Prompting students to talk about mathematics is an important goal of mathematics education reform. Teachers are encouraged to provide opportunities for students to discuss their ideas about mathematics and to listen closely to what students say. Yet managing such discourse is no easy task and can present teachers with many new challenges (Ball 1991; NCTM 1991).

This article focuses on the efforts of one middle school teacher, David Louis, as he attempted to develop a discourse community in his mathematics classroom. David envisioned an environment in which students would share, explain, and build on one another’s ideas and, as a result, would learn mathematics (Sherin, Mendez, and Louis 2000). Throughout the year, David chose activities that would encourage students’ thinking and generate plenty of ideas for students to discuss.

As David experimented with different ways to facilitate discourse, he encountered a dilemma. On the one hand, he was trying to create a classroom environment in which students’ ideas were the basis of discussion. On the other hand, he wanted to ensure that these discussions were productive mathematically. At times, David found it difficult to balance these two goals, that is, to have a classroom that is both open to students’ ideas and focused on learning specific mathematical content. In a journal that David kept throughout the year, he explained this dilemma:

Today I was forced to consider an interesting issue. The issue is, “Do I sacrifice some . . . content in order to foster discussions during class?” . . . There were several different places today where discussion arose . . . I should have expected that, considering I’m trying to set a culture of expressing one’s ideas, but it caught me by surprise a little. At first I tried to press on [to the content I had planned to cover], but students still had [new] ideas. In fact at one point, their ideas [about the content] were quite different than mine . . . [and] when I wanted to move on, they didn’t.

A Structure for Whole-Class Discussion

David struggled with this tension throughout the school year. At times, students shared ideas but did not talk about the mathematics that David had in mind. At other times, the class seemed to reach a comfortable balance between discussing student ideas and moving along mathematically. Looking back at the school year, David found that the discussions that appeared to be most successful in achieving both goals shared a similar format consisting of three main parts: generating ideas, comparing and evaluating ideas, and focusing the range of ideas.

The initial part of a discussion is concerned with generating ideas. Here, the teacher’s goal is for students to share their ideas and to get as many ideas as possible out in the open. To meet this goal, David often asked such questions as “What do you think? Why? What do other folks think?” David was not concerned about taking control or focusing the content of the discussion. Instead, he tried to elicit students’ ideas and give students a sense of ownership of the discussion. The Professional Standards...
for Teaching Mathematics (NCTM 1991) notes that consistently asking “Why?” can help stimulate discourse by promoting norms of inquiry and interest in students’ ideas.

In the next phase of the discussion, the class moves into comparing and evaluating ideas. Rather than raise new ideas, students are encouraged to talk about ones that have been mentioned previously. In particular, students are asked to compare and evaluate different ideas that have been offered. “Is your idea the same as Sam’s? What do you think about Robert’s idea?” In this phase, the teacher is beginning to think more about the content of the discussion, but open-ended discourse is still encouraged.

In the third, or focusing, phase of the discussion, the range of ideas being considered is narrowed. Usually, the teacher accomplishes this focusing by asking the class to look closely at a particular idea that has been raised. Thus, David might say, “Okay, let’s focus on Jeff’s idea.” In addition, the teacher often introduces a new mathematical idea that is intended to help the class make progress on the issue being investigated. During the focusing phase, the teacher is actively concerned with content issues and is willing to take some control of the discussion. The Professional Standards for Teaching Mathematics discusses this issue explicitly, suggesting that teachers must “filter and direct students’ explorations” and “at times, provide information and lead students” (NCTM 1991, 36).

After the focusing phase, the class frequently returns to generating ideas, and the cycle continues.

**Reflection:**
Can you think of a discussion in your class that used one or more of these structures? How did they help move the discussion along?

**Discussion in Action:**
**The Slingshot Lesson**

THE SLINGSHOT LESSON SERVES AS AN EXAMPLE of a discussion that reflects this structure. The lesson comes from one of David’s eighth-grade-mathematics classes during the second month of school. The class was in the middle of a unit on functions, in which students were asked to explore data, interpret graphs, and write simple linear equations. Note that the class was not a pre-algebra or an algebra class.

For the lesson, students were given an apparatus, shown in figure 1, that resembled a slingshot sitting on a wooden board with a rubber band strung between two nails. The apparatus rested on the floor, and students measured the distance that a tinfoil ball traveled along the floor after being released from the slingshot. Students were instructed to pull the rubber band back 1, 2, and 3 centimeters and to record the distance traveled for several trials at each length.

David selected this activity for a number of reasons. First, he expected the students’ data to be somewhat “messy”; that is, the tinfoil balls were not precisely the same size, and students might not release the balls in exactly the same way each time. Second, because the data might be inconsistent, David thought that it would be interesting for students to try to decide whether the data represented a linear function. Third, because of the variety that
David expected in the data, he anticipated that the lesson would provide an interesting context for discussion. After students collected their data in groups, David brought the class together for a brief discussion of the activity. The next day, David continued the discussion by explaining that a student in a different class, Patrice, had come up with the following equation to fit her data: \[ y = 120x, \]
where \( x \) represents how much the rubber band was stretched and \( y \) represents how far the ball traveled. David asked the students whether they thought that Patrice’s equation was correct, and they began to offer a variety of responses. One student suggested that the initial distance traveled was not exactly 120 centimeters because the floor caused some friction. Another student said that the balls did not always travel in a straight line. Other students commented that the distance traveled depends on the tightness of the particular rubber band used. At this point in the discussion, the class was engaged in generating ideas; students proposed a number of reasons why their own data were not as uniform as Patrice’s equation suggested.

After a few minutes, a student named Ben joined the conversation. Ben suggested that the distance might not actually increase by 120 centimeters, or something close to that, each time the rubber band is pulled back another centimeter.

**Reflection:**

*How did David’s question about the validity of Patrice’s equation help to open up the conversation for generating ideas?*

*Ben.* I think it depends, like, on how far you pull it back.

*Teacher.* What do you mean?

*Ben.* Like, if you pull it back . . . 1 centimeter, it might be 120 centimeters. But then [when] you pull it back to . . . the second one, it might be farther than 120 centimeters. It might just keep going at a steady rate, but it might be larger than 120 centimeters apart.

Ben introduced an idea that was somewhat different from other students’ comments. Instead of offering another physical factor that affected data collection, Ben wondered whether the function might not have a constant growth. David encouraged other students to comment on Ben’s idea, as well as the other ideas that had been suggested. In response, Robert rephrased Ben’s idea and explained that Ben was suggesting that the graph of the data might not be a straight line.

*Robert.* I think he means that the graph might not be linear. If you made a graph out of it, it might not go at a constant rate.

*Teacher.* Is that what you’re saying?

*Ben.* Yeah.

*Teacher.* What do other people think about that?

Other students added to the conversation. Jeff responded in agreement with Ben: “The change [in the distance traveled] between 0 centimeters and 1 centimeter will be less than the change between 1 centimeter and 2 centimeters.” Sam disagreed, saying that the variation resulted from human error, not from an increase in the distance traveled. This part of the discussion illustrates the comparing and evaluating phase. The students were commenting on, and responding to, ideas that had been raised, and David encouraged their comments by asking students whether they agreed or disagreed with the ideas of their classmates.

**Reflection:**

*In what ways has the discussion progressed mathematically? How have the contributions of the students changed?*

Next, David moved into the focusing phase of the discussion. He highlighted two ideas that had been raised by the class: “I hear people saying two things. One group of people [is] saying that . . . each time, it goes 120 centimeters farther . . . the same amount farther each time. I hear another group of people saying that possibly . . . each time, it goes a little farther. What do you guys think about that idea?”

Students began to look at their data to see which of these two patterns fit more accurately. To aid in this process, David suggested that students use the graphing calculator to create a scatterplot. He demonstrated this technique using data from one group, as shown in figure 2, and the class tried to estimate visually which line or curve would match the data. The class then talked about the notion of a “line of best fit” and debated whether any data points had to be located on such a line and whether the same number of data points had to be located above and below the line.

![Fig. 2 Scatterplot based on one group’s data](image-url)
In this part of the discussion, David focused the class’s attention on two ideas that had been previously mentioned and offered new mathematical content and techniques that he thought would help the class make progress mathematically. By focusing the discussion, David was able to help the students address a key concept of the day’s lesson: What do you do when you have a set of data that does not fit neatly into a specific equation? At the same time that David was focusing the discussion, he also allowed the students to return to generating ideas when they talked about which line would represent the “line of best fit.”

**Conclusion**

THIS SAMPLE LESSON ILLUSTRATES A DISCUSSION THAT is mathematically productive and allows for a great deal of open-ended discourse. How did David manage to achieve this outcome? One way that we can understand David’s success is by looking at the three structures, or phases, that he used to organize the whole-class discussion. Through generating ideas, comparing and evaluating, and focusing, David is able to funnel students’ ideas. The term **funneling** describes a process in which a teacher highlights specific student-generated ideas for the class to pursue. This description differs from Wood, Cobb, and Yackel’s (1991) description of funneling, in which a teacher leads students through a series of questions until a specific answer is produced. In this classroom, the funneling process, shown in **figure 3**, begins when David solicits and discusses many ideas. In the next stage in the process, David focuses the class on only a few ideas. The process continues as David returns to eliciting ideas, comparing and evaluating the ideas, and so on. This process allows extensive open-ended discourse. At the same time, at each of the funneling points, David exerts control over how the class proceeds mathematically, and the content of the lesson becomes more and more focused. In this way, David ensures that the students discuss relevant mathematical ideas.

These three structures also allow David to explicitly move back and forth in his emphasis on students’ ideas and the mathematical content of the lesson. During the first phase, he is clearly focused on eliciting students’ ideas and not concerned with the direction of the content. Later in the discussion, this emphasis changes; during the focusing phase, David carefully selects which ideas he wants the class to pursue. The entire process leaves room for students’ ideas but permits the teacher to guide those ideas in a particular direction and insert additional ideas about mathematics.

Mathematics reform calls for teachers to engage students in discussing, explaining, and justifying their ideas. Although teachers are asked to use students’ ideas as the basis for instruction, they must also keep in mind the mathematics that the class is expected to explore. This article has examined one way that a teacher can structure whole-class discussion to balance the need to include students’ ideas with the goal of helping students learn mathematics.

**References**


---

**Reflection:**

In what ways was David able to achieve a balance between discussing students’ ideas and directing the mathematical content of the lesson?

---

![Fig. 3 The process of funneling students’ ideas](image-url)