forall t, \text{ states}, \tag{3}$$

where c is consumption, a is assets, h is hours worked, w is the wage rate, β is the impatience parameter, and r is the interest rate. Utility is time separable, and separable in consumption and hours worked. While Ziliak and Kniesner [2005] finds that consumption and hours worked are complements, the separability assumption allows for a closed form estimation strategy, and is especially convenient since not all datasets have consumption information. The way that utility depends on h is convenient and commonly used, and given separability between c and h, the form of $u(c)$ does not matter. $\boldsymbol{Z}_{it}$ (a vector) and U_{it} are observed and unobserved shifters of the taste for work. Implicit in Equation (1) is an assumption that the father does not value child or mother consumption. We might be willing to believe this is the case, since both child and mother no longer live with the father. In Section 4.5, I explore sensitivity of estimates to incorporating mother and child consumption into the father's utility function.

The budget constraint (2) that the father faces is typical, except that the father has to pay S in child support. The way that child support enters the budget constraint reflects how child support is determined in reality. Child support is formally an amount to be paid, with the amount determined periodically either by the court or an agency with authority. As mentioned in Section 2.1, this amount is determined based on a previous period's income; this is reflected in Equation (3). s_{it} is the support rate, which depends on the age of the youngest eligible child (among other factors unobserved to the econometrician). Lastly, $\hat{E}(\cdot|\Phi_{it})$ is father i's subjective expectation given the information set Φ_{it}. Φ_{it} is known to the father at time t, includes the wage, support rate, and taste shifters, and follows a stochastic process that depends only on past information. In particular, this rules out dependence of future wages (which are in $\Phi_{i,t+1}$) on current choice of labor supply, a topic studied by Altuğ and Miller [1998], Imai and Keane [2004], Blundell et al. [2016], among others.

It is straightforward to show that in this model, the Frisch elasticity, defined as $\frac{\partial \log h_{it}}{\partial \log w_{it}}\Big|_{\lambda_{it}}$ where λ_{it} is the marginal utility of wealth in period t, is γ. In Appendix B.1, I show the steps to derive the following:[10]

$$\log h_{it} = \gamma t \log \frac{1}{\beta(1+r)} + \gamma \log(1 - \dot{s}_{it}) + \gamma \log \lambda_{i0} + \gamma \log w_{it} - \gamma \boldsymbol{Z}'_{it}\boldsymbol{\alpha} - \gamma U_{it} + \gamma \sum_{\tau=1}^{t} \log(1 + \epsilon_{i\tau}), \tag{4}$$

where $\dot{s}_{it} \equiv \frac{s_{it}}{(1+r)}$, λ_{i0} is the marginal utility of wealth in period 0, and $\epsilon_{i,t+1} \equiv \beta(1+r)\frac{\lambda_{i,t+1}}{\lambda_{it}} - \hat{E}\left[\beta(1+r)\frac{\lambda_{i,t+1}}{\lambda_{it}}|\Phi_{it}\right]$ is the innovation to the ratio of marginal utility of wealth (henceforth, shocks to MU) that is not predicted in advance by the father. Equation (4) shows that the elasticity of labor supply with respect to the support rate (a reduced form parameter) is equivalent to the Frisch elasticity (a structural parameter). The support rate is discounted one period because the effect of a change in hours is only felt in the next period; this means that the estimated Frisch elasticity is attenuated by a discounting factor.[11] As a

[10] A technical assumption required for this and a subsequent step is that $|\epsilon_{it}| < 1$, so that we can take logs and perform a Taylor expansion.

[11] In practice, I ignore the discounting factor in this paper and treat $\dot{s}_{it}$ as equivalent to s_{it}.

Lagrangian multiplier, the marginal utility of wealth at time 0 is a function of all parameters and information in that period, and hence will be correlated in the cross-section with what fathers use to predict about the support rate. Holding this constant accounts for the fact that different fathers have different initial information sets (at the minimum, they may have different initial support rates); the most straightforward way to implement this is to include individual fixed effects. Shocks to MU are a major threat to identification. If the future support rate is not perfectly predictable in advance, positive shocks to $\log\left(1-\dot{s}_{it}\right)$ are unexpected increases in income, and hence are negatively correlated with shocks to MU, leading to an income effect. One role of the instrument is thus to capture only the response of hours to predictable variation (Keane, 2011, p. 1012).

For analysis of policy, it is more direct to examine the elasticity of earnings with respect to $1-\dot{s}_{it}$ instead. For this, we can rewrite Equation (4) so that we have log earnings on the left:

$$\log w_{it}h_{it} = \gamma t \log \frac{1}{\beta\left(1+r\right)} + \gamma \log\left(1-\dot{s}_{it}\right) + \gamma \log \lambda_{i0} + \left(1+\gamma\right)\log w_{it} - \gamma \boldsymbol{Z}'_{it}\boldsymbol{\alpha} - \gamma U_{it} + \gamma \sum_{\tau=1}^{t} \log\left(1+\epsilon_{i\tau}\right). \tag{5}$$

A causal regression of log earnings on $\log\left(1-\dot{s}_{it}\right)$ would also give an "earnings-Frisch" elasticity in this model. More importantly, if fathers have some control over the wages that they receive—ruled out in the model for simplicity—examining earnings gives a more general margin of response than hours to the price of labor.

The model yields a linear specification that makes it convenient when interpreting regression coefficients, but it describes the behavior of a father who would never choose to supply zero hours—such cases are implicitly assumed to be driven by exogenous factors. These extensive margin responses are also important when explaining aggregate labor supply movement at the economy level [Rogerson and Wallenius, 2009, Chetty et al., 2013], and can be estimated by replacing $\log h_{it}$ or $\log w_{it}h_{it}$ with indicator variables for whether h_{it} or $w_{it}h_{it}$ are positive.[12] The sum of the intensive and extensive margin responses then map to an economy-level aggregate response based on a statistical decomposition [Blundell et al., 2011].[13]

Because the error term is derived as a model-based object, we can motivate exogeneity of the instrument—whether the youngest eligible child is emancipated or not—using the

[12] Estimates are full elasticities (as opposed to semi-elasticities) in the following way: a 10% increase in after-support income $1-s_{it}$ leads to a γ percentage point increase in the probability of working, which implies a γ% increase in hours worked or earnings for the aggregate economy.

[13] This requires an assumption of homogenous preferences [Chetty et al., 2013, note 27].

following assumptions. The first concerns unobserved taste shifters, and the next three are about exogeneity with respect to the $\log(1+\epsilon_{i\tau})$ terms.[14] I show the derivation in Appendix B.2.

1. First, the instrument must be uncorrelated with unobserved taste shifters U_{it} conditional on covariates. This is the usual exogeneity assumption applied in a specific way. A concern in the literature along this vein has centered on aggregate labor demand shocks that are correlated with age or education [Altuğ and Miller, 1990]; in this context, I require them to be uncorrelated with the *child's* age.[15]

2. Second, we assume that the instrument is known to the father in advance. Intuitively, if fathers did not know the ages of the children that they are paying support for, they cannot shift labor supply across the emancipation age threshold.

3. Third, we assume that fathers have rational expectations regarding changes in MU. As an example of a violation, consider the case where fathers systematically underestimate the size of the support rate change on emancipation. In this case, the drop surprises fathers, they feel richer and work less, and this is not captured by fixed effects in estimation. Rational expectations implies that even if the drop surprises some fathers, other fathers expect a larger drop than objectively observed, and the income effects cancel. The saliency of the large drop in statutory rate and the fact that I use long-term variation for identification are factors that likely mitigate concerns about how much fathers understand.

4. Fourth, we assume that the instrument must be uncorrelated with $\sum_{k=2}^{\infty} \epsilon_{it}^{k}$ conditional on covariates; this is again an extension of the usual exogeneity assumption. To understand this assumption better, let $\boldsymbol{z}_{it}$ denote the vector of all included and excluded instruments, and note that we can rewrite the first element in the sum as

$$E\left[\boldsymbol{z}_{it}\epsilon_{it}^{2}\right] = E\left[\boldsymbol{z}_{it}E\left[\epsilon_{it}^{2}|\Phi_{i0}\right]\right]$$

under the second assumption. The term $E\left[\epsilon_{it}^{2}|\Phi_{i0}\right]$ is the conditional variance with respect to the initial information set; hence, one way this assumption can be satisfied is through homoskedasticity, as well as zero conditional skewness, and further moments.

[14] The latter follows Altonji [1986] and related papers in exploiting the full structure of the model to motivate exogeneity.

[15] Altuğ and Miller point out that this concern is exacerbated by the use of short panels. Even if the correlation was zero in expectation, aggregate shocks do not average out over individuals, and short panels do not provide enough averaging over time. However, one could make this argument about any experiment-based estimate, and hence this is a replicability issue rather than a concern regarding the instrument.

3.2 Empirical specification

Equations (4) and (5) are easy to implement as an IV regression. To do this, I use the specification

$$y_{it} = \psi_i + \zeta_{d(i)t} + \boldsymbol{Z}_{it}'\boldsymbol{\alpha}_{d(i)} + \gamma \log(1 - s_{it}) + \varepsilon_{it} \tag{6}$$
$$\log(1 - s_{it}) = \tilde{\psi}_i + \tilde{\zeta}_{d(i)t} + \boldsymbol{Z}_{it}'\tilde{\boldsymbol{\alpha}}_{d(i)} + \tilde{\gamma}_{d(i)} IV_{it} + \nu_{it},$$

where s_{it} is the child support rate of individual i in year t, and IV_{it} takes a value of one if the father's youngest eligible child is older than or at the emancipation age of his jurisdiction, half in the year before emancipation, and zero for years before.[16] I consider four main outcomes for y_{it}: log of annual hours worked, log of annual labor earnings, whether the father worked any positive hours, and whether he had any positive labor earnings. The main coefficient of interest, γ, is the Frisch elasticity.

I combine estimates from the five datasets $d(i)$ to improve statistical power. Individual fixed effects (ψ_i and $\tilde{\psi}_i$) and year-dataset fixed effects ($\zeta_{d(i)t}$ and $\tilde{\zeta}_{d(i)t}$) are included in all specifications, and a vector of covariates $\boldsymbol{Z}_{it}$ is included in my main specification. $\boldsymbol{Z}_{it}$ includes age-education fixed effects because age and education are obvious candidates for taste shifters of work [Pistaferri, 2003], and father age is likely to be correlated with the age of the youngest eligible child. It also includes the log of the father's wage since that is in Equations (4) and (5); because wage is unlikely to be correlated with the age of the youngest eligible child, I impute missing log wage values with a constant number and include a dummy variable in parallel to reduce arbitrary loss in data. I cluster standard errors at the individual level because residuals are correlated within individuals in Equations (4) and (5).

The only cofficient that is not heterogeneous across datasets in my specification is the main coefficient of interest, γ. In particular, this implies that there are five instruments in the first stage, one for each dataset. While combining the five instruments might give a more powerful first stage, using five instruments is a more accurate reflection of the differences in institutional factors and survey question wording. All IV results shown in this paper are based on two-stage least squares (2SLS).

The estimated labor supply elasticity can be interpreted as an elasticity that is common across all datasets. This assumes no heterogeneity in the responses across datasets or countries, and is known as the fixed treatment effects model or full pooling equilibrium model in the meta-analysis literature [Borenstein et al., 2010, Meager, 2018]. Alternatively, we can

[16] The price of labor changes at least one year before emancipation because support is determined based on past year income. I use half to account for the annual nature of the data.

interpret the estimated elasticity as a weighted average of the dataset-specific labor supply elasticities, induced by differences in questionnaire wording or institutional factors. In Appendix C, I show that the weights are a function of the size of the dataset, the strength of the first stages (the $\tilde{\gamma}_{d(i)}$'s), and the variation of the instrument within each dataset after partialling out the covariates. Empirically, there is little difference in the second and third factors across datasets; consequently, the averaging is almost entirely based on the size of the dataset.[17]

The meta-analysis literature also provides formal estimators that summarize dataset-specific treatment effects when there is heterogeneity in responses across datasets. Known as the random treatment effects model or the no pooling equilibrium model in the literature, the main difference for the estimated average is that the random treatment effects estimators asymptote towards a simple average of the dataset-specific estimates as the degree of heterogeneity across datasets increases. Along with the estimated average, random treatment effects models are usually used in order to understand the variance of the heterogeneity across datasets. Given the few number of datasets used in this paper, I abstract from this. The most important implication of this is that estimates from this study are limited in their generality [Borenstein et al., 2009, p. 83–84]; despite the fact that I use five datasets in this paper, we should exercise the same level of caution regarding external validity as we would for results from any one single study.[18]

In a few sections below, it is useful to consider the reduced form specification

$$y_{it} = \psi_i + \zeta_{d(i)t} + \boldsymbol{Z}'_{it}\boldsymbol{\alpha}_{d(i)} + \tilde{\tilde{\gamma}} IV_{it} + \varepsilon_{it}, \tag{7}$$

with $\tilde{\tilde{\gamma}}$ interpretable as the labor supply effect of emancipation of the youngest child eligible for child support. A "true" reduced form specification for (4) will require five equations, one for each dataset; the only difference between this specification and the "true" specification is that $\tilde{\tilde{\gamma}}$ is averaged across datasets.

[17] Fixed treatment effects estimators frequently weight estimates by their precision. This is done because the variance of the estimate is a summary measure of the number of observations as well as the study design (e.g. some studies match observations and others do not; Borenstein et al., 2009, p. 52). With microdata, both concerns are eliminated, and hence there is no further need to weight any dataset.

[18] In fact, the main estimates in this paper are qualitatively similar if we used the most common random treatment effects estimators from the meta-analysis literature. However, in order to perform inference, these estimators rely on the assumption that the heterogeneous effects are drawn from a normal distribution. If we relaxed this assumption using methods from the literature on clustering of standard errors, conservatively estimated 95% confidence intervals do not cover zero. Note that this does not invalidate the fixed treatment effects interpretation of the elasticity in any way; in fact, formal tests usually conclude that the heterogeneity in effects between datasets does not warrant using random treatment effects estimators.

4 Frisch elasticity estimation results

4.1 First stage

Figure 1 shows the average rate paid by fathers in each year relative to emancipation of the youngest eligible children. In the years before the change in the price of labor (marked by the vertical dashed line), fathers pay an average of 7 percent of their income in child support, a figure lower than statutory rates reported in each country.[19] This is mainly due to two factors. First, fathers may be delinquent or paying less than the full amount required. Second, some non-eligible children might be misclassified as eligible, since identification of eligibility in the data is based on the timing of births and marriages. Unfortunately, the data does not allow separation of the two factors.[20] After emancipation age, the average rate drops to an average of 1 percent in the ten years after emancipation.

Table 1 formally shows that the first stage is strong in four out of five of the datasets, and is strong overall.[21] On average, crossing the threshold corresponds to a 5 percent rise in $1 - s_{it}$, or—exploiting the log approximation for small s_{it}—a 5 percentage points drop in the support rate. The first stage F-statistic is somewhat weaker for Switzerland. There are two reasons for this. First, the Swiss sample is much smaller than the rest. Second, most of the youngest eligible children are in vocational education in the Swiss sample, and vocational education is heterogeneous in length according to the course of study (not observed in the data). This in turn induces heterogeneity in the age at which support payments end, since payments are required while the child in vocational education. I include Switzerland in the meta-estimate of the labor supply elasticity despite the weaker first stage since, to my knowledge, all other aspects of the institutional setting are similar to those in other countries that I use. Results are robust to excluding Switzerland.

4.2 Frisch elasticity

Panel A, columns 1 to 4, of Table 2 show estimates of the intensive-margin Frisch elasticity based on an OLS regression of Equation (6). Columns 1 and 2 show the Frisch elasticity estimated using log work hours as the dependent variable—this is the specification closest to what papers on the Frisch elasticity typically use—and columns 3 and 4 show the intensive-margin earnings-Frisch elasticity estimated using log earnings as the dependent variable.

[19]Statutory rates differ across jurisdictions and number of children supported, but are generally above 10%.

[20]To separate the two, I require a question on whether the father is required to pay child support. This is only available in the UK Household Longitudinal Study.

[21]Appendix Figure A1 shows the first stage for each dataset graphically.

Intensive margin estimates range between 0.1 and 0.7, with hours-based estimates smaller than those based on earnings. Columns 5 to 8 of the same panel show that corresponding estimates for the extensive margin are small and not statistically significantly different from zero. These OLS estimates are biased for at least three reasons. First, measurement error in the child support rate likely attenuates the estimate.[22] Second, to the extent that the support rate is not perfectly predictable, we have a negative bias from an income effect. A third factor, the influence of taste shifters and other institutional factors like the ability of fathers to (illegally) avoid support payment, could bias the elasticity upwards or downwards.

Causal estimates based on the IV specification (6) are shown in panel B of Table 2. Including all controls, intensive margin estimates range between 0.7 and 0.8, and extensive margin estimates range between 0.1 (not statistically significant) and 0.2 (significant at 5 percent). The implied total-margin Frisch elasticity is between 0.8 and 1.0. Causal point estimates are larger than non-causal estimates from panel A, and are similar in magnitude to quasi-experimental estimates in the literature.[23] The similarity in estimates across the first four columns suggests that fathers either have little control over their wage or do not respond on this margin. In Appendix Tables A2 and A3, I show intensive and extensive margin estimates estimated separately for each dataset. IV point estimates are positive in general, and suggest some heterogeneity across datasets, but standard errors are large and we cannot reject that the smallest estimates are different from the largest estimates at the 5 percent level.

Panel A of Figure 2 shows the average annual number of hours worked by fathers in each year relative to emancipation of the youngest eligible children, after partialling out individual and age fixed effects.[24,25] In the years before the change in the price of labor, after adjusting for individual and age effects, employed fathers worked 1,930 hours anually on average; in the years after, they worked 1,980 hours. Panel B shows the analogous plot for the average annual earnings of fathers. In the years before the change in the price of labor, employed fathers earned 47,400 dollars annually on average; in the years after, they

[22]A related possibility is division bias [Borjas, 1980], since the regressor of interest is computed by dividing support amounts by total income, and measurement errors in total income might be correlated with measurement errors in hours or earnings. However, this correlation is likely to be stronger for earnings than hours, and hence the downward bias should be larger in columns 3, 4, 7, and 8.

[23]In a meta-analysis paper, Chetty et al. [2013] finds two and six quasi-experimental studies reporting intensive-margin and extensive-margin Frisch elasticities respectively. I update their list by replacing Bianchi et al. [2001] with Sigurdsson [2018]—they use the same event for identification—and include Martinez et al. [2018]. This yields simple averages of 0.38 for the intensive margin and 0.27 for the extensive margin.

[24]Life cycle profiles make interpretation of non-residualized figures difficult (Appendix Figure A2). Nonetheless, work hours exhibit similar increases around the emancipation age. Earnings appears to be continuous across emancipation age, indicating that identification relies more on fixed effects.

[25]Analogous dataset-specific figures are less precise, and are shown in Appendix Figure A3.

earned 49,000 dollars. Hence, emancipation of the youngest eligible child is associated with an increase of labor supply of around 3 percent. Furthermore, residualized hours worked and earnings appear to be relatively constant before and after emancipation respectively, which suggests that the effect is constant at different time horizons. This is reassuring on two counts: first, it supports the use of long-term variation rather than focusing on the years just around emancipation, and second, it lessens the concern that the result is due to events that happen just around emancipation. Appendix Table A4 provides formal estimates of the reduced form specification (7); the total margin change for both hours and earnings is 4 percent, driven mainly by the intensive margin.

In Table 3, I test the robustness of the estimated Frisch elasticity to various changes in model specifications. In the table, each cell shows the estimated elasticity with the change specified at the start of the row. First, as the youngest eligible children approach emancipation age, fathers might expect that they could increase labor supply without an accompanying increase in amount paid in future once the child is emancipated. Hence, in the first row, I exclude the three years before emancipation of the youngest eligible child. The estimated elasticities remain similar. Next, in the second row, I investigate the sensitivity of estimates to using only variation around emancipation in a RD specification. To implement this, I restrict the sample to five years before and four years after emancipation, exclude the emancipation year and the year before, and include linear trends in the age of the youngest eligible child before and after emancipation.[26] Point estimates are relatively similar, but estimates are imprecise. In the third row, I exclude observations with imputed support rates. In the fourth row, I exclude Switzerland from the sample, since we might be worried about the weak first stage in the SHP. In the fifth row, I exclude divorces that occur after the youngest eligible child turns 10, to address concerns about possible endogenous choice in the timing of divorce. In the last row, I exploit the full variation in the support-age profile shown in Figure 1 for identification. To do this, I compute the leave-self-out average of the support rate at every child age relative to emancipation by dataset, and construct the instrument as the log of one less this average. Standard errors do not improve appreciably over those in my main specification. Estimates in the third to sixth rows are similar to the main estimates in Table 2.

Table 4 shows heterogeneity of the estimated intensive-margin Frisch elasticity by socioeconomic status.[27] Estimates are based on IV specifications, with both $\log(1 - s_{it})$ and

[26] I exclude the year before emancipation since that is the year in which the price of labor changes. I exclude the emancipation year because Figure 1 suggests that many fathers are still paying support in that year. This leaves four years before and four years after emancipation in the sample.

[27] Differences in child support compliance by income or race is well documented [Graham and Beller, 1996, Nelson, 2004]; here, I examine differences in the effect of having to pay support on labor supply. I focus on

the instruments additionally interacted with the heterogeneity variable.[28] In the first two columns, the heterogeneity variable is whether the father's average hourly wage before emancipation of the child is below the median in each dataset. In the next two columns, the heterogeneity variable is whether the father completed high school. In the last two columns, the heterogeneity variable is whether the father belongs to a minority race or ethnicity. In general, point estimates suggest that the Frisch elasticity might be larger among the more disadvantaged socioeconomic group, but the difference in effect is only significant in two out of the six columns. This result is consistent with some degree of heterogeneity in the Frisch elasticity or taste shifters (falsification tests and other robustness checks notwithstanding), and is inconsistent with binding credit constraint for the more disadvantaged group.[29]

4.3 Other outcomes

In Appendix Table A5, I report results for several other outcomes of interest. Columns 1 and 2 show results for the elasticity of food consumption of the father's family with respect to the child support rate.[30] Because the support rate is not a price on consumption, and because the IV strategy uses only variation predictable in advance by the father, the separability assumption in model (1) between consumption and labor implies that there should no effect on food consumption. The estimated elasticity is significant at the 5 percent level after all controls are included (column 2), indicating that there might be some complementarity between labor and consumption, as emphasized by Ziliak and Kniesner [2005]. The next four columns examine two mutually exclusive components of earnings: income earned as salaried employees, and income from self-employment.[31] Estimates of the elasticity of employee earnings are similar to that of all earnings, while point estimates of the elasticity of self-employment earnings are much larger. Note, however, that the latter is estimated based on a much smaller sample, and is not significant.

the intensive margin since fathers are mostly responding on this margin.

[28] Control variables are not fully interacted. Estimates based on fully interacted models are qualitatively similar, but less precise.

[29] Intuitively, credit constraints restrict the ability of fathers to move future increases in take-home income to earlier periods. The post-emancipation drop in support rate then leads to a negative income effect that is not captured by fixed effects in estimation; hence, the credit constrained group should have a smaller estimated elasticity.

[30] I examine food consumption because it is available with a relatively stable definition across time in in three out of five datasets (PSID, BHPS, HILDA).

[31] I examine the earnings breakdown rather than hours because the employee-self employment hours breakdown is not available for all datasets.

4.4 Falsification

We might worry that the effect is due to taste shifters that are correlated with child age. For example, if fathers still maintain contact with these children who live outside their household, and if emancipation of these children positively affects factors that are complements in utility with labor, the estimated elasticity would be biased upwards.[32] I perform two falsification exercises that help reassure that the results are not driven by this. Both rely on the same idea: if taste shifters were correlated with child age, we should detect a similar change in labor supply when *any* youngest child is emancipated, and in particular, when the youngest child from an intact marriage is emancipated.

For the first falsification test, I restrict attention to the subsample of fathers who are supposed to pay child support for at least one child, but whose youngest children are never eligible. Barring errors due to imperfect identification of children and marriages, these youngest children are born within marriages that are intact at the time of observation, and have older half-siblings who *are* eligible. Panel B of Table 5 estimates the reduced form specification (7) with the addition of an indicator variable for emancipation of the youngest ineligible child (panel A repeats the reduced form estimates from the main sample for comparison). Estimates are imprecise because the sample is much smaller, but point estimates for emancipation of the ineligible child are smaller than that for emancipation of the child eligible for support. Appendix Figure A4 shows this visually; while precision and outliers limit the conclusion we can draw, fathers do not appear to be increasing labor supply on emancipation of their ineligible children (solid line), but might be doing so for their eligible children (dashed line).

For the second falsification test, I examine fathers whose oldest children are born at least one year after the fathers' last marriages, and whose last marriages are intact at the last interview. Barring errors due to imperfect identification of children and marriages, these fathers do not have to pay child support for any of their children. Panel C of Table 5 show that these fathers do not work more on emancipation of these children. We might worry about comparability of the sample—these fathers certainly seem to be working and earning more on average than fathers with children eligible for support. Hence, in panel D and Figure 3, I show propensity score reweighted estimates based on the procedure in DiNardo et al. [1996]. Estimates are still insignificant and small in magnitude.

[32] Note that the bias has to be through taste shifters that affect the like or dislike for labor. In particular, lump sum transfers like having to pay for the child's college education are not taste shifters. Because these enter the budget constraint directly, they are not prices on labor.

4.5 Incorporating mother and child consumption in utility

The main estimate of the Frisch elasticity presented above are based on a model which assumes that the father does not value mother or child consumption. In this section, I present a model that incorporates these two variables, and explore sensitivity to this assumption. Intuitively, if the father resents the fact that the mother benefits from child support, estimates obtained in previous sections could be a reflection of this unwillingness to let the mothers benefit at his expense. The causally-estimated true Frisch elasticity (intertemporal response of hours work to *wages*) is then smaller than my main estimates. Conversely, if the father values child or mother consumption, the Frisch elasticity estimated in previous section is biased downwards.

The main difficulty in estimating this model is data-related—even combining datasets, only about one-fifth of father-years have corresponding observations for the mother. As such, estimates in this section are based on calibration of parameters that are difficult to obtain directly in the data. Intuitively, emancipation induces a labor supply response, which is then split between "resentment" and "dislike for work" based on the parameterization of the model.

The augmented model is

$$\max_{\{c_{it},a_{i,t+1},h_{it}\}_{t=0,1,\ldots,\text{all states}}} \hat{E}\left[\sum_{t=0}^{T}\beta^{t}\left[u_i\left(c_{it}\right)-\frac{e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}}}{1+\frac{1}{\gamma}}h_{it}^{1+\frac{1}{\gamma}}+\mathfrak{M}\left(\mathfrak{m}_{it}\right)+\mathfrak{C}\left(\mathfrak{c}_{it}\right)\right]|\Phi_{i0}\right] \tag{8}$$

$$\text{s.t. } c_{it}+\frac{1}{1+r}a_{i,t+1}=a_{it}+w_{it}h_{it}-S_{it},$$

$$S_{i,t+1}=s_{it}w_{it}h_{it},$$

$$\mathfrak{m}_{it}=\left(1-k\right)S_{it},$$

$$\mathfrak{c}_{it}=kS_{it},$$

where $\mathfrak{M}(\cdot)$ is a disutility function and $\mathfrak{C}(\cdot)$ is a utility function that capture the father's preferences for mother's ($\mathfrak{m}_{it}$) and child's ($\mathfrak{c}_{it}$) consumption out of child support respectively, and k is the child's share of consumption. The last two constraints model mother and child consumption as static shares of the support amount.

In Appendix B.3, I show that the estimating equation becomes

$$\log h_{it} = \gamma t \log \frac{1}{\beta\left(1+r\right)} + \gamma \log\left(1 - \dot{s}_{it} + \dot{s}_{it}\xi_{it}\right) + \gamma \log \lambda_{i0}$$
$$+ \gamma \log w_{it} - \gamma \boldsymbol{Z}'_{it}\boldsymbol{\alpha} - \gamma U_{it} + \gamma \sum_{\tau=1}^{t} \log\left(1 + \epsilon_{i\tau}\right), \quad (9)$$

where

$$\xi_{it} \equiv \beta\left(1+r\right)\frac{1}{\lambda_{it}}\hat{E}\left[\left(1-k\right)\mathfrak{M}'\left(\left(1-k\right)S_{i,t+1}\right) + k\mathfrak{C}'\left(kS_{i,t+1}\right)|\Phi_{it}\right] \quad (10)$$

captures how much the father values expected mother and child consumption out of child support relative to his own. The only difference between (9) and (4) is that the $\log\left(1 - \dot{s}_{it} + \dot{s}_{it}\xi_{it}\right)$ term now includes an additional $\dot{s}_{it}\xi_{it}$ term.

I parameterize $\mathfrak{M}\left(\cdot\right)$ and $\mathfrak{C}\left(\cdot\right)$ using a linearization of the father's own family consumption utility:

$$\mathfrak{M}\left(\mathfrak{m}_{it}\right) = -\kappa^{*}_{\mathfrak{m}}\tilde{u}_i\left(\mathfrak{m}_{it}\right),$$
$$\mathfrak{C}\left(\mathfrak{c}_{it}\right) = -\kappa^{*}_{\mathfrak{c}}\tilde{u}_i\left(\mathfrak{c}_{it}\right),$$

where $\tilde{u}_i\left(c_{it}\right) \equiv u_i\left(\tilde{c}_i\right) + u'_i\left(\tilde{c}_i\right)\left(c_{it} - \tilde{c}_i\right)$ is the first order Taylor expansion of $u_i\left(c_{it}\right)$ around a $\tilde{c}_i$ specified below, and $\kappa^{*}_{\mathfrak{m}}$ and $\kappa^{*}_{\mathfrak{c}}$ are parameters that capture the intensity of like or dislike for mother or child consumption out of child support. While incorporating concavity in $\mathfrak{M}\left(\cdot\right)$ and $\mathfrak{C}\left(\cdot\right)$ might be a better reflection of preferences over mother and child consumption, linearization removes the need for consumption data, yields an easily interpretable form for ξ_{it}, and in any case is required for $\mathfrak{M}\left(\cdot\right)$ to represent either a concave utility or concave disutility term.[33] A convenient point for linearization is the $\tilde{c}_i$ such that $\tilde{u}'_i\left(\tilde{c}_i\right) = e^{\frac{1}{T}\sum_t \log \lambda_{it}}$: the point that yields the individual's geometric mean of his marginal utility of wealth. At this point, straightforward derivation yields

$$\xi_{it} = \beta\left(1+r\right)\left(\left(1-k\right)\kappa^{*}_{\mathfrak{m}} + k\kappa^{*}_{\mathfrak{c}}\right)\frac{e^{\frac{1}{T}\sum_t \log \lambda_{it}}}{\lambda_{it}}, \quad (11)$$

in other words, ξ_{it} is the average-to-contemporaneous ratio in the marginal valuation of the father's own consumption, scaled by the intensity terms. In practice, I assume that $\beta = 1+r$, and set k to 0.3 based on estimates of the share of household expenditures attributable to children [Lino et al., 2017].

The child support rate provides some information that can be used to bound the intensity of like for child consumption $\kappa^{*}_{\mathfrak{c}}$. Intuitively, ignoring household size effects, a father who

[33] In particular, the negative of the commonly-used constant relative risk aversion function is convex and hence produces corner solutions.

values the consumption of his child as much as his own consumption should be consuming half of his income and sending the other half to the child, which is rejected by the data. In Appendix B.4, I show that the statutory support rate of around 10 to 15 percent per child implies that $\kappa_{\mathfrak{c}}^*$ should be below 0.25. For mothers, I bound $\kappa_{\mathfrak{m}}^*$ by symmetry.

I obtain an estimate of the Frisch elasticity after accounting for mother and child consumption at various values of the intensity parameters $\kappa_{\mathfrak{m}}^*$ and $\kappa_{\mathfrak{c}}^*$. Because the marginal utility of wealth λ_{it} is in the ξ_{it} term in Equation (11), the solution is iterative. Specifically, starting with a guess for the Frisch elasticity $\hat{\gamma}$, I compute the log of the marginal utility of wealth $\hat{\lambda}_{it}$ using

$$\frac{1}{\hat{\gamma}} \log h_{it} = \log (1 - \dot{s}_{it}) + \log \hat{\lambda}_{it} + \log w_{it},$$

which arises from the first order condition of h_{it} after we normalize $e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}}$ to 1.[34,35] The estimated $\hat{\lambda}_{it}$ is then used to compute $\hat{\xi}_{it}$ using Equation (11), and new 2SLS estimates of $\hat{\gamma}$ are obtained using Equation (9). The above process is iterated until $\hat{\gamma}$ converges.

Table 6 shows the estimated Frisch elasticity at various values of $\kappa_{\mathfrak{m}}^*$ and $\kappa_{\mathfrak{c}}^*$. As the intensity of like for child or mother consumption out of child support increases, the estimated Frisch elasticity increases. The highlighted cells show the values that are consistent with the bounds provided by the statutory child support rate, $\kappa_{\mathfrak{m}}^* \in [-0.3, 0.3]$ and $\kappa_{\mathfrak{c}}^* \in [0, 0.3]$. At the upper bound, when the father likes both mother and child consumption out of child support 30 percent as much as his own family's consumption, the true Frisch elasticity rises to 1.2. At the lower bound, when the father dislikes the mother's consumption out of child support 30 percent as much as he likes his own family's consumption, the true Frisch elasticity drops to 0.6.[36]

5 The intertemporal income effect for mothers

The above strategy can be used to estimate the labor supply response of mothers in response to receiving child support. In this section, I describe the issues involved, estimates, and the interpretation.

[34] The normalization is done because we do not have causal estimates of $\boldsymbol{\alpha}$. It is almost without loss of generality since the marginal utility of wealth appears in both the numerator and denominator of Equation (11).

[35] The distribution of $\hat{\lambda}_{it}$ is highly skewed, which causes problems for convergence since it enters as a ratio in Equation (11). Because of this, I winsorize $\hat{\lambda}_{it}$ at the 5 and 95th percent before the next step.

[36] Fathers might resent the mother's consumption out of child support more if they earned less than the mothers. In Appendix Table A6, I show that this is not true for the 15% of fathers that can be linked to the mothers; while imprecise, point estimates suggest that the response to child support is smaller in this case. This in turn suggests that $\kappa_{\mathfrak{m}}^*$ is more than zero.

5.1 Data

I use the US Census Bureau's 1990 to 2008 SIPP panels to estimate the intertemporal income effect for mothers.[37] To my knowledge, the SIPP is the largest panel data source with marriage and fertility information for mothers.[38] Each panel's interviews are conducted every four months for between two to five years, with questions on income and public transfers for each of the past four months. Each panel also includes a set of topical modules that are asked only in specific interviews; in particular, the second wave of all panels include a marital history module and a fertility history module. The 1992 and 1993 panels also include a follow-up panel on the same individuals between 1998 to 2002 known as the Survey of Program Dynamics; I include this survey in the sample as well to increase the sample size and to be exhaustive in treatment.

I treat the SIPP data as similarly as possible to the fathers panel. SIPP panels before 2014 do not include questions on middle children; hence, identification of child support eligible children and mothers uses only the birth information for the first- and last-born children and the timing of the last marriage of each mother. The short panel nature of the SIPP also implies that I cannot incorporate earlier-age residency history of the children in this step. I do not encounter problems in handling the other key variables in the same way as in the fathers panel.

I use all female observations between ages 26 and 59. Of the other restrictions imposed on the fathers panel, I include only the first and the last. The last restriction—excluding observations when the youngest eligible child is younger than 5—is applied directly. For the first restriction, I use only mothers who ever had an eligible child; due to the short panel nature of the SIPP, I do not follow the same mothers before and after emancipation. The other restrictions are not relevant for the SIPP due to limited fertility information and the short panel nature.

5.2 Interpretation and estimation strategy

In contrast to the situation for fathers, child support received by mothers allows estimation of an income effect. In an intertemporal setting, this is also a weak test of rationality. Suppose

[37] Estimates based on the annual panels used for fathers are similar to those in the SIPP, but less precise. I do not combine these panels with the SIPP because the latter sample is much larger than the former. In principle, the CPS might be used as well; however, few individuals appear in both the March Annual Social and Economic Supplement (required for child support income receipt) and the June Fertility and Marriage Supplement (required for eligibility status).

[38] The SIPP only started including fertility questions for male respondents in the the 2014 panel, and hence cannot be used to estimate the Frisch elasticity for fathers. I exclude SIPP panels before 1990 because the first stage for these panels are weak, suggesting that the enaction of presumptive guidelines by the Family Support Act of 1988 is important in the US (see online institutional details appendix).

that surprise increases in wealth lead to a decrease in labor supply, and that mothers do not expect the drop in child support as the youngest eligible children are emancipated. In this case, they should work more upon emancipation of the child. A lack of response on emancipation is indicative that the marginal propensity to earn out of unearned income (henceforth, the unanticipated income effect) is zero, or that rational expectations hold for mothers.

Estimation of the intertemporal income effect for mothers in the child support setting is complicated by the fact that the child lives with the mother, and is expected to leave home after emancipation. Because of this, the estimation strategy for fathers cannot be used wholesale for mothers. Instead, I use a local-linear RD specification to yield a causal estimate of the income effect. Specifically, I restrict the sample to observations within 24 months of emancipation on both sides, and estimate the specification

$$\log y_{it} = \psi_i + \zeta_{d(i)t} + \boldsymbol{Z}'_{it}\boldsymbol{\alpha}_{d(i)} + \gamma \log(1 - s_{it}) + \rho^{pre}_{d(i)} r_{it} + \rho^{post}_{d(i)} r_{it} IV_{it} + \varepsilon_{it} \tag{12}$$
$$\log(1 - s_{it}) = \tilde{\psi}_i + \tilde{\zeta}_{d(i)t} + \boldsymbol{Z}'_{it}\tilde{\boldsymbol{\alpha}}_{d(i)} + \tilde{\gamma} IV_{it} + \tilde{\rho}^{pre}_{d(i)} r_{it} + \tilde{\rho}^{post}_{d(i)} r_{it} IV_{it} + \nu_{it}.$$

Equation (12) is almost the same as Equation (6), and except for IV_{it}, all repeated variables have the same definition. In particular, I use the same outcome $\log y_{it}$ and key regressor $\log(1 - s_{it})$ for comparability with earlier sections, and show robustness of the estimates to other specifications that yield dimensionless regression coefficients. The RD running variable r_{it} is the age of the youngest eligible child relative to the emancipation age of the jurisdiction, and IV_{it} takes a value of one if r_{it} is greater than zero, and zero if smaller. I exclude the observation when r_{it} is exactly zero. The linear slopes for the RD are allowed to be different before and after emancipation, and across SIPP panels—I treat each panel as a different dataset. Different from Equation (6), I do not allow the effect of the instrument to be heterogeneous across SIPP panels in the first stage. I do this to improve power in the first stage, and because the SIPP's questionnaire wording and the US's child support system are relatively stable over the period of analysis. Similar to before, I cluster standard errors at the individual level. The main coefficient of interest is again γ; an intertemporal income effect exists if γ is positive.

5.3 First stage and key confounders

Figure 4 shows the key variables involved in obtaining a causal estimate of the intertemporal labor supply income effect. The solid purple line shows the average amount of child support received by mothers in each month relative to emancipation of the youngest eligible child. On emancipation, the amount of child support received drops, falling from 1,700 dollars per year in the month before emancipation to 800 dollars per year twelve months after emancipation. In the same figure, I show how three potentially confounding variables that are likely to change with relative age. The first is whether the child has left home, which potentially confounds because of changes in consumption needs (dash-dotted line). The second is the earnings of the children, since changes in labor supply could be due to intra-family substitution of labor (short-dashed line). The third is the amount of government transfers that depend on the presence of children (long-dashed line). Visually, all three variables have different levels before and after emancipation, but do not exhibit a discontinuity on emancipation, which supports the RD strategy. Appendix Table A7 formally shows that the first stage is statistically significant, and that the potential confounders do not change sharply on emancipation.[39]

5.4 Estimates of the intertemporal income effect

Table 7 shows estimates of the intertemporal income effect, the main estimates of interest in this section. The first two columns show estimates based on work hours, and the last two columns show estimates based on earnings, and the last four columns show estimates for the extensive margin. In my preferred specification, all estimates of the intertemporal income effect are not significantly different from zero. At the 5 percent level, we can reject estimates of 0.4 for the intensive margin (columns 2 and 4) and 0.1 for the extensive margin (columns 6 and 8). Several papers have found negative unanticipated income effects, which suggests that rational expectations hold for mothers in the child support setting. Appendix Table A8 shows estimates based on a level-level specification for earnings as well as estimates based on a regression of log outcome on the inverse hyperbolic sine of the child support received. Estimates in these specifications are also not significantly different from zero. Figure 5 and Appendix Table A9 show the the reduced form estimates graphically and formally.

Table 8 shows that the estimates are not significantly different from zero in several robustness specifications. The layout is similar to Table 3: each cell shows the estimate based on the change specified at the start of the row. The first three rows show standard robustness checks when using an RD design: doubling the bandwidth, halving the bandwidth, and increasing the polynomial order. The next two rows exclude observations around emancipation and just after emancipation respectively; this is motivated by shape of the child support receipt graph in Figure 4. Implicitly, estimates in these rows rely on an extrapolation assumption. In the last column, I allow heterogeneity across SIPP panels in the effect of emancipation

[39] In principle, fixed effects and covariates should be uncorrelated with IV_{it} conditional on the other RD variables (r_{it} and $r_{it}IV_{it}$) in an RD specification. Hence, I show results from a specification that includes only panel fixed effects alongside my preferred specfication.

on the support rate in the first stage, similar to my treatment of the instrument for fathers.

Appendix Figure A5 and Appendix Table A10 show results from a falsification test estimated on a sample of mothers whose oldest children are born after their last marriages, and thus should not be receiving child support for any of their children.[40] For these mothers, emancipation of the youngest child does not lead to changes in labor supply. Standard errors imply that the main reduced form estimates in Appendix Table A9 are not significantly different from the falsification estimates, which helps to reassure that the main results are not driven by a potential confounder associated with child emancipation.

6 Conclusion

In this paper, I obtain causal estimates of the Frisch elasticity based on child support. Because child support is dependent on the father's income, and because this dependence is known to fathers well in advance, the child support system provides a good setting for estimation of the Frisch elasticity. For identification, I exploit the fact that fathers are not legally required to make payments after all eligible children are emancipated in many jurisdictions. On emancipation of the youngest eligible child, the amount of child support paid by fathers as a percentage of income drops; the amount of income left over increases by 5 percent. At the same time, fathers increase the number of hours that they work and their annual earnings by 3 percent. Based on this, the estimated intensive-margin Frisch elasticity is 0.8 based on work hours, and 0.7 based on earnings, similar in magnitude to quasi-experimental in the literature. On the extensive margin, I obtain estimates of 0.2 based on work hours, significant at the 5 percent level, and a statistically insigificant 0.1 based on earnings. Modifying the above strategy for mothers, I do not find an intertemporal income effect, which is consistent with mothers smoothing the anticipated change in unearned income across emancipation.

To make sense of the difference between my estimates and others in the literature, it is useful to consider the ways in which the settings and methodology differ. First, the instrument I use depends on the ages of children not living with the father rather than the individual's age and education. The variation in labor prices induced by the instrument in this setting is large, and fathers likely understand this; these likely remove income effects in estimating the Frisch elasticity. The instrument here is also likely to be more exogenous with respect to taste shifters for labor. Second, estimates of the Frisch elasticity in this paper relies more on long run variation rather than short run variation in first difference

[40] This is analogous to the second falsification test done for fathers. I do not perform the first falsification test because the SIPP lacks information on middle children.

specifications. Point estimates that use only the short run variation are slightly smaller, which suggests that there could be some frictions in free adjustment of labor supply, but low statistical power limits the conclusion that we can draw. Third, estimation in this setting relies on an assumption that the father ignores the consumption of the mother or the child. I address this by considering estimates of the Frisch elasticity given various intensities of like or dislike for mother and child consumption, but better matched consumption data is required to investigate this properly.

The intensive-margin earnings-Frisch elasticity of 0.8 estimated in this paper is the intertemporal child support analog of the elasticity of taxable income with respect to the net of tax share in Feldstein [1999]. The pre-emancipation support rate in the 5 datasets is 7 percent, and the average earnings is $55,000 in 2016 US dollars. Applying Feldstein's deadweight loss formula ($DWL = 0.5\frac{s^2}{1-s}\gamma wh$, where notation follows that introduced in Section 3), we obtain a DWL of $120 per father. This number is a lower bound on the deadweight loss for two reasons. First the formula is sensitive to the support rate used, and the pre-emancipation support rate estimated in this paper is attenuated by misclassification of child eligibility. Second, this calculations ignores interactions with the tax system, which would raise deadweight loss.

Data acknowledgements:

[PSID] The collection of data used in this study was partly supported by the National Institutes of Health under grant number R01 HD069609 and R01 AG040213, and the National Science Foundation under award numbers SES 1157698 and 1623684.

[NLSY] The NLSY79 survey is sponsored and directed by the U.S. Bureau of Labor Statistics and conducted by the Center for Human Resource Research at The Ohio State University. Interviews are conducted by the National Opinion Research Center at the University of Chicago.

[BHPS+] Understanding Society and BHPS are funded by the Economic and Social Research Council and various Government Departments, with scientific leadership by the Institute for Social and Economic Research, University of Essex, and survey delivery by NatCen Social Research and Kantar Public. The research data are distributed by the UK Data Service.

[HILDA] This paper uses unit record data from the Household, Income and Labour Dynamics in Australia (HILDA) Survey. The HILDA Survey was initiated and is funded by the Australian Government Department of Social Services (DSS) and is managed by the Melbourne Institute of Applied Economic and Social Research (Melbourne Institute). The findings and views reported in this paper, however, are those of the author and should not be attributed to the Australian Government, DSS or the Melbourne Institute.

[SHP] This study has been realized using the data collected by the Swiss Household Panel (SHP), which is based at the Swiss Centre of Expertise in the Social Sciences FORS. The project is financed by the Swiss National Science Foundation.

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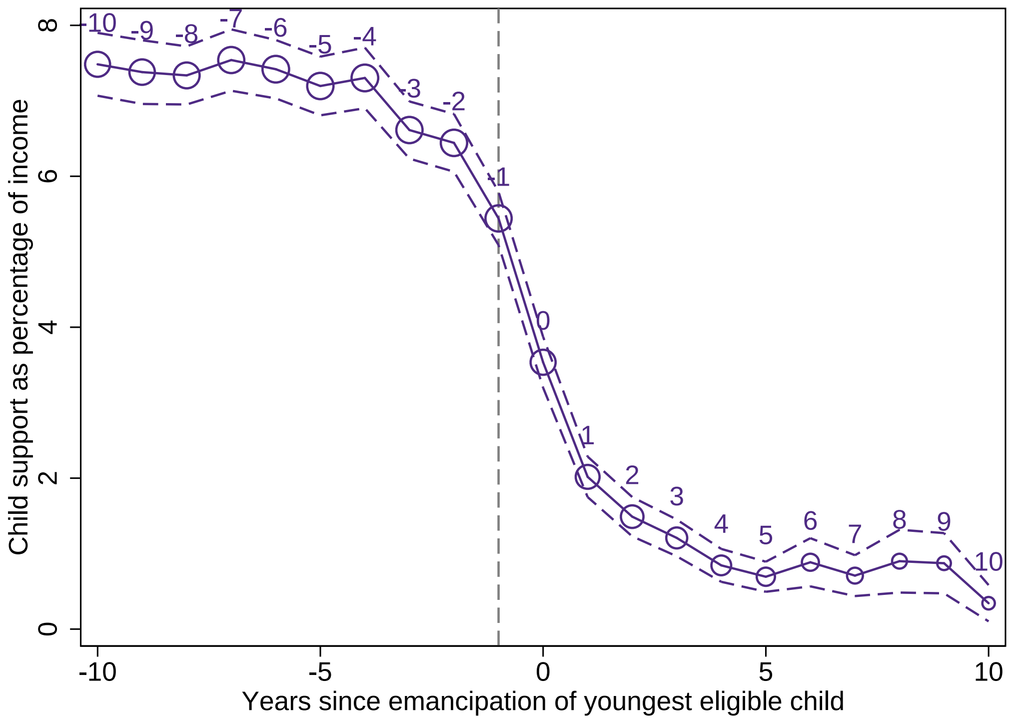
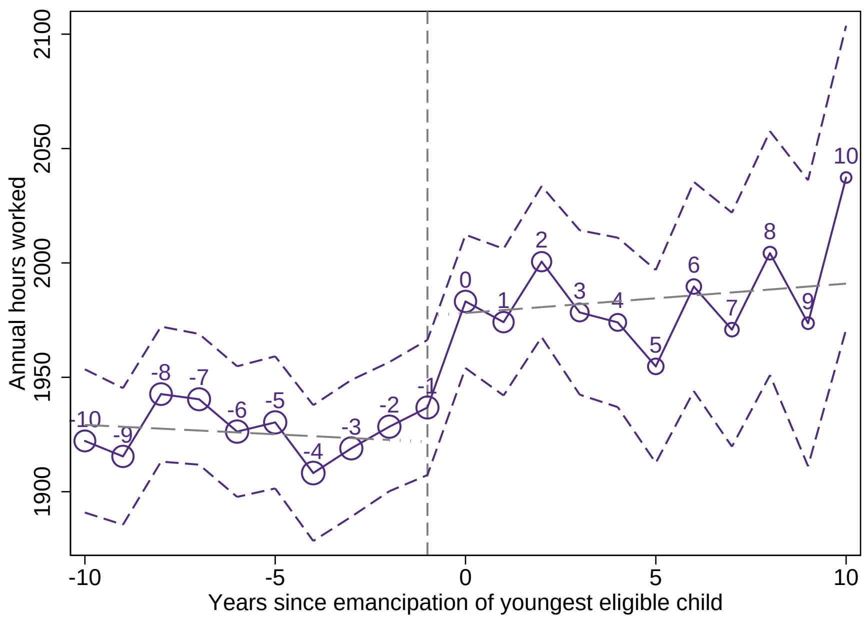
Figure 1: Average child support rate in each year relative to emancipation



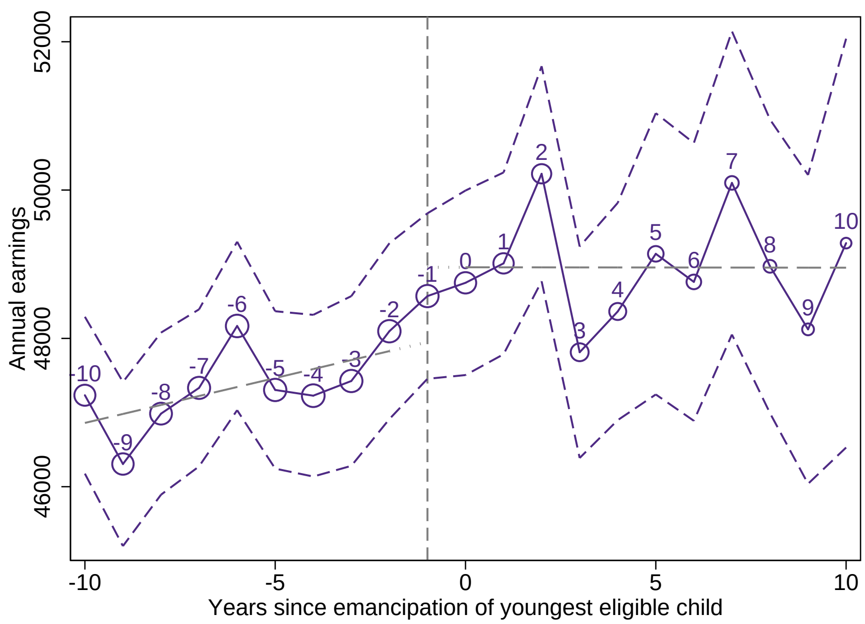
Notes: The sample comprises all father-years with non-missing child support rates. Each point on the figure is the average support rate in the year relative to emancipation of the youngest eligible child. Marker size is proportional to number of observations of each cell. Dashed line shows 95% confidence interval of the estimated mean. Vertical dashed line marks the year before emancipation.

Figure 2: Average residualized hours worked and earnings among fathers in each year relative to emancipation

(a) Hours worked



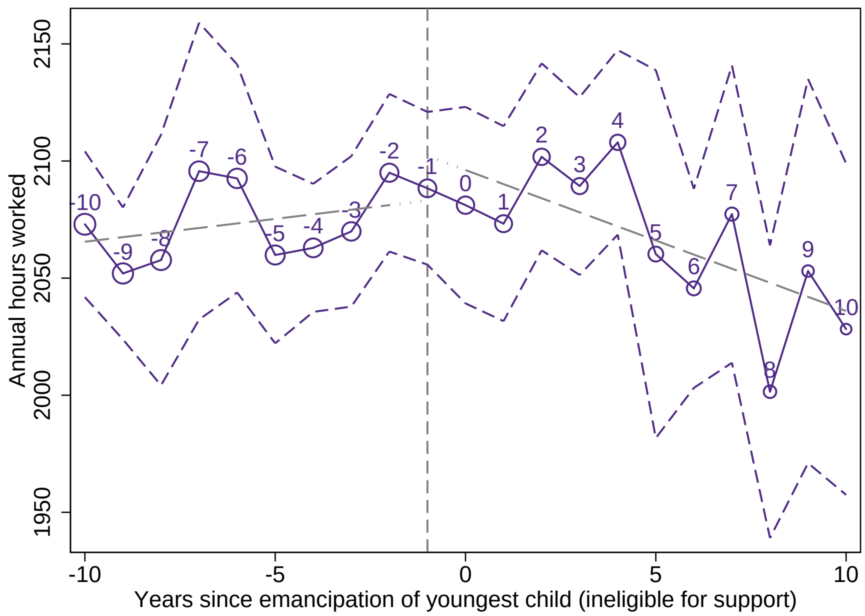
(b) Earnings



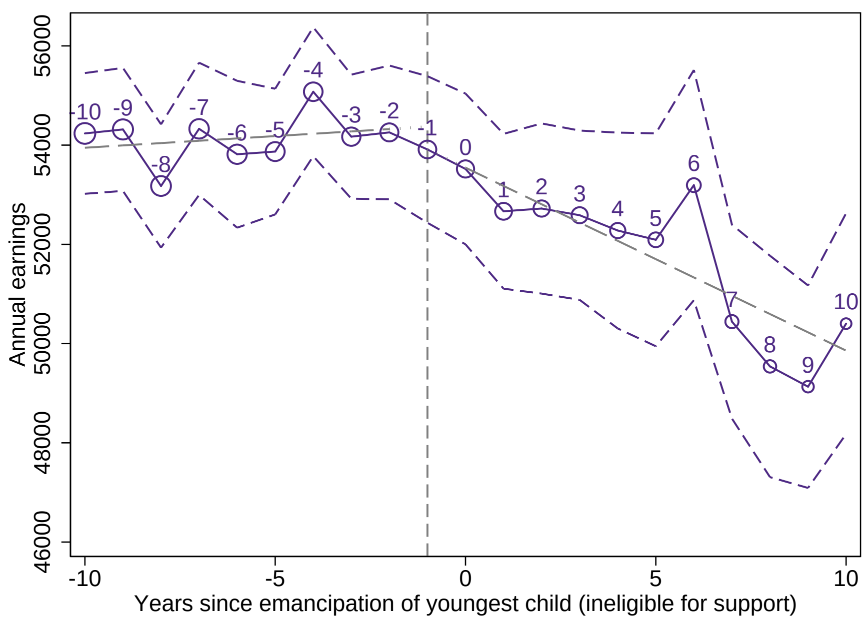
Notes: The sample comprises all father-years with non-missing child support rates. Each point on each figure is the average residualized annual hours worked or earnings (including zero hours or dollars) in the year relative to emancipation of the youngest eligible child, after individual and age fixed effects are partialled out from the outcome. Marker size is proportional to number of observations of each cell. Dashed line shows 95% confidence interval of the estimated mean. Vertical dashed line marks the year before emancipation. Horizontal dashed lines are fitted linear predictions before and after the year before emancipation respectively.

Figure 3: Falsification sample: Average residualized hours worked and earnings in each year relative to emancipation of youngest child among fathers who do not have child support obligations

(a) Hours worked

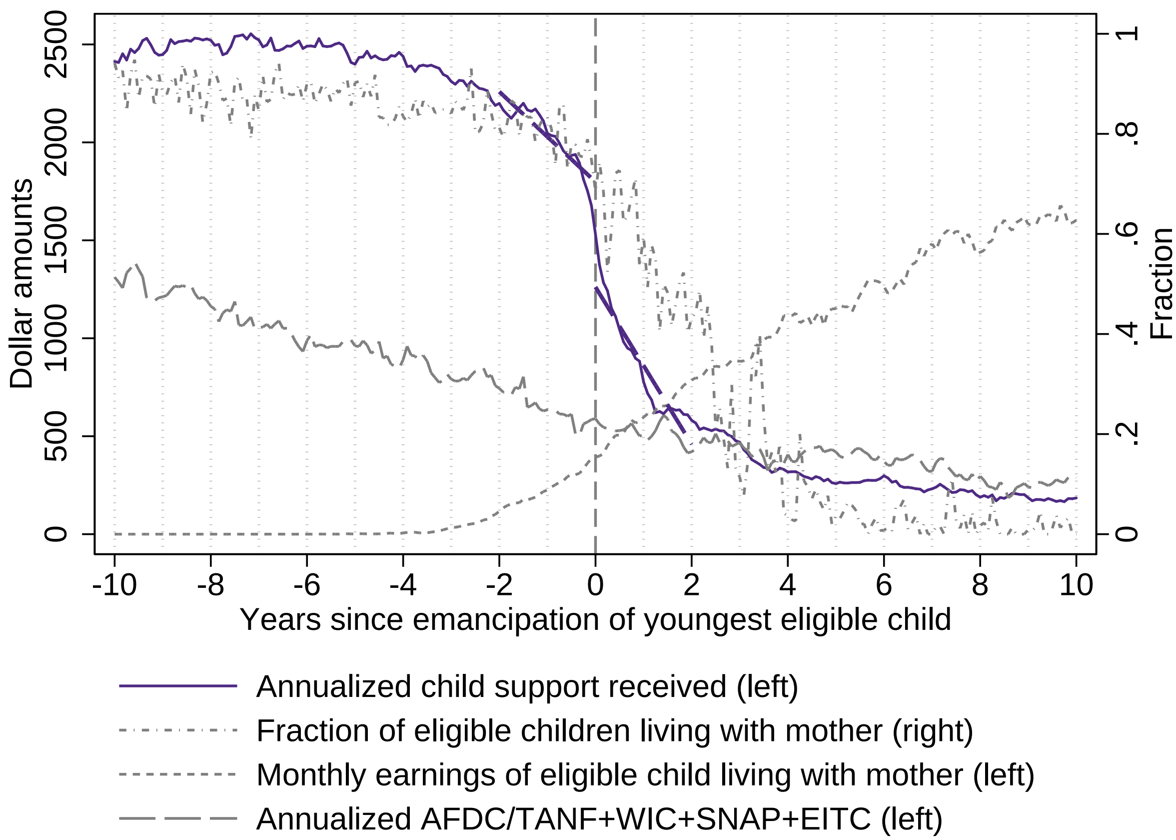


(b) Earnings



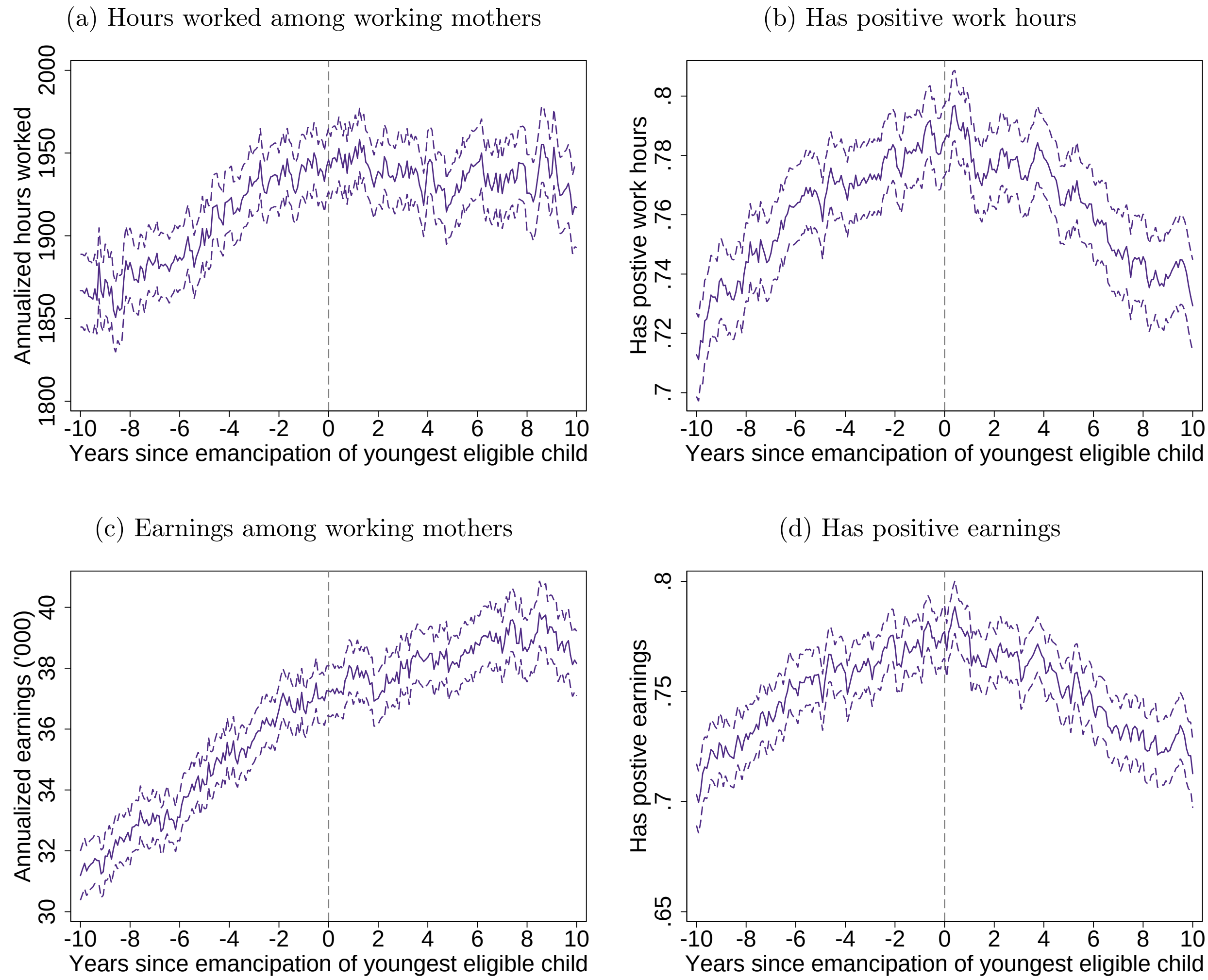
Notes: The sample comprises all father-years for fathers who do not have child support obligations. Each point on each figure is the propensity-score weighted average residualized annual hours worked or earnings (including zero hours or dollars) in the year relative to emancipation of the youngest child, after individual and age fixed effects are partialled out from the outcome. Propensity score weights are the the weights used in the last panel of Table 5. Marker size is proportional to propensity-score weight and number of observations of each cell. Dashed line shows 95% confidence interval of the estimated mean. Vertical dashed line marks the year before emancipation. Horizontal dashed lines are fitted linear predictions before and after the year before emancipation respectively.

Figure 4: First stage of RD for mothers



Notes: The sample comprises all mother-year-months with non-missing child support rates. Each point on the figure is the average value (among non-missing observations) for all mothers at the specified month relative to emancipation of the child. Vertical dashed line marks the month of emancipation.

Figure 5: Average hours worked and earnings among mothers in each month relative to emancipation



Notes: The sample comprises all mother-year-months with non-missing child support rates. Each point on the figure is the average value (among non-missing observations) for all mothers at the specified month relative to emancipation of the child. Vertical dashed line marks the month of emancipation.

Table 1: Effect of emancipation of the youngest eligible child on the child support rate

	Dependent variable: Log of 100% − CS rate					
	PSID (USA)	NLSY (USA)	BHPS+ (GBR)	HILDA (AUS)	SHP (CHE)	Pooled
	(1)	(2)	(3)	(4)	(5)	(6)
Post-emancipation	0.033***	0.064***	0.037***	0.041***	0.054***	0.045***
	(0.0029)	(0.0049)	(0.0041)	(0.0035)	(0.016)	(0.0021)
Observations	13,527	8,821	4,126	8,019	1,623	36,116
No. of fathers	2,030	1,128	544	1,013	254	4,969
Mean pre-threshold CS rate	5.6	10.4	6.5	6.5	15.4	7.3
F-statistic on instrument	130	175	79	135	12	480
Emancipation age	18–21	18–21	16	18	18	16–21

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample for each column is composed of all father-years in the dataset specified in the column title; country is indicated in parentheses. CS rate is the child support rate (in percentages), computed as the ratio of the child support amount paid to the income of the father. Post-emancipation is an indicator variable that takes a value of one if the youngest eligible child is at emancipation age or older, half in the year before emancipation, and zero if younger. All specifications include as control variables individual fixed effects, dataset-year fixed effects, age-education-dataset fixed effects, and the log of the hourly wage interacted with dataset indicators.

Table 2: OLS and IV estimates of the Frisch elasticity

	Dependent variable:							
	Log of work hours		Log of earnings		Has positive work hours		Has positive earnings	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Panel A: OLS							
Log(100% − CS rate)	0.11***	0.15***	0.74***	0.60***	-0.00070	-0.0059	0.013	0.013
	(0.041)	(0.042)	(0.068)	(0.063)	(0.021)	(0.016)	(0.022)	(0.019)
Observations	32,836	32,831	33,441	33,436	36,246	36,241	36,399	36,394
No. of fathers	4,729	4,729	4,800	4,800	4,995	4,995	5,009	5,009
Mean hours/earnings/frac.	2212.1	2212.2	53977.3	53977.0	0.91	0.91	0.92	0.92
	Panel B: 2SLS							
Log(100% − CS rate)	0.79***	0.76***	0.81**	0.72**	0.32**	0.21**	0.21*	0.11
	(0.21)	(0.23)	(0.32)	(0.30)	(0.13)	(0.098)	(0.12)	(0.10)
Observations	32,423	32,418	33,024	33,019	35,793	35,788	35,944	35,939
No. of fathers	4,686	4,686	4,756	4,756	4,949	4,949	4,963	4,963
Mean hours/earnings/frac.	2214.1	2214.2	53900.8	53900.6	0.91	0.91	0.92	0.92
First stage F-statistic	118	102	122	105	123	106	124	107
Individual & year FEs	x	x	x	x	x	x	x	x
Other controls		x		x		x		x

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years. Work hours are the annual work hours of the father. Earnings are the annual gross income from work of the father, in 2016 US dollars; currency conversions are based on 2016 exchange rates. CS rate is the child support rate (in percentages), computed as the ratio of the child support amount paid to the income of the father. Instruments used are the post-emancipation variable from Table 1 interacted with dataset indicators. Individual & year FEs are individual fixed effects and dataset-year fixed effects. Other controls are age-education-dataset fixed effects, and the log of the hourly wage interacted with dataset indicators.

Table 3: Frisch estimates based on robustness specifications

	Dependent variable:			
	Log of work hours	Log of earnings	Has positive work hours	Has positive earnings
	(1)	(2)	(3)	(4)
Main estimates	0.76***	0.72**	0.21**	0.11
	(0.23)	(0.30)	(0.098)	(0.10)
Exclude three years before emancipation	0.66***	0.66**	0.20*	0.043
	(0.25)	(0.32)	(0.11)	(0.11)
Regression discontinuity specification	0.55	0.90	0.24	0.57*
	(0.60)	(0.76)	(0.27)	(0.30)
Exclude imputed support rate	0.71***	0.74**	0.16*	0.049
	(0.23)	(0.31)	(0.093)	(0.084)
Exclude Switzerland	0.80***	0.74**	0.22**	0.13
	(0.24)	(0.32)	(0.10)	(0.11)
Exclude divorces after child age 10	0.73***	0.74**	0.21*	0.16
	(0.27)	(0.36)	(0.12)	(0.12)
IV uses full support-age variation	0.51**	0.52*	0.25***	0.16
	(0.22)	(0.29)	(0.096)	(0.099)

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years. Work hours are the annual work hours of the father. Earnings are the annual gross income from work of the father, in 2016 US dollars; currency conversions are based on 2016 exchange rates. Each cell is the result of a 2SLS regression of the dependent variable on log of 100% less the child support rate, with all controls from Table 2 included. Except in the last row, instruments used are the post-emancipation variable from Table 1 interacted with dataset indicators. Row 1: Estimates are the same as in Table 2. Row 2: The sample excludes three years before emancipation of the youngest eligible child. Row 3: The sample is restricted to two intervals: five to two years before emancipation, and one to four years after emancipation. The specification additionally includes the running variable—age of the youngest eligible child less emancipation age plus one (to center the running variable around the year before emancipation)—and the running variable interacted with post-emancipation indicator, both interacted with dataset indicators. Row 4: The sample excludes observations for which the child support rate were imputed. Row 5: The sample excludes the Swiss Household Panel. Row 6: The sample excludes observations for which the month of divorce occurs after the youngest eligible child turns 10. Row 7: The instrument is $\log(1-\bar{s}_{it})$, where $\bar{s}_{it} = \sum_j \mathbb{1}\left[j \neq i, relage_{jt} = relage_{it}, d(j) = d(i)\right] s_{jt}$ is the leave-self-out average of the child support rate s_{it} over all observations with the same child age relative to emancipation $relage_{it}$ and in the same dataset $d(i)$.

Table 4: Heterogeneity by initial socioeconomic conditions

	Dependent variable: Log of work hours or earnings. Heterogeneity variable is:					
	Pre-emancipation wage below median		Did not complete high school		Minority race	
	Work hours	Earnings	Work hours	Earnings	Work hours	Earnings
	(1)	(2)	(3)	(4)	(5)	(6)
Log(100% − CS rate)	0.75***	0.76***	0.68***	0.19	0.43*	0.40
	(0.23)	(0.29)	(0.24)	(0.31)	(0.23)	(0.30)
Log(100% − CS rate) × het. variable	0.15	0.16	0.51	3.11***	0.91***	0.65*
	(0.27)	(0.34)	(0.64)	(0.97)	(0.32)	(0.38)
Observations	31,442	31,963	32,418	33,019	29,573	30,130
No. of fathers	4,462	4,512	4,686	4,756	4,254	4,322
Mean hours/earnings	2,217	54,006	2,214	53,901	2,212	51,107
First stage F-statistic	40	42	10	8	43	47

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years, unless the heterogeneity variable is not available. Columns 1 and 2: The heterogeneity variable is an indicator variable equal to 1 if the father's pre-emancipation wage is below the median among all fathers in each dataset. Fathers with no wage observations before emancipation are excluded. Columns 3 and 4: The heterogeneity variable is an indicator variable equal to 1 if the father did not complete high school or equivalent. Columns 5 and 6: The heterogeneity variable is an indicator variable equal to 1 if the father is nonwhite in the PSID and NLSY, nonwhite and non-British in the BHPS+, and of Aboriginal or Torres Strait Islander origin in the HILDA. The SHP has no clear race or ethnicity variable and is excluded. Estimates shown are based on 2SLS specifications in which IVs are fully interacted with the heterogeneity variable. All columns include all controls in Table 2, and definitions not specified are the same as in that table.

Table 5: Falsification regressions

	Dependent variable:			
	Log of work hours	Log of earnings	Has positive work hours	Has positive earnings
	(1)	(2)	(3)	(4)
Panel A: Sample: Fathers with CS obligations (main sample)				
Post-emancipation of child support child	0.031***	0.031**	0.012***	0.0069
	(0.010)	(0.014)	(0.0045)	(0.0049)
Observations	33,087	33,537	38,102	38,242
No. of fathers	4,781	4,881	5,186	5,257
Mean hours/earnings/fraction	2212.1	53679.7	0.87	0.88
Panel B: Sample: Main sample with subsequent children				
Post-emancipation of child support child	0.030	0.060*	0.011	0.015
	(0.026)	(0.033)	(0.0097)	(0.010)
Post-emancipation of ineligible child	0.022	0.019	-0.0093	0.0076
	(0.031)	(0.041)	(0.012)	(0.013)
Observations	7,014	7,115	8,210	8,234
No. of fathers	1,035	1,053	1,126	1,142
Mean hours/earnings/fraction	2218.6	50604.7	0.86	0.87
Panel C: Sample: Fathers with no CS obligations (unweighted)				
Post-emancipation of ineligible child	-0.0024	-0.0073	0.000012	0.00071
	(0.0060)	(0.0088)	(0.0032)	(0.0032)
Observations	105,923	107,141	113,929	115,637
No. of fathers	12,776	13,296	13,385	14,018
Mean hours/earnings/fraction	2295.0	69655.7	0.93	0.93
Panel D: Sample: Fathers with no CS obligations (weighted)				
Post-emancipation of ineligible child	-0.013	-0.0089	-0.00036	-0.00033
	(0.012)	(0.015)	(0.0051)	(0.0066)
Observations	105,923	107,141	113,929	115,637
No. of fathers	12,776	13,296	13,385	14,018
Mean hours/earnings/fraction	2264.2	58224.5	0.91	0.91

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. Panel A: The sample is the main sample from Table 2. Panel B: The sample is the main sample restricted to fathers whose youngest children are not eligible for child support. Panel C and D: The sample comprises all father-years for fathers whose oldest children are born at least one year after the fathers' last marriages, and whose last marriages are intact at the last interview. Post-emancipation of child support child and ineligible child are indicator variables equal to one if the indicated child is at emancipation age or older, half in the year before emancipation, and zero if younger. The child support child is the youngest child eligible for child support used in the main tables, and the ineligible child is the youngest child of the father. Dependent variables are those used in Table 2, and all control variables in that table are included. Weights in panel D are propensity score weights computed separately by dataset using the procedure in DiNardo et al. [1996]. Propensity scores are based on a probit regression of being a father with an eligible child on the following covariates: indicators for birth year deciles fully interacted with a linear function in birth year, indicators for the education level, indicators for age at first birth in decades fully interacted with a linear function in the age at first birth, indicators for the number of children by age 30, the log of average earnings between 25 and 30, and the log of average work hours between 25 and 30. For each of the last two, missing values are imputed with a constant and an indicator variable is included in parallel.

Table 6: Sensitivity of the Frisch elasticity to including child and mother consumption in father's utility

Intensity of like for child consumption ($\kappa^*_{\mathfrak{c}}$)	Intensity of like for mother consumption ($\kappa^*_{\mathfrak{m}}$)										
	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5
0	0.41***	0.45***	0.58***	0.64***	0.70***	0.76***	0.83***	0.92***	1.0***	1.1***	1.2***
	(0.15)	(0.15)	(0.17)	(0.19)	(0.21)	(0.23)	(0.25)	(0.27)	(0.30)	(0.34)	(0.38)
0.1	0.43***	0.54***	0.60***	0.66***	0.72***	0.79***	0.87***	0.95***	1.1***	1.2***	1.3***
	(0.15)	(0.16)	(0.18)	(0.19)	(0.21)	(0.24)	(0.26)	(0.29)	(0.32)	(0.35)	(0.40)
0.2	0.44***	0.57***	0.63***	0.69***	0.75***	0.82***	0.90***	0.99***	1.1***	1.2***	1.4***
	(0.15)	(0.16)	(0.18)	(0.20)	(0.22)	(0.25)	(0.27)	(0.30)	(0.33)	(0.37)	(0.42)
0.3	0.52***	0.59***	0.65***	0.72***	0.78***	0.86***	0.94***	1.0***	1.2***	1.3***	1.5***
	(0.15)	(0.17)	(0.19)	(0.21)	(0.23)	(0.26)	(0.28)	(0.31)	(0.35)	(0.39)	(0.44)
0.4	0.56***	0.62***	0.68***	0.74***	0.81***	0.89***	0.98***	1.1***	1.2***	1.4***	1.5***
	(0.16)	(0.18)	(0.20)	(0.22)	(0.24)	(0.27)	(0.29)	(0.33)	(0.37)	(0.41)	(0.47)
0.5	0.58***	0.64***	0.71***	0.77***	0.85***	0.93***	1.0***	1.1***	1.3***	1.4***	1.6***
	(0.17)	(0.19)	(0.21)	(0.23)	(0.25)	(0.28)	(0.31)	(0.34)	(0.38)	(0.44)	(0.50)

Notes: Standard errors in parentheses are for the last iteration of the procedure, and are clustered by individual. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. $\kappa^*_{\mathfrak{m}}$ and $\kappa^*_{\mathfrak{c}}$ capture the intensity of like for mother or child consumption out of child support, respectively. Highlighted cells are the values that are consistent with the bounding on $\kappa^*_{\mathfrak{m}}$ and $\kappa^*_{\mathfrak{c}}$ provided by the statutory child support rate.

Table 7: Estimates of the intertemporal income effect for mothers

	Dependent variable:							
	Log of work hours		Log of earnings		Has positive work hours		Has positive earnings	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log(100% − CS rate)	0.13	0.034	0.57	-0.13	0.16	0.037	0.11	0.0029
	(0.25)	(0.17)	(0.49)	(0.29)	(0.12)	(0.043)	(0.12)	(0.050)
Observations	168,744	168,467	171,938	171,621	215,697	215,433	222,891	222,619
No. of mothers	9,367	9,098	9,456	9,146	10,837	10,579	10,928	10,662
Mean hours/earnings/fraction	1939.9	1940.2	37012.9	37024.7	0.78	0.78	0.77	0.77
First stage F-stat.	18	25	41	29	11	11	12	11
All controls		x		x		x		x

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all mother-year-months. Work hours are the annualized work hours of the mother. Earnings are the annualized gross income from work of the mother, in 2016 US dollars. CS rate is the ratio of child support amount received to the income of the mother. The specification used is a uniform-kernel local-linear fuzzy RD design with the month relative to the emancipation month as the running variable. The bandwidth used is two years (inclusive) on each side of the emancipation month; the emancipation month itself is excluded. The linear specification is allowed to differ pre- and post-emancipation, and across panels. All columns include a panel fixed effect. All controls are individual fixed effects, panel-year-month fixed effects, age-education-panel fixed effects, and the log of the hourly wage interacted with panel indicators. Sample sizes and numbers of clusters reported are effective numbers used in the RD.

Table 8: Intertemporal income effect estimates for mothers based on robustness specifications

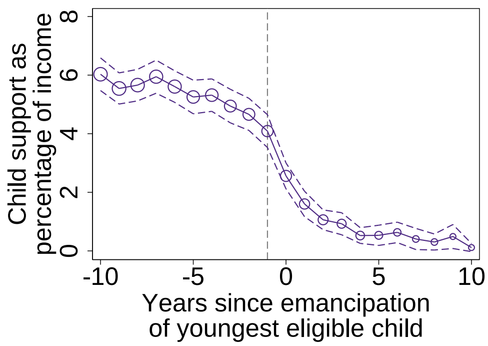
	Dependent variable:			
	Log of work hours	Log of earnings	Has positive work hours	Has positive earnings
	(1)	(2)	(3)	(4)
Double bandwidth	0.11	-0.18	0.00014	-0.013
	(0.15)	(0.26)	(0.025)	(0.035)
Halve bandwidth	-0.41	-0.74	0.025	-0.024
	(0.30)	(0.54)	(0.067)	(0.051)
Local-quadratic RD	-0.59*	-1.44	0.22	0.078
	(0.35)	(0.95)	(0.44)	(0.22)
Exclude 6 months on both sides of emancipation	0.16	0.022	-0.0043	0.016
	(0.25)	(0.30)	(0.032)	(0.039)
Exclude 12 months after emancipation	0.016	-0.17	-0.0047	-0.019
	(0.22)	(0.26)	(0.031)	(0.039)
IV fully interacted with SIPP panel	0.019	-0.14	0.027	0.030
	(0.15)	(0.25)	(0.028)	(0.024)

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all mother-year-months. Work hours are the annualized work hours of the mother. Earnings are the annualized gross income from work of the mother, in 2016 US dollars. Each cell is the result of a fuzzy RD design analogous to that in Table 7, with the specified change in specification. Row 1: Estimates are the same as in Table 7. Row 2: The bandwidth used is four years (inclusive) on each side of the emancipation month; the emancipation month itself is excluded. Row 3: The bandwidth used is one year (inclusive) on each side of the emancipation month; the emancipation month itself is excluded. Row 4: The specification is a local-quadratic fuzzy RD design. The quadratic specification is allowed to differ pre- and post-emancipation, and across panels. Row 5: The bandwidth used is 2.5 years (inclusive) on each side of the emancipation month; the sample excludes the six months (inclusive) on each side of the emancipation month. Row 6: The bandwidth used is 2 years before and three years after the emancipation month; the sample excludes observations between 0 and 1 year (inclusive) of the emancipation month. Row 7: The instrument is interacted with panel fixed effects in the first stage. All cells include all controls in Table 7, and definitions not specified are the same as in that table.

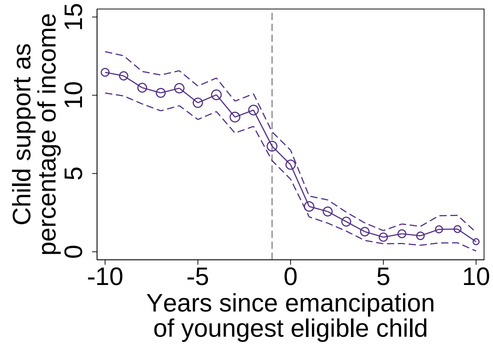
A Appendix tables and figures

Appendix Figure A1: Average child support rate in each year relative to emancipation, by dataset

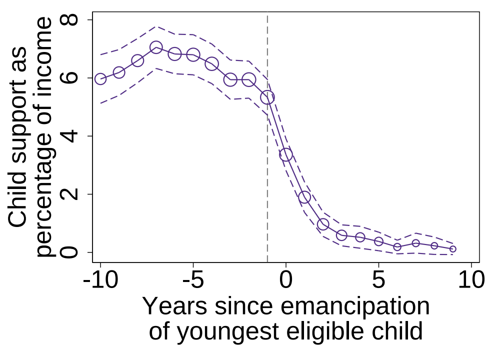
(a) PSID



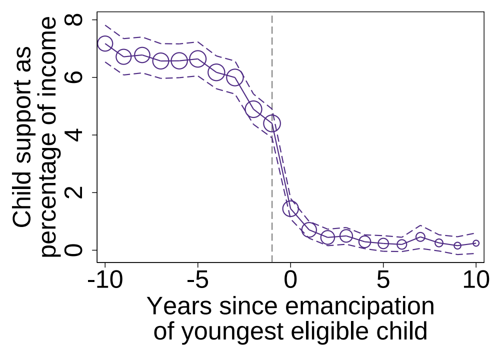
(b) NLSY



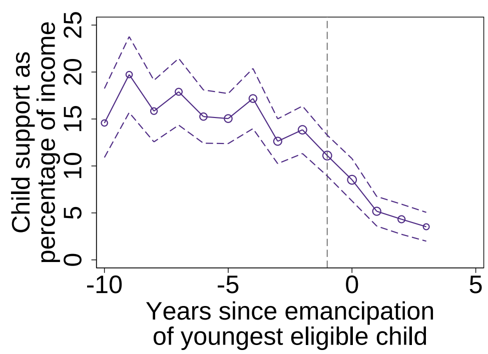
(c) BHPS+



(d) HILDA



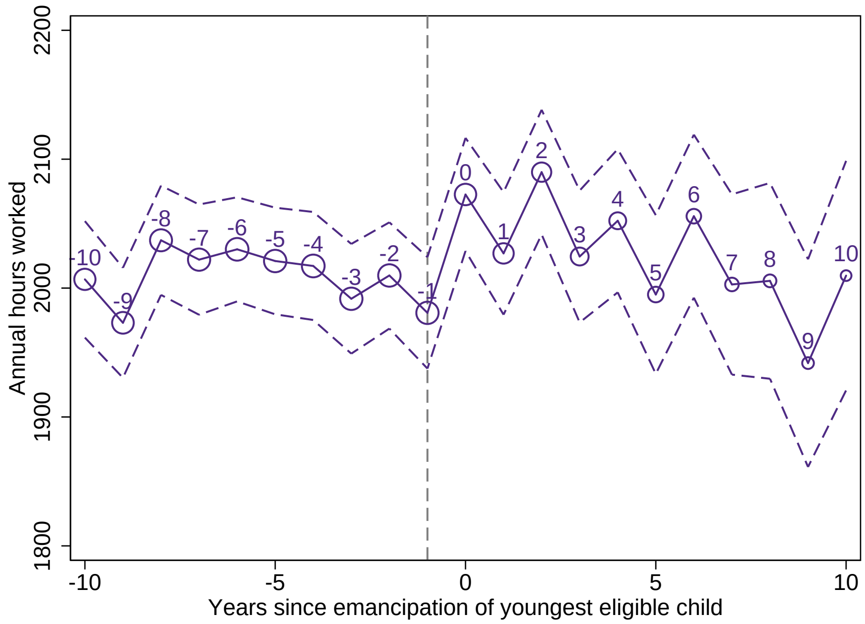
(e) SHP



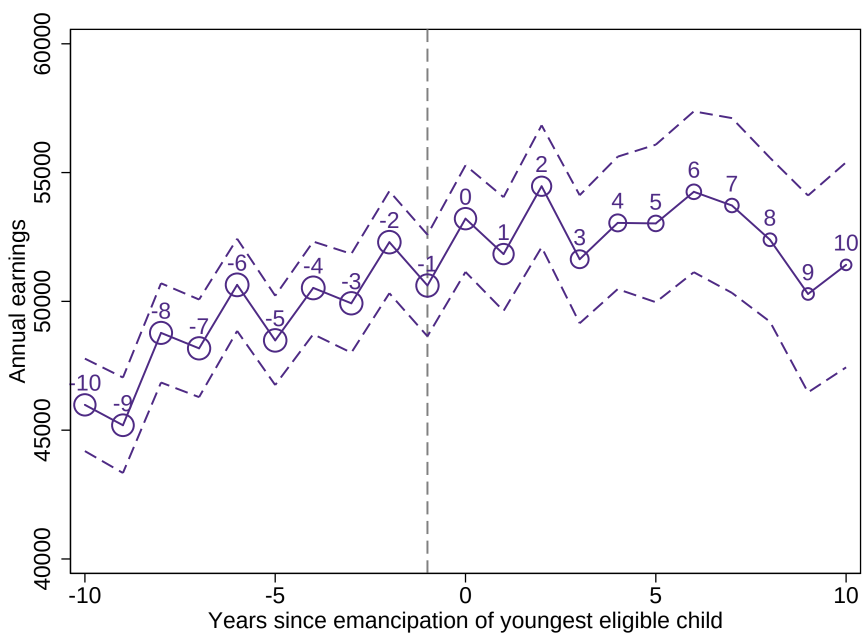
Notes: The sample comprises all father-years with non-missing child support rates. Each point on the figure is the average support rate in the year relative to emancipation of the youngest eligible child in the specified dataset. Marker size is proportional to number of observations of each cell. Dashed line shows 95% confidence interval of the estimated mean. Cells with fewer than 50 observations are not shown. Vertical dashed line marks the year before emancipation.

Appendix Figure A2: Average (non-residualized) hours worked and earnings among fathers in each year relative to emancipation

(a) Hours worked



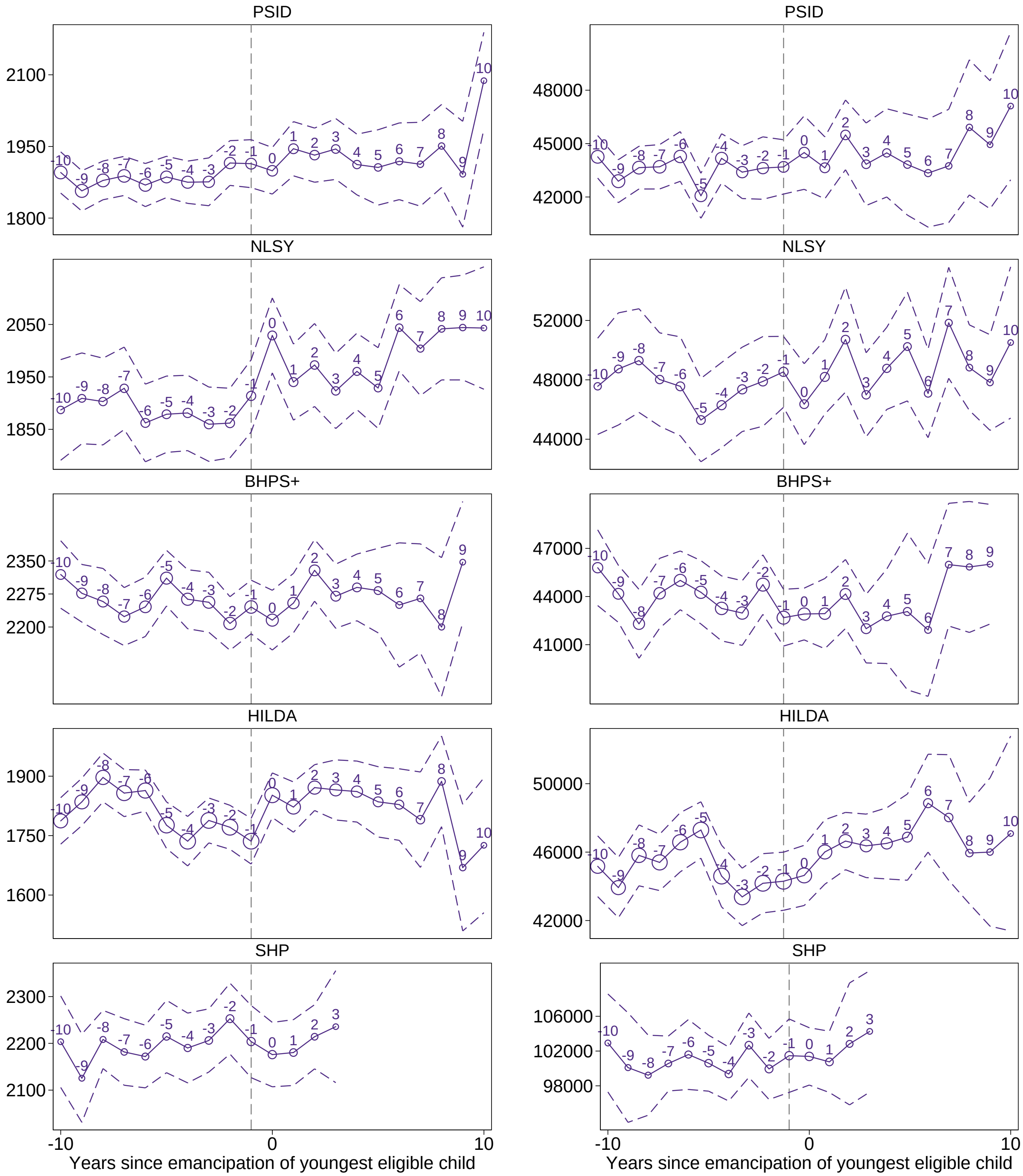
(b) Earnings



Notes: The sample comprises all father-years with non-missing child support rates. Each point on each figure is the average annual hours worked or earnings (including zero hours or dollars) in the year relative to emancipation of the youngest eligible child. Marker size is proportional to number of observations of each cell. Dashed line shows 95% confidence interval of the estimated mean. Vertical dashed line marks the year before emancipation.

Appendix Figure A3: Average residualized hours worked and earnings among fathers in each year relative to emancipation, by dataset

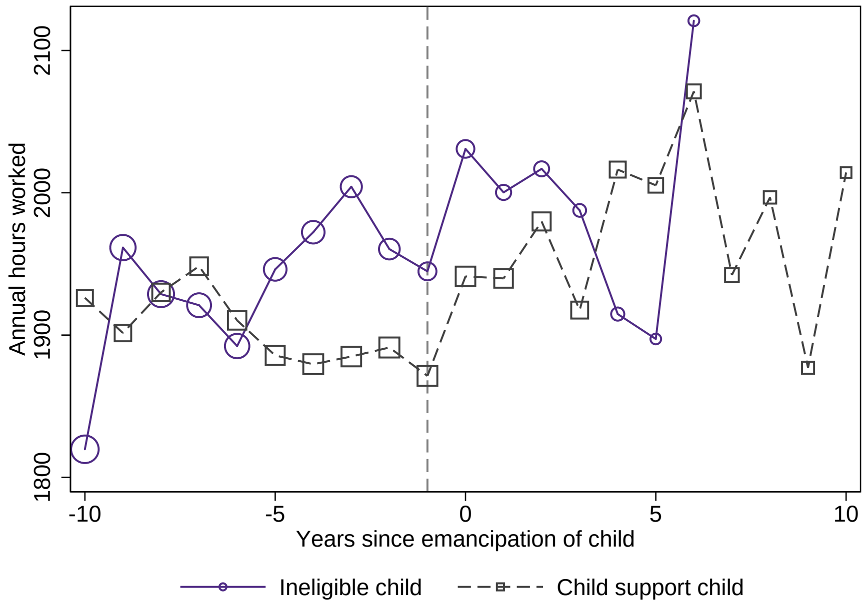
(a) Hours worked (b) Earnings



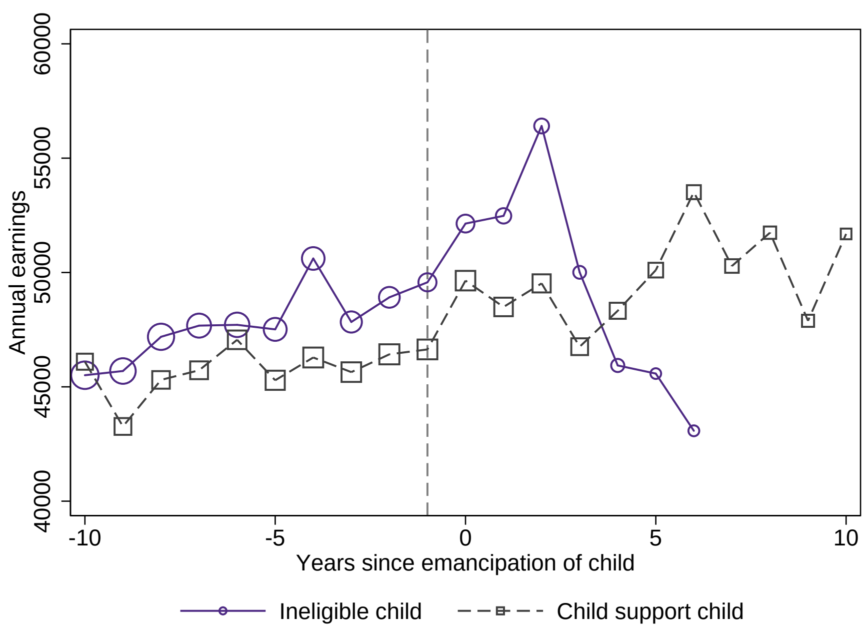
Notes: The sample comprises all father-years with non-missing child support rates. Each point on each figure is the average residualized annual hours worked or earnings (including zero hours or dollars) in the year relative to emancipation of the youngest eligible child for observations in the specified dataset, after individual and age fixed effects are partialled out from the outcome. Marker size is proportional to number of observations of each cell. Dashed line shows 95% confidence interval of the estimated mean. Cells with fewer than 50 observations are not shown. Vertical dashed line marks the year before emancipation.

Appendix Figure A4: Falsification sample: Average residualized hours worked and earnings among fathers in each year relative to emancipation of noneligible and eligible children

(a) Hours worked



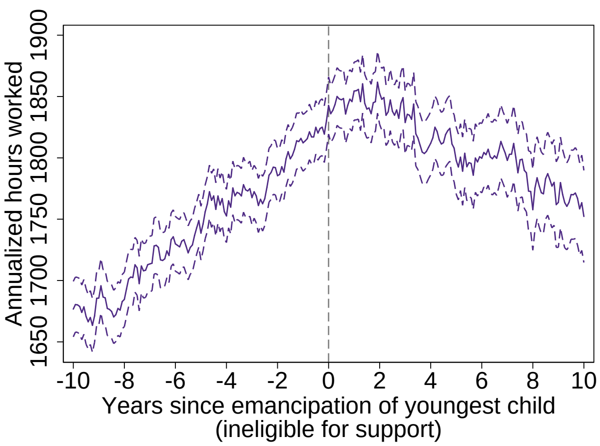
(b) Earnings



Notes: The sample comprises all father-years with non-missing child support rates for fathers who have at least one eligible child and whose youngest children are never eligible for child support. Each point on each figure is the average residualized annual hours worked or earnings (including zero hours or dollars) in the year relative to emancipation of the youngest child and youngest eligible child respectively, after individual and age fixed effects are partialled out from the outcome. Marker size is proportional to number of observations of each cell. Cells with fewer than 50 observations are not shown. Vertical dashed line marks the year before emancipation.

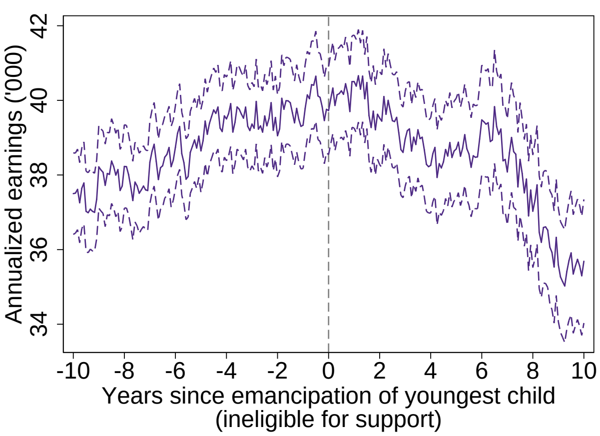
Appendix Figure A5: Falsification sample: Average hours worked and earnings in each month relative to emancipation of youngest child among mothers who should not be receiving support

(a) Hours worked among working mothers

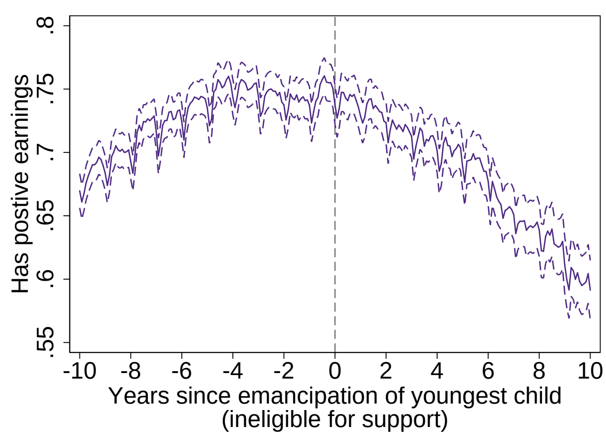


(b) Has positive work hours

(c) Earnings among working mothers



(d) Has positive earnings



Notes: The sample comprises all mother-year-months for mothers whose oldest children are born after the mother's last marriage, and whose last marriages are intact at the last interview. Each point on the figure is the average value (among non-missing observations) for all mothers at the specified month relative to emancipation of the child. Vertical dashed line marks the month of emancipation.

Appendix Table A1: Characteristics of fathers in the sample and for all men in the datasets

	Fathers with support obligations	All men in the five datasets		P-value for test of difference
		Unweighted	Weighted	
	(1)	(2)	(3)	(4)
Number of individuals				
Full sample	6,857	51,203	47,575	
Observed before age 30	4,034	25,280	22,823	
Economic characteristics between ages 25 and 30				
Any work hours (%)	94	96	96	0.00
	[23]	[19]	[19]	
Average hourly wage (USD/hour)	14	19	19	0.00
	[14]	[45]	[40]	
Average annual earnings (USD)	33,900	40,600	44,500	0.00
	[22,800]	[27,400]	[28,600]	
Average annual work hours	1,820	1,960	1,990	0.00
	[826]	[761]	[747]	
Demographic characteristics				
Birth year	1965	1964	1965	0.11
	[11]	[14]	[13]	
Completed high school (%)	80	85	87	0.00
	[40]	[36]	[34]	
Has college education (%)	28	38	40	0.00
	[45]	[49]	[49]	
Age at first marriage (if ever married)	24	25	24	0.00
	[9.3]	[10]	[11]	
Age at birth of first child (if a father)	25	28	28	0.00
	[5]	[6]	[6]	
Number of children by age 30	1.7	0.89	0.83	0.00
	[1.2]	[1.2]	[1.1]	

Notes: Standard deviations reported in brackets below means. The sample comprises all men aged 26 to 59, with the dataset-specific restrictions specified in the data appendix. Each observation is an individual. The sample of fathers with support obligations is the main sample used in the text. Weighted counts exclude observations with zero weight. Weighted means and standard deviations are cross-sectional sampling weights, normalized to place equal weight on every dataset. Weights for the NLSY are for 1979; weights for all other datasets are for the last observation of each individual. P-values reported in column 4 are for a test of difference in means between fathers with support obligations and all other men. Dollar values are in 2016 US dollars.

Appendix Table A2: Estimates of the Frisch elasticity by dataset (intensive margin)

	PSID (USA)	NLSY (USA)	BHPS+ (GBR)	HILDA (AUS)	SHP (CHE)	Pooled
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Dependent variable: Log of work hours						
Log(100% − CS rate)	0.51	0.99***	0.44	0.65	0.14	0.76***
	(0.59)	(0.35)	(0.46)	(0.48)	(0.51)	(0.23)
Observations	12,647	7,935	3,830	6,450	1,556	32,418
No. of fathers	1,963	1,095	523	857	248	4,686
Mean hours	2,116	2,270	2,364	2,235	2,276	2,214
First stage F-stat.	119	165	82	128	12	102
Panel B: Dependent variable: Log of earnings						
Log(100% − CS rate)	0.36	0.71	-0.080	1.54*	0.31	0.72**
	(0.70)	(0.44)	(0.55)	(0.83)	(0.49)	(0.30)
Observations	12,540	8,105	3,979	6,876	1,519	33,019
No. of fathers	1,958	1,113	537	904	244	4,756
Mean earnings	49,037	55,539	46,386	53,249	107,940	53,901
First stage F-stat.	119	178	85	133	10	105

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years. Work hours are the annual work hours of the father. Earnings are the annual gross income from work of the father, in 2016 US dollars; currency conversions are based on 2016 exchange rates. CS rate is the child support rate (in percentages), computed as the ratio of the child support amount paid to the income of the father. Instruments used are the post-emancipation variable from Table 1 interacted with dataset indicators. All columns include all control variables from Table 2.

Appendix Table A3: Estimates of the Frisch elasticity by dataset (extensive margin)

	PSID (USA)	NLSY (USA)	BHPS+ (GBR)	HILDA (AUS)	SHP (CHE)	Pooled
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Dependent variable: Has positive work hours						
Log(100% − CS rate)	0.33*	0.11	0.074	0.54*	0.16	0.21**
	(0.18)	(0.14)	(0.14)	(0.32)	(0.21)	(0.098)
Observations	13,525	8,679	3,989	7,972	1,623	35,788
No. of fathers	2,029	1,125	533	1,008	254	4,949
Fraction	0.94	0.92	0.96	0.81	0.96	0.91
First stage F-stat.	130	175	81	132	12	106
Panel B: Dependent variable: Has positive earnings						
Log(100% − CS rate)	0.25	0.12	-0.26	0.20	-0.21	0.11
	(0.16)	(0.14)	(0.26)	(0.36)	(0.19)	(0.10)
Observations	13,526	8,669	4,126	8,019	1,599	35,939
No. of fathers	2,030	1,125	544	1,013	251	4,963
Fraction	0.93	0.94	0.97	0.86	0.95	0.92
First stage F-stat.	130	181	79	135	11	107

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years. Work hours and earnings are the annual work hours and gross income from work respectively of the father. CS rate is the child support rate (in percentages), computed as the ratio of the child support amount paid to the income of the father. Instruments used are the post-emancipation variable from Table 1 interacted with dataset indicators. All columns include all control variables from Table 2.

Appendix Table A4: Reduced form effect of emancipation of the youngest eligible child on labor supply

	Dependent variable:			
	Log of work hours	Log of earnings	Has positive work hours	Has positive earnings
	(1)	(2)	(3)	(4)
Post-emancipation	0.031***	0.031**	0.012***	0.0069
	(0.010)	(0.014)	(0.0045)	(0.0049)
Observations	33,087	33,537	38,102	38,242
No. of fathers	4,781	4,881	5,186	5,257
Mean hours/earnings/fraction	2212.1	53679.7	0.87	0.88

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years. Work hours are the annual work hours of the father. Earnings are the annual gross income from work of the father, in 2016 US dollars; currency conversions are based on 2016 exchange rates. CS rate is the child support rate (in percentages), computed as the ratio of the child support amount paid to the income of the father. Post-emancipation is an indicator variable that takes a value of one if the youngest eligible child is at emancipation age or older, half in the year before emancipation, and zero if younger. All columns include all control variables from Table 2.

Appendix Table A5: Elasticities of other outcomes with respect to the child support rate

	Dependent variable:					
	Log of food expenditure		Log of employee earnings		Log of self-employment earnings	
	(1)	(2)	(3)	(4)	(5)	(6)
Log(100% − CS rate)	0.57	0.81**	0.77**	0.68**	4.83	2.83
	(0.38)	(0.40)	(0.31)	(0.29)	(3.07)	(2.14)
Observations	22,912	22,912	30,740	30,729	3,066	2,994
No. of fathers	3,432	3,432	4,569	4,568	694	681
Expenditure/earnings	8.42	8.42	54.7	54.7	36.5	36.6
First stage F-stat.	128	110	114	100	9	8
Individual & year FEs	x	x	x	x	x	x
Other controls		x		x		x

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years. Columns 1 and 2: The dependent variable is the log of annualized food expenditure of the father. Only the PSID, BHPS, and HILDA have information on food consumption and are used in these regressions. Columns 3 and 4: The dependent variable is the log of annual income earned as salaried employees of the father. Columns 5 and 6: The dependent variable is the log of annual income from self-employment of the father. CS rate is the child support rate (in percentages), computed as the ratio of the child support amount paid to the income of the father. Instruments used are the post-emancipation variable from Table 1 interacted with dataset indicators. Individual & year FEs are individual fixed effects and dataset-year fixed effects. Other controls are age-education-dataset fixed effects, and the log of the hourly wage interacted with dataset indicators. Dollar values are in thousands of 2016 US dollars; currency conversions are based on 2016 exchange rates.

Appendix Table A6: Heterogeneity by father's earning power relative to the mother

	Dependent variable: Log of work hours or earnings. Heterogeneity variable is:			
	Father earns less than mother before emancipation		Father has lower wages than mother before emancipation	
	Work hours (1)	Earnings (2)	Work hours (3)	Earnings (4)
Log(100% − CS rate)	1.22***	1.22**	0.88**	0.79
	(0.45)	(0.60)	(0.43)	(0.56)
Log(100% − CS rate) × heterogeneity variable	-1.31	-1.47*	-1.06*	-0.69
	(0.82)	(0.84)	(0.60)	(0.71)
Observations	4,946	5,018	4,664	4,719
No. of fathers	740	749	699	706
Mean hours/earnings	2,247	58,971	2,247	59,549
First stage F-statistic	35	30	14	14

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all father-years that can be matched to the mother. Columns 1 and 2: The heterogeneity variable is an indicator variable equal to 1 if the father's annual earnings is less on average than the mother's annual earnings, both before emancipation. Columns 3 and 4: The heterogeneity variable is an indicator variable equal to 1 if the father's hourly wage is less on average than the mother's hourly wage, both before emancipation. Estimates shown are based on 2SLS specifications in which IVs are fully interacted with the heterogeneity variable. All columns include all controls in Table 2, and definitions not specified are the same as in that table.

Appendix Table A7: RD first stage and reduced form estimates of potential confounders

	Dependent variable:						
	Log of 100% − CS rate		Child lives with mother	IHS of AFDC/TANF+ WIC+SNAP+ EITC		IHS of earnings of child in family	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Post-emancipation	0.045***	0.042***	-0.025	-0.019	-0.016	-0.11	-0.14
	(0.013)	(0.013)	(0.025)	(0.11)	(0.067)	(0.12)	(0.11)
Observations	222,891	222,619	5,796	51,150	51,071	78,133	78,042
No. of mothers	10,928	10,662	5,796	3,954	3,883	3,896	3,813
Percent/fraction/earnings	93.52	93.52	0.673	590.9	590.7	4544.6	4544.8
With controls		x			x		x

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all mother-year-months. CS rate is the ratio of child support amount received to the income of the mother. Child lives with mother is an indicator variable that takes a value of one if the child is living with the mother, and zero otherwise. The source of this variable is the fertility history module of the SIPP, which occurs once per individual. AFDC/TANF+WIC+SNAP+EITC is the annualized amount of transfers in 2016 dollars from the following government programs: Aid to Families with Dependent Children or Temporary Assistance for Needy Families (depending on panel), Special Supplemental Nutrition Program for Women, Infants, and Children, Supplemental Nutrition Assistance Program, and Earned Income Tax Credit (EITC). EITC amounts are only available in the taxation topical modules (not available in all waves), and the annual amount is added to the annualized amounts of the other transfers. Earnings of child in family is the annualized gross income from work of the youngest eligible child if he is living with the mother, if the child is identifiable based on the birth month. IHS is the inverse hyperbolic sine transformation. Post-emancipation is an indicator variable that takes a value of 1 if the youngest eligible child is older than emancipation age, and zero if younger. The specification used is a uniform-kernel local-linear sharp RD design with the month relative to the emancipation month as the running variable. The bandwidth used is two years (inclusive) on each side of the emancipation month; the emancipation month itself is excluded. The linear specification is allowed to differ pre- and post-emancipation, and across panels. All columns include a panel fixed effect. All controls are individual fixed effects, panel-year-month fixed effects, age-education-panel fixed effects, and the log of the hourly wage interacted with panel indicators. Sample sizes and numbers of clusters reported are effective numbers used in the RD.

Appendix Table A8: Level-level and log-IHS estimates of the intertemporal income effect for mothers

	Dependent variable:					
	Earnings		Log of work hours		Log of earnings	
	(1)	(2)	(3)	(4)	(5)	(6)
CS amount received	-0.73	-0.11				
	(0.60)	(0.31)				
IHS of CS amount received			-0.0027	-0.00096	-0.017	0.0032
			(0.0073)	(0.0061)	(0.014)	(0.0073)
Observations	232,068	231,807	169,136	168,853	171,938	171,621
No. of mothers	11,202	10,947	9,398	9,123	9,456	9,146
Mean earnings/work hours	27,423	27,422	1,940	1,940	37,013	37,025
First stage F-stat.	133	140	134	159	140	156
All controls		x		x		x

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all mother-year-months. Work hours are the annualized work hours of the mother. Earnings are the annualized gross income from work of the mother, in 2016 US dollars. CS amount received is the annualized amount of child support received of the mother, in 2016 dollars. IHS is the inverse hyperbolic sine transformation. The specification used is a uniform-kernel local-linear fuzzy RD design with the month relative to the emancipation month as the running variable. The bandwidth used is two years (inclusive) on each side of the emancipation month; the emancipation month itself is excluded. The linear specification is allowed to differ pre- and post-emancipation, and across panels. All columns include a panel fixed effect. All controls are individual fixed effects, panel-year-month fixed effects, age-education-panel fixed effects, and the log of the hourly wage interacted with panel indicators. Sample sizes and numbers of clusters reported are effective numbers used in the RD.

Appendix Table A9: Reduced form estimates of the effect of emancipation on labor supply for mothers

	Dependent variable:			
	Log of work hours	Log of earnings	Has positive work hours	Has positive earnings
	(1)	(2)	(3)	(4)
Post-emancipation	0.00075	-0.0025	0.0012	-0.00010
	(0.0048)	(0.0057)	(0.0017)	(0.0021)
Observations	168,853	171,621	224,492	231,807
No. of mothers	9,123	9,146	10,871	10,947
Mean hours/earnings/fraction	1939.9	37024.7	0.75	0.74

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all mother-year-months. Work hours are the annualized work hours of the mother. Earnings are the annualized gross income from work of the mother, in 2016 US dollars. Post-emancipation is an indicator variable that takes a value of 1 if the youngest eligible child is older than emancipation age, and zero if younger. The specification used is a uniform-kernel local-linear sharp RD design with the month relative to the emancipation month as the running variable. The bandwidth used is two years (inclusive) on each side of the emancipation month; the emancipation month itself is excluded. The linear specification is allowed to differ pre- and post-emancipation, and across panels. All columns include all controls from Table 7. Sample sizes and numbers of clusters reported are effective numbers used in the RD.

Appendix Table A10: Falsification regressions for mothers

	Dependent variable:			
	Log of work hours	Log of earnings	Has positive work hours	Has positive earnings
	(1)	(2)	(3)	(4)
After ineligible child emancipated	0.0056	-0.0010	0.0025	-0.0021
	(0.0060)	(0.0073)	(0.0023)	(0.0023)
Observations	115,592	115,703	151,471	156,384
No. of mothers	5,933	5,903	7,210	7,263
Mean hours/earnings/fraction	1825.5	39691.3	0.76	0.74

Notes: Standard errors clustered by individual in parentheses. Asterisks denote significance: * $p < .10$, ** $p < .05$, *** $p < .01$. The sample comprises all mother-year-months for mothers whose oldest children are born after the mother's last marriage, and whose last marriages are intact at the last interview. Work hours are the annualized work hours of the mother. Earnings are the annualized gross income from work of the mother, in 2016 US dollars. After ineligible child emancipated is an indicator variable that takes a value of 1 if the youngest child is older than emancipation age, and zero if younger. The specification used is a uniform-kernel local-linear sharp RD design with the month relative to the emancipation month as the running variable. The bandwidth used is two years (inclusive) on each side of the emancipation month; the emancipation month itself is excluded. The linear specification is allowed to differ pre- and post-emancipation, and across panels. All columns include all controls from Table 7. Sample sizes and numbers of clusters reported are effective numbers used in the RD.

B Solutions to models

B.1 Main model

The recursive version of the problem is

$$V_t\left(a_{it,}\Phi_{it},S_{it}\right)=\max_{c_{it},a_{i,t+1},h_{it}}\left\{u\left(c_{it}\right)-\frac{e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}}}{1+\frac{1}{\gamma}}h_{it}^{1+\frac{1}{\gamma}}\right.$$
$$\left.+\beta\hat{E}\left[V_{t+1}\left(a_{i,t+1},\Phi_{i,t+1},S_{i,t+1}\right)|\Phi_{it}\right]\right\} \tag{13}$$

$$\text{s.t. } c_{it}+\frac{1}{1+r}a_{i,t+1}=a_{it}+w_{it}h_{it}-S_{it},$$
$$S_{i,t+1}=s_{it}w_{it}h_{it}.$$

The first order condition (FOC) for $a_{i,t+1}$ is

$$\beta\left(1+r\right)\hat{E}\left[\frac{\partial}{\partial a_{i,t+1}}V_{t+1}\left(a_{i,t+1},\Phi_{i,t+1},S_{i,t+1}\right)|\Phi_{it}\right]=\lambda_{it}, \tag{14}$$

where λ_{it} is the marginal utility of wealth (and the Lagrangian multiplier for the budget constraint), and the FOC for h_{it} is

$$e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}}h_{it}^{\frac{1}{\gamma}}=\lambda_{it}w_{it}+\beta\hat{E}\left[\frac{\partial}{\partial h_{it}}V_{t+1}\left(a_{i,t+1},\Phi_{i,t+1},S_{i,t+1}\right)|\Phi_{it}\right].$$

Since $a_{i,t+1}$ is a function of state variables and parameters (and not other choice variables), and $\Phi_{i,t+1}$ does not depend on h_{it}, this simplifies slightly to

$$e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}}h_{it}^{\frac{1}{\gamma}}=\lambda_{it}w_{it}+\beta s_{it}w_{it}\hat{E}\left[\frac{\partial}{\partial S_{i,t+1}}V_{t+1}\left(a_{i,t+1},\Phi_{i,t+1},S_{i,t+1}\right)|\Phi_{it}\right]. \tag{15}$$

The envelope theorem gives

$$\frac{\partial}{\partial a_{it}}V_t\left(a_{it},\Phi_{it},S_{it}\right)=\lambda_{it}, \tag{16}$$

and

$$\frac{\partial}{\partial S_{it}}V_t\left(a_{it},\Phi_{it},S_{it}\right)=-\lambda_{it}. \tag{17}$$

Substituting (16) and (17) in (14) and (15), we have

$$\beta\left(1+r\right)\hat{E}\left[\lambda_{i,t+1}|\Phi_{it}\right]=\lambda_{it}, \tag{18}$$

and

$$e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}}h_{it}^{\frac{1}{\gamma}}=\lambda_{it}w_{it}-\beta s_{it}w_{it}\hat{E}\left[\lambda_{i,t+1}|\Phi_{it}\right]. \tag{19}$$

Substituting (18) in (19), and defining $\dot{s}_{it} \equiv \frac{s_{it}}{(1+r)}$, we have

$$e^{\boldsymbol{Z}'_{it}\boldsymbol{\alpha}+U_{it}} h_{it}^{\frac{1}{\gamma}} = (1 - \dot{s}_{it})\, \lambda_{it} w_{it}, \tag{20}$$

or in logs,

$$\log h_{it} = \gamma \log (1 - \dot{s}_{it}) + \gamma \log \lambda_{it} + \gamma \log w_{it} - \gamma \boldsymbol{Z}'_{it}\boldsymbol{\alpha} - \gamma U_{it}. \tag{21}$$

The Frisch elasticity is defined as $\frac{\partial \log h_{it}}{\partial \log w_{it}}\Big|_{\lambda_{it}}$; examining equation (21) shows this is γ in this model.

If we assumed that λ_{it} is independent of s_{it} or w_{it}, we could estimate Equation (21). However, this assumption is inconsistent with the model, since the Lagrangian multiplier is a function of all parameters. It is also a function of the state variables, including the information set at time t, and hence a function of any instrument we can use. To get around this problem, I implement the strategy of MaCurdy [1981] (among others).[41] Rewrite the marginal utility of wealth equation of motion (18) as

$$\beta (1 + r) \frac{\lambda_{i,t+1}}{\lambda_{it}} = 1 + \epsilon_{i,t+1} \tag{22}$$

where $\epsilon_{i,t+1} \equiv \beta (1 + r) \frac{\lambda_{i,t+1}}{\lambda_{it}} - \hat{E}\left[\beta (1 + r) \frac{\lambda_{i,t+1}}{\lambda_{it}} | \Phi_{it}\right]$ is an expectation error that captures the unpredicted components of changes in future wages, child support rates, and preference shifters. Assuming that $\epsilon_{i,t+1} > -1$, we can take logs of (22) to get

$$\log \lambda_{i,t+1} = \log \lambda_{it} + \log \frac{1}{\beta (1 + r)} + \log (1 + \epsilon_{i,t+1}), \tag{23}$$

and repeat substitution of (23) gives

$$\log \lambda_{it} = \log \lambda_{i0} + t \log \frac{1}{\beta (1 + r)} + \sum_{\tau=1}^{t} \log (1 + \epsilon_{i\tau}), \quad \text{all } t \geq 1. \tag{24}$$

Substituting (24) into (21), we get

$$\begin{aligned} \log h_{it} = {} & \gamma t \log \frac{1}{\beta (1 + r)} + \gamma \log (1 - \dot{s}_{it}) + \gamma \log \lambda_{i0} \\ & + \gamma \log w_{it} - \gamma \boldsymbol{Z}'_{it}\boldsymbol{\alpha} - \gamma U_{it} + \gamma \sum_{\tau=1}^{t} \log (1 + \epsilon_{i\tau}), \quad \text{all } t \geq 0. \end{aligned} \tag{25}$$

This equation says that an optimizing father plans his future labor supply in the following way. First, he takes into consideration his baseline marginal utility of wealth, $\log \lambda_{i0}$, which progresses deterministically over time ($\gamma t \log \frac{1}{\beta(1+r)}$). Next, there is a contribution from wages ($\log w_{it}$), child support ($\log (1 - \dot{s}_{it})$), observable characteristics ($\boldsymbol{Z}_{it}$), and unobservable characteristics (U_{it}), all of which could be random variables from the perpective of time 0.

[41] In particular, the formulation of the expectation error follows that of Altuğ and Miller [1990], which has the advantage of not requiring a Taylor series expansion until we try to interpret the exogeneity condition.

Random shocks to the marginal utility of wealth cumulate over time (the $\log(1+\epsilon_{i\tau})$ terms), and explain the differences between what happens at t and what the person expects (at time 0) to happen at t. Note that while (25) reflects father i's planned labor supply choices at time 0, the planned choice is still the optimal one upon realization of $(w_{it}, s_{it}, \boldsymbol{Z}_{it}, U_{it}, \epsilon_{i1}, \ldots \epsilon_{it})$ at time t (since we solved by backward induction). Hence, our implicit assumption that fathers optimize ensures that the planned choice coincides with the actual choice observed in the data, which allows us to estimate (25) using data on $(w_{it}, s_{it}, \boldsymbol{Z}_{it})$ and h_{it}.

B.2 Model-based interpretation of instrument exogeneity

Suppose we have a relevant instrument IV_{it}. For notational simplicity, let $\boldsymbol{z}_{it}$ be the vector of included instruments and IV_{it} (i.e. all observable variables we would put in the first stage of an IV regression). Suppose that (i) the instruments are known in advance, i.e. $\boldsymbol{z}_{it}$ is in Φ_{i0}, (ii) fathers have rational expectations regarding future marginal utility of wealth changes, i.e. $\hat{E}\left[\frac{\lambda_{it}}{\lambda_{i,t-1}}|\Phi_{i0}\right] = E\left[\frac{\lambda_{it}}{\lambda_{i,t-1}}|\Phi_{i0}\right]$; (iii) $\boldsymbol{z}_{it}$ is uncorrelated with U_{it}; and (iv) $\boldsymbol{z}_{it}$ is uncorrelated with the second and further powers of ϵ_{it}. Also, assume that $\epsilon_{it} < 1$ (we assumed $\epsilon_{it} > -1$ earlier). Note that assumption (ii) aligns expectations so that we can use a well-known property of expectations errors:

$$
\begin{aligned}
E\left[\epsilon_{it}|\Phi_{i0}\right] &= \beta\left(1+r\right) E\left[\frac{\lambda_{it}}{\lambda_{i,t-1}} - \hat{E}\left[\frac{\lambda_{it}}{\lambda_{i,t-1}}|\Phi_{i,t-1}\right]|\Phi_{i0}\right] \\
&= \beta\left(1+r\right) E\left[\frac{\lambda_{it}}{\lambda_{i,t-1}} - E\left[\frac{\lambda_{it}}{\lambda_{i,t-1}}|\Phi_{i,t-1}\right]|\Phi_{i0}\right] \\
&= 0. \qquad (26)
\end{aligned}
$$

Then,

$$
\begin{aligned}
E\left[\boldsymbol{z}_{it}\varepsilon_{it}\right] &= -\gamma E\left[\boldsymbol{z}_{it}U_{it}\right] + \gamma\sum_{\tau=1}^{t} E\left[\boldsymbol{z}_{it}\log\left(1+\epsilon_{i\tau}\right)\right] \\
&= \gamma\sum_{\tau=1}^{t} E\left[\boldsymbol{z}_{it}\sum_{k=1}^{\infty}\left(-1\right)^{k+1}\frac{\epsilon_{i\tau}^{k}}{k}\right] \\
&= \gamma\sum_{\tau=1}^{t} E\left[\boldsymbol{z}_{it}\epsilon_{i\tau}\right] \\
&= \gamma\sum_{\tau=1}^{t} E\left[\boldsymbol{z}_{it}E\left[\epsilon_{i\tau}|\Phi_{i0}\right]\right] \\
&= \boldsymbol{0},
\end{aligned}
$$

where the second equality follows from assumption (iii) and a Taylor series expansion around $\epsilon_{i\tau} = 0$, the third equality follows from assumption (iv), the fourth equality follows from iterated expectations and assumption (i), and the last equality follows from (26).

B.3 Model incorporating mother's and child's consumption

The recursive version of the problem is now

$$V_t\left(a_{it,}\Phi_{it},S_{it}\right)=\max_{c_{it},a_{i,t+1},h_{it}}\left\{u_i\left(c_{it}\right)-\frac{e^{\boldsymbol{Z}_{it}'\boldsymbol{\alpha}+U_{it}}}{1+\frac{1}{\gamma}}h_{it}^{1+\frac{1}{\gamma}}+\mathfrak{M}\left(\mathfrak{m}_{it}\right)+\mathfrak{C}\left(\mathfrak{c}_{it}\right)\right.$$
$$\left.+\beta\hat{E}\left[V_{t+1}\left(a_{i,t+1},\Phi_{i,t+1},S_{i,t+1}\right)|\Phi_{it}\right]\right\} \tag{27}$$

$$\begin{aligned}\text{s.t. } c_{it}+\frac{1}{1+r}a_{i,t+1}&=a_{it}+w_{it}h_{it}-S_{it},\\ S_{i,t+1}&=s_{it}w_{it}h_{it},\\ \mathfrak{m}_{it}&=\left(1-k\right)S_{it},\\ \mathfrak{c}_{it}&=kS_{it}.\end{aligned}$$

The FOCs and $\frac{\partial}{\partial a_{it}}V_t\left(a_{it},\Phi_{it},S_{it}\right)$ are unchanged, but now we have

$$\frac{\partial}{\partial S_{it}}V_t\left(a_{it},\Phi_{it},S_{it}\right)=\left(1-k\right)\mathfrak{M}'\left(\left(1-k\right)S_{it}\right)+k\mathfrak{C}'\left(kS_{it}\right)-\lambda_{it}. \tag{28}$$

Substituting (16) and (28) in (14) and (15), and after some manipulation, we get

$$e^{\boldsymbol{Z}_{it}'\boldsymbol{\alpha}+U_{it}}h_{it}^{\frac{1}{\gamma}}=\lambda_{it}w_{it}-\dot{s}_{it}\lambda_{it}w_{it}+\dot{s}_{it}\xi_{it}\lambda_{it}w_{it} \tag{29}$$

in place of (20), where

$$\xi_{it}\equiv\beta\left(1+r\right)\frac{1}{\lambda_{it}}\hat{E}\left[\left(1-k\right)\mathfrak{M}'\left(\left(1-k\right)S_{i,t+1}\right)+k\mathfrak{C}'\left(kS_{i,t+1}\right)|\Phi_{it}\right] \tag{30}$$

captures how much the father values mother and child consumption out of child support relative to his own. Finally, following the rest of the steps in the main solution, we obtain

$$\log h_{it}=\gamma t\log\frac{1}{\beta\left(1+r\right)}+\gamma\log\left(1-\dot{s}_{it}+\dot{s}_{it}\xi_{it}\right)+\gamma\log\lambda_{i0}$$
$$+\gamma\log w_{it}-\gamma\boldsymbol{Z}_{it}'\boldsymbol{\alpha}-\gamma U_{it}+\gamma\sum_{\tau=1}^{t}\log\left(1+\epsilon_{i\tau}\right) \tag{31}$$

in place of (4).

In this paper, I parameterize $\mathfrak{M}\left(\cdot\right)$ and $\mathfrak{C}\left(\cdot\right)$ using the following:

$$\mathfrak{M}\left(\mathfrak{m}_{it}\right)=\kappa_{\mathfrak{m}}^{*}\tilde{u}_i\left(\mathfrak{m}_{it}\right),$$

and

$$\mathfrak{C}\left(\mathfrak{c}_{it}\right)=-\kappa_{\mathfrak{c}}^{*}\tilde{u}_i\left(\mathfrak{c}_{it}\right),$$

where $\tilde{u}_i(c_{it}) \equiv u_i(\tilde{c}_i) + u_i'(\tilde{c}_i)(c_{it} - \tilde{c}_i)$ is the first order taylor expansion of $u_i(c_{it})$ around a $\tilde{c}_i$, and $\kappa_{\mathfrak{m}}^* \in [-1,1]$ and $\kappa_{\mathfrak{c}}^* \in [0,1]$ are parameters that capture the intensity of like or dislike for mother or child consumption out of child support. For each father, $\tilde{c}_i$ is such that $\tilde{u}_i'(\tilde{c}_i) = e^{\frac{1}{T}\sum_t \log \lambda_{it}}$, i.e. the point that yields the individual's geometric mean of his marginal utility of wealth.

Since $\mathfrak{M}(\cdot)$ and $\mathfrak{C}(\cdot)$ are linear functions, their derivatives are constant for each father:

$$\mathfrak{M}'(\mathfrak{m}_{it}) = \kappa_{\mathfrak{m}}^* \tilde{u}_i'(\tilde{c}_i) = \kappa_{\mathfrak{m}}^* e^{\frac{1}{T}\sum_t \log \lambda_{it}},$$

$$\mathfrak{C}'(\mathfrak{c}_{it}) = \kappa_{\mathfrak{c}}^* \tilde{u}_i'(\tilde{c}_i) = \kappa_{\mathfrak{c}}^* e^{\frac{1}{T}\sum_t \log \lambda_{it}}.$$

Substituting into (30), we have

$$\xi_{it} = \left((1-k)\kappa_{\mathfrak{m}}^* + k\kappa_{\mathfrak{c}}^*\right) \frac{e^{\frac{1}{T}\sum_t \log \lambda_{it}}}{\lambda_{it}}.$$

B.4 Bound on $\kappa_{\mathfrak{c}}^*$

Suppose the father valued his family's and child's consumption directly.[42] If he liked the child consumption out of support $\kappa_{\mathfrak{c}}^*$ times as much as his own, correcting for household size, we have

$$\frac{\mathfrak{c}}{numofchildren} = \kappa_{\mathfrak{c}}^* \frac{c}{famsize},$$

where $\mathfrak{c}$ is the child consumption out of support (the notation follows the main text as far as possible), c is his own family's consumption, and $numofchildren$ and $famsize$ are equivalence scale-corrected number of children supported and own family size, respectively. In a static setting, the child support rate s is

$$s = \frac{\mathfrak{c}}{\mathfrak{c} + c}.$$

Solving the two, we have

$$\kappa_{\mathfrak{c}}^* = \frac{famsize}{numofchildren} \frac{s}{1-s}.$$

In the data, the average pre-emancipation own-family size is 2.6, and the average number of children supported is 1.5. For simplicity, I use a square root equivalence scale when correcting for family size. I use the statutory rate per child of around 10 to 15% for s. This yields $\kappa_{\mathfrak{c}}^* = \sqrt{\frac{2.6}{1.5}} \frac{0.15}{0.85} = 0.24$.

[42]We could motivate this formally using a constant relative risk aversion utility function, which yields even tighter bounds.

C The weighting scheme of the meta-analysis estimator

C.1 The weighting scheme

The labor supply elasticity estimated in this paper is a weighted average of the labor supply elasticities estimated separately in each dataset. In this section, I derive the weighting scheme based on least squares geometry.[43] Intuitively, in OLS, the weights should increase with dataset size—larger datasets contribute more to the sum of squared residuals in the pooled dataset—and variation in the regressor's distribution—regressors further from the mean are more influential in OLS. The weighting scheme for 2SLS is similar, except that the strength of the first stage relationship plays a role as well. Because the estimator is more general than the context of this paper, this section uses more conventional regression notation, different from other sections of the paper.

The setting I consider is a 2SLS specification with covariates. We are interested in the partial effect $\boldsymbol{\beta}$ (a K-vector of coefficients) of a K-vector of endogenous variables $\boldsymbol{x}_{di}$ on a scalar outcome y_{di}, estimated using 2SLS with the L_d-vector of (excluded) instruments $\boldsymbol{z}_{di}$, where d indexes the dataset and i indexes an observation. We have an M_d-vector of covariates $\boldsymbol{w}_{di}$ (included instruments); we are not interested in the partial effect of these covariates, $\boldsymbol{\alpha}_d$, which differs by dataset. The number and identities of instruments and covariates can vary across datasets; for example, we might control for age in one dataset and not in another. Thus, we have the 2SLS specification

$$y_{di} = \boldsymbol{x}_{di}'\boldsymbol{\beta} + \boldsymbol{w}_{di}'\boldsymbol{\alpha}_d + \varepsilon_{di}, \tag{32}$$

$$\boldsymbol{x}_{di} = \boldsymbol{\theta}_d \boldsymbol{z}_{di} + \boldsymbol{\kappa}_d \boldsymbol{w}_{di} + \boldsymbol{\nu}_{di},$$

where, in addition to the objects defined above, ε_{di} and $\boldsymbol{\nu}_{di}$ are error terms, and $\boldsymbol{\theta}_d$ and $\boldsymbol{\kappa}_d$ are $K \times L_d$ and $K \times M_d$ matrices of first stage coefficients that differ based on dataset d.

We have D datasets, with each dataset d having n_d observations, for a total of n observations. We can obtain D estimates of $\boldsymbol{\beta}$ by sequentially restricting the sample to dataset d and estimating (32) by 2SLS; denote each restricted-dataset estimate by $\hat{\boldsymbol{\beta}}_d$. We can also obtain a meta-analysis estimate of $\boldsymbol{\beta}$ by estimating (32) on the full sample; denote this estimate by $\hat{\boldsymbol{\beta}}$. We want to understand the relationship between $\hat{\boldsymbol{\beta}}$ and the $\hat{\boldsymbol{\beta}}_d$'s.

For each dataset d, let $\boldsymbol{y}_d \equiv \begin{bmatrix} y_{d1} & \dots & y_{dn_d} \end{bmatrix}'$, $\boldsymbol{X}_d \equiv \begin{bmatrix} \boldsymbol{x}_{d1} & \dots & \boldsymbol{x}_{dn_d} \end{bmatrix}'$, $\boldsymbol{Z}_d \equiv \begin{bmatrix} \boldsymbol{z}_{d1} & \dots & \boldsymbol{z}_{dn_d} \end{bmatrix}'$, and $\boldsymbol{W}_d \equiv \begin{bmatrix} \boldsymbol{w}_{d1} & \dots & \boldsymbol{w}_{dn_d} \end{bmatrix}'$ be the $n_d \times 1$, $n_d \times K$, $n_d \times L_d$, and $n_d \times M_d$ data matrices (or vector) from the dataset. Application of the Frisch-Waugh-Lovell (FWL) theorem yields[44]

$$\hat{\boldsymbol{\beta}}_d = \left(\tilde{\boldsymbol{X}}_d' \boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d} \tilde{\boldsymbol{X}}_d\right)^{-1} \tilde{\boldsymbol{X}}_d' \boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d} \tilde{\boldsymbol{y}}_d, \tag{33}$$

where $\tilde{\boldsymbol{y}}_d \equiv \boldsymbol{M}_{\boldsymbol{W}_d} \boldsymbol{y}_d$, $\tilde{\boldsymbol{X}}_d \equiv \boldsymbol{M}_{\boldsymbol{W}_d} \boldsymbol{X}_d$, and $\tilde{\boldsymbol{Z}}_d \equiv \boldsymbol{M}_{\boldsymbol{W}_d} \boldsymbol{Z}_d$, and where for any arbitrary matrix $\boldsymbol{C}$, $\boldsymbol{P}_{\boldsymbol{C}} \equiv \boldsymbol{C}\left(\boldsymbol{C}'\boldsymbol{C}\right)^{-1}\boldsymbol{C}$ is the projection matrix and $\boldsymbol{M}_{\boldsymbol{C}} \equiv \boldsymbol{I} - \boldsymbol{P}_{\boldsymbol{C}}$ is the annihilator

[43] One implication of this is that this relationship is exact in the finite sample.

[44] To my knowledge, no formal derivation of the theorem exists for the overidentified 2SLS case exists, although the result has been used in econometric software. I provide this derivation in Section C.2.

matrix.

Next, stack the outcome and endogenous variables from the D datasets to obtain the n-vector $\boldsymbol{y}_d \equiv \begin{bmatrix} y_{d1} & \dots & y_{dn_d} \end{bmatrix}'$ and the $n \times K$ matrix $\boldsymbol{X} \equiv \begin{bmatrix} \boldsymbol{X}_1' & \dots & \boldsymbol{X}_D' \end{bmatrix}'$. Since we fully interact the instruments and covariates with dataset indicators, the corresponding data matrix of instruments is the $n \times \sum_{d=1}^{D} L_d$ block diagonal matrix

$$\boldsymbol{Z} \equiv \begin{bmatrix} \boldsymbol{Z}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{Z}_D \end{bmatrix}$$

and the $n \times \sum_{d=1}^{D} M_d$ block diagonal matrix

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{W}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{W}_D \end{bmatrix}.$$

The FWL theorem again yields

$$\hat{\boldsymbol{\beta}} = \left(\tilde{\boldsymbol{X}}' \boldsymbol{P}_{\tilde{\boldsymbol{Z}}} \tilde{\boldsymbol{X}} \right)^{-1} \tilde{\boldsymbol{X}}' \boldsymbol{P}_{\tilde{\boldsymbol{Z}}} \tilde{\boldsymbol{y}},$$

where $\tilde{\boldsymbol{y}} \equiv \boldsymbol{M}_{\boldsymbol{W}} \boldsymbol{y}$, $\tilde{\boldsymbol{X}} \equiv \boldsymbol{M}_{\boldsymbol{W}} \boldsymbol{X}$, and $\tilde{\boldsymbol{Z}} \equiv \boldsymbol{M}_{\boldsymbol{W}} \boldsymbol{Z}$.

To proceed, note that straightforward block diagonal matrix algebraic manipulation gives[45]

$$\boldsymbol{M}_{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{M}_{\boldsymbol{W}_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{M}_{\boldsymbol{W}_D} \end{bmatrix},$$

which implies that $\tilde{\boldsymbol{Z}}$ (a product of two block diagonal matrices with dimensions that match up) is itself block diagonal, with each block given by $\boldsymbol{M}_{\boldsymbol{W}_d} \boldsymbol{Z}_d = \tilde{\boldsymbol{Z}}_d$. This in turn implies that

$$\boldsymbol{P}_{\tilde{\boldsymbol{Z}}} = \begin{bmatrix} \boldsymbol{P}_{\tilde{\boldsymbol{Z}}_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{P}_{\tilde{\boldsymbol{Z}}_D} \end{bmatrix}.$$

[45] See, for example, Petersen and Pedersen [2012, p. 46].

Hence, we have

$$
\begin{aligned}
\hat{\boldsymbol{\beta}} &= \left(\tilde{\boldsymbol{X}}'\boldsymbol{P}_{\tilde{\boldsymbol{Z}}}\tilde{\boldsymbol{X}}\right)^{-1}\tilde{\boldsymbol{X}}'\boldsymbol{P}_{\tilde{\boldsymbol{Z}}}\tilde{\boldsymbol{y}} \\
&= \left(\sum_{d=1}^{D}\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d\right)^{-1}\sum_{d=1}^{D}\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{y}}_d \\
&= \left(\sum_{d=1}^{D}\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d\right)^{-1}\sum_{d=1}^{D}\left(\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d\right)\left(\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d\right)^{-1}\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{y}}_d \\
&= \left(\sum_{d=1}^{D}\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d\right)^{-1}\sum_{d=1}^{D}\left(\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d\right)\hat{\boldsymbol{\beta}}_d,
\end{aligned}
$$

where the second equality exploits the block diagonal nature of $\boldsymbol{P}_{\tilde{\boldsymbol{Z}}}$ and the ordering of the d datasets in $\tilde{\boldsymbol{X}}$ and $\tilde{\boldsymbol{y}}$ (so that dimensions match up), and the fourth equality follows from (33). Hence, $\hat{\boldsymbol{\beta}}$ is a weighted average of the $\hat{\boldsymbol{\beta}}_d$'s, with weights given by the $K \times K$ matrices $\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d$.

We can decompose the weights into component parts:

$$
\begin{aligned}
\tilde{\boldsymbol{X}}'_d\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_d}\tilde{\boldsymbol{X}}_d &= \tilde{\boldsymbol{X}}'_d\tilde{\boldsymbol{Z}}_d\left(\tilde{\boldsymbol{Z}}'_d\tilde{\boldsymbol{Z}}_d\right)^{-1}\tilde{\boldsymbol{Z}}'_d\tilde{\boldsymbol{X}}_d \\
&= \tilde{\boldsymbol{X}}'_d\tilde{\boldsymbol{Z}}_d\left(\tilde{\boldsymbol{Z}}'_d\tilde{\boldsymbol{Z}}_d\right)^{-1}\tilde{\boldsymbol{Z}}'_d\tilde{\boldsymbol{Z}}_d\left(\tilde{\boldsymbol{Z}}'_d\tilde{\boldsymbol{Z}}_d\right)^{-1}\tilde{\boldsymbol{Z}}'_d\tilde{\boldsymbol{X}}_d \\
&= n_d\hat{\boldsymbol{\theta}}_d\left(\frac{1}{n_d}\tilde{\boldsymbol{Z}}'_d\tilde{\boldsymbol{Z}}_d\right)\hat{\boldsymbol{\theta}}'_d, \qquad (34)
\end{aligned}
$$

where in the last equality, $\hat{\boldsymbol{\theta}}_d$ is the OLS estimator of $\boldsymbol{\theta}_d$ in the first stage of (32). Thus, the weights are a function of the size of the dataset, the size of the first stage estimates (which captures the strength of the first stage relationships), and the sample covariance matrix of the partialled instruments (assuming that a constant is included in $\boldsymbol{w}_{di}$). Note that the last two factors are influenced by the institutional setting, the dataset sampling rules, and the choice of instruments and covariates; to the extent that these three factors are similar across datasets, the weights are essentially a function of the sample size.

I conclude this section with two remarks. First, if OLS were used instead of 2SLS, $\hat{\boldsymbol{\theta}}_d$ becomes the identity matrix and (34) collapses to weighting by dataset size and the sample covariance matrix of the regressors of interest. Second, we might want to relax the requirement that all the first stage coefficients ($\boldsymbol{\theta}_d$) and the partial effect of covariates ($\boldsymbol{\alpha}_d$ and $\boldsymbol{\kappa}_d$) differ across datasets in the meta-analysis estimate. This might be justified by the econometrician bringing in additional information to discipline the model; we might know that the life cycle profile of work hours is the same in several datasets after adjusting for a level effect, or that emancipation leads to the same drop in the child support rate in several datasets. Suppose we imposed that the last element of $\boldsymbol{\alpha}_1$ is equal to the last element of $\boldsymbol{\alpha}_2$, and similar for $\boldsymbol{\kappa}_1$ and $\boldsymbol{\kappa}_2$. This is equivalent to pooling datasets 1 and 2, applying an additional FWL partialling step at the start, and then proceeding as before. Hence, $\hat{\boldsymbol{\beta}}$ is still interpretable as a weighted average of the $\hat{\boldsymbol{\beta}}_d$'s, but with $\hat{\boldsymbol{\beta}}_1$ and $\hat{\boldsymbol{\beta}}_2$ estimated based on transformed data. Next, suppose that we imposed that the first stage effect of the last instruments in $\boldsymbol{z}_{1i}$ and $\boldsymbol{z}_{2i}$ are the same. In this case, there is no general weighting scheme that relates $\hat{\boldsymbol{\beta}}$ to the $\hat{\boldsymbol{\beta}}_d$'s because information based off dataset 1 restricts the first stage of dataset 2, and vice versa. Instead, $\hat{\boldsymbol{\beta}}$ is a weighted average of of $\hat{\boldsymbol{\beta}}_d, d = 3, 4, \dots D$, and $\hat{\boldsymbol{\beta}}_{1*}$, where $\hat{\boldsymbol{\beta}}_{1*}$ is the coefficient from pooling datasets 1 and 2 together and interacting dataset indicators with all covariates and all instruments other than the last. In either case (restriction of $\boldsymbol{\theta}_d$ or $\boldsymbol{\alpha}_d$ and $\boldsymbol{\kappa}_d$), bringing in some cross-dataset information changes the interpretation of the meta-analysis estimate slightly.

C.2 Derivation of FWL for overidentified 2SLS

Section C.1 uses the FWL theorem applied in an overidentified 2SLS setting. To my knowledge, no formal derivation exists, although Stata's user-created package *ivreg2* uses the result in its *partial* option, and a proof exists for the exactly-identified 2SLS case [Baum et al., 2007, Giles, 1984]. Here, I extend the method in Giles to show the result for the overidentified setting.

Suppose we estimated the 2SLS specification

$$y_i = \boldsymbol{x}_i'\boldsymbol{\beta} + \boldsymbol{w}_i'\boldsymbol{\alpha} + \varepsilon_i, \tag{35}$$

$$\boldsymbol{x}_i = \boldsymbol{\theta}\boldsymbol{z}_i + \boldsymbol{\kappa}\boldsymbol{w}_i + \boldsymbol{\nu}_i,$$

where y_i is a scalar outcome variable, $\boldsymbol{x}_i$ is a K-vector of endogenous variables, $\boldsymbol{z}_i$ is an L-vector of (excluded) instruments, $\boldsymbol{w}_i$ is an M-vector of covariates (included instruments), ε_i and $\boldsymbol{\nu}_i$ are error terms, and $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$, $\boldsymbol{\theta}$, and $\boldsymbol{\kappa}$ are coefficients. The 2SLS estimator of $\begin{bmatrix} \boldsymbol{\beta}' & \boldsymbol{\alpha}' \end{bmatrix}$ based on a dataset of size n is

$$\begin{bmatrix} \hat{\boldsymbol{\beta}} \\ \hat{\boldsymbol{\alpha}} \end{bmatrix} = \left(\begin{bmatrix} \boldsymbol{X}' \\ \boldsymbol{W}' \end{bmatrix} \boldsymbol{P}_{\begin{bmatrix} \boldsymbol{Z} & \boldsymbol{W} \end{bmatrix}} \begin{bmatrix} \boldsymbol{X} & \boldsymbol{W} \end{bmatrix} \right)^{-1} \begin{bmatrix} \boldsymbol{X}' \\ \boldsymbol{W}' \end{bmatrix} \boldsymbol{P}_{\begin{bmatrix} \boldsymbol{Z} & \boldsymbol{W} \end{bmatrix}} \boldsymbol{y}, \tag{36}$$

where $\boldsymbol{y} \equiv \begin{bmatrix} y_1 & \dots & y_n \end{bmatrix}'$, $\boldsymbol{X} \equiv \begin{bmatrix} \boldsymbol{x}_1 & \dots & \boldsymbol{x}_n \end{bmatrix}'$, $\boldsymbol{Z} \equiv \begin{bmatrix} \boldsymbol{z}_1 & \dots & \boldsymbol{z}_n \end{bmatrix}'$, and $\boldsymbol{W} \equiv \begin{bmatrix} \boldsymbol{w}_1 & \dots & \boldsymbol{w}_n \end{bmatrix}'$, and $\boldsymbol{P}_{.}$ is the projection matrix operator defined in Section C.1 (i.e. $\boldsymbol{P}_{\boldsymbol{C}} \equiv \boldsymbol{C} \left(\boldsymbol{C}'\boldsymbol{C}\right)^{-1} \boldsymbol{C}$). We want to show that

$$\hat{\boldsymbol{\beta}} = \left(\tilde{\boldsymbol{X}}' \boldsymbol{P}_{\tilde{\boldsymbol{Z}}} \tilde{\boldsymbol{X}} \right)^{-1} \tilde{\boldsymbol{X}}' \boldsymbol{P}_{\tilde{\boldsymbol{Z}}} \tilde{\boldsymbol{y}},$$

where $\tilde{\boldsymbol{y}} \equiv \boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{y}$, $\tilde{\boldsymbol{X}} \equiv \boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{X}$, and $\tilde{\boldsymbol{Z}} \equiv \boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}$, and $\boldsymbol{M}_{.}$ is the annihilator matrix operator defined in Section C.1 (i.e. $\boldsymbol{M}_{\boldsymbol{C}} \equiv \boldsymbol{I} - \boldsymbol{P}_{\boldsymbol{C}}$).

First, block matrix algebraic manipulation gives

$$\left(\begin{bmatrix} \boldsymbol{Z}' \\ \boldsymbol{W}' \end{bmatrix} \begin{bmatrix} \boldsymbol{Z} & \boldsymbol{W} \end{bmatrix} \right)^{-1} = \begin{bmatrix} \boldsymbol{C}_1 & \boldsymbol{C}_2 \\ \boldsymbol{C}_2' & \boldsymbol{C}_4 \end{bmatrix},$$

where

$$\boldsymbol{C}_1 \equiv \left(\boldsymbol{Z}'\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}\right)^{-1},$$

$$\boldsymbol{C}_2 \equiv -\boldsymbol{C}_1\boldsymbol{Z}'\boldsymbol{W}\left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1},$$

and

$$\boldsymbol{C}_4 \equiv \left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1} + \left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1}\boldsymbol{W}'\boldsymbol{Z}\boldsymbol{C}_1\boldsymbol{Z}'\boldsymbol{W}\left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1}.$$

Expanding the definition of $\boldsymbol{P}_{\left[\begin{array}{cc}\boldsymbol{Z} & \boldsymbol{W}\end{array}\right]}$, and after some agebraic manipulation, we get

$$\boldsymbol{P}_{\left[\begin{array}{cc}\boldsymbol{Z} & \boldsymbol{W}\end{array}\right]} = \boldsymbol{P}_{\boldsymbol{W}} + \boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}.$$

Hence,

$$\left[\begin{array}{c}\boldsymbol{X}' \\ \boldsymbol{W}'\end{array}\right]\boldsymbol{P}_{\left[\begin{array}{cc}\boldsymbol{Z} & \boldsymbol{W}\end{array}\right]} = \left[\begin{array}{c}\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{W}} + \boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}} \\ \boldsymbol{W}'\end{array}\right], \tag{37}$$

where I have used the fact that $\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}\boldsymbol{W} = 0$ and $\boldsymbol{W}\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}} = 0$ due to the annihilating property of $\boldsymbol{M}_{\boldsymbol{W}}$.

Next, the above and block matrix inversion gives

$$\left(\left[\begin{array}{c}\boldsymbol{X}' \\ \boldsymbol{W}'\end{array}\right]\boldsymbol{P}_{\left[\begin{array}{cc}\boldsymbol{Z} & \boldsymbol{W}\end{array}\right]}\left[\begin{array}{cc}\boldsymbol{X} & \boldsymbol{W}\end{array}\right]\right)^{-1} = \left[\begin{array}{cc}\boldsymbol{C}_5 & \boldsymbol{C}_6 \\ \boldsymbol{C}_6' & \boldsymbol{C}_8\end{array}\right], \tag{38}$$

where

$$\boldsymbol{C}_5 \equiv \left(\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}\boldsymbol{X}\right)^{-1}, \tag{39}$$

$$\boldsymbol{C}_6 \equiv -\boldsymbol{C}_5\boldsymbol{X}'\boldsymbol{W}\left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1}, \tag{40}$$

and

$$\boldsymbol{C}_8 \equiv \left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1} + \left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1}\boldsymbol{W}'\boldsymbol{X}\boldsymbol{C}_5\boldsymbol{X}'\boldsymbol{W}\left(\boldsymbol{W}'\boldsymbol{W}\right)^{-1}.$$

Substituting (37) and (38) into (36), we get

$$\left[\begin{array}{c}\hat{\boldsymbol{\beta}} \\ \hat{\boldsymbol{\alpha}}\end{array}\right] = \left[\begin{array}{c}\boldsymbol{C}_5\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{W}}\boldsymbol{y} + \boldsymbol{C}_5\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}\boldsymbol{y} + \boldsymbol{C}_6\boldsymbol{W}'\boldsymbol{y} \\ \boldsymbol{C}_6'\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{W}}\boldsymbol{y} + \boldsymbol{C}_6'\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}\boldsymbol{y} + \boldsymbol{C}_8\boldsymbol{W}'\boldsymbol{y}\end{array}\right],$$

and simplifying the upper block gives

$$\begin{aligned}\hat{\boldsymbol{\beta}} &= \boldsymbol{C}_5\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{W}}\boldsymbol{y} + \boldsymbol{C}_5\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}\boldsymbol{y} - \boldsymbol{C}_5\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{W}}\boldsymbol{y} \\ &= \left(\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{P}_{\boldsymbol{M}_{\boldsymbol{W}}\boldsymbol{Z}}\boldsymbol{y} \\ &= \left(\tilde{\boldsymbol{X}}'\boldsymbol{P}_{\tilde{\boldsymbol{Z}}}\tilde{\boldsymbol{X}}\right)^{-1}\tilde{\boldsymbol{X}}'\boldsymbol{P}_{\tilde{\boldsymbol{Z}}}\tilde{\boldsymbol{y}},\end{aligned}$$

where the first and second equalities follow from (39) and (40) respectively, and the third equality uses the idempotency of $\boldsymbol{M}_{\boldsymbol{W}}$.