First-Best Dynamic Assignment of Commuters with Endogenous Heterogeneities in a Corridor Network

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**Background**

- *Departure-time choice equilibrium model* for commute problems has long been studied.
  - Vickrey, 1969; Hendrickson & Kocur, 1981; etc..

- In the long term, *travel demand patterns* vary over time because of commuters’ choices (e.g. residential locations and jobs) and inevitably affect short-term dynamic.

*An effective framework incorporating the short term and long term is worth of studying.*
Propose a framework that synthesizes long-term travel demand forecasting with short-term dynamic traffic assignment.

- Long-term equilibrium: residential location; job
- Short-term equilibrium: departure time

Investigate properties of equilibrium.

- Temporal/spatial patterns
Contributions

► Integration of dynamic bottleneck congestion and Urban Economics
  
  
  ▪ In a corridor network with multiple bottlenecks
    cf. Shen & Zhang, 2009; Tian, et al., 2012; etc.
  
  ▪ With endogenous heterogeneities

► Policy implication for investment on bottleneck capacity expansions
Outline

► Model
  ▪ Settings: Network, Commuters’ Behavior
  ▪ Equilibria: Short-term and Long-term
  ▪ Integrated Model

► Analysis of Short-term Equilibrium
  ▪ Special Case I: Corridor without Heterogeneities
  ▪ Special Case II: Single Bottleneck with Heterogeneities
  ▪ General Case: Corridor with Heterogeneities

► Analysis of Long-term Equilibrium
  ▪ Spatial Sorting Patterns
  ▪ Policy Implications

► Concluding Remarks
Network

A corridor with multiple bottlenecks

- Residential location
- Land supply
- Bottleneck (BN)
- Labor demand
- Jobs at CBD

\[ A_I \]

\[ A_i \]

\[ \mu_I \]

\[ \mu_i \]

\[ \mu_2 \]

\[ \mu_1 \]

\[ L_1 \]

\[ L_2 \]

\[ L_j \]
Commuters’ Behavior

► Commuters’ choices

▪ In the long term
  - Residential location \(i\) with endogenous land rent \(r_i\)
  - Job \(j\) with endogenous wage \(w_j\)

▪ In the short term
  - Departure time (arrival time \(t\) at the CBD) with commuting cost \(C_{i,j}(t)\)

► Commuters’ objective

▪ Maximize own utility (minimize own cost)

\[
\max_{i,j,t} v_{i,j}(t) = \max_{i,j} \{ w_j - r_i - \min_t C_{i,j}(t) \}
\]
Interactions between Short and Long Terms

Feedback structure

Long-term policies:
land supply \( \{A_i\} \), labor demand \( \{L_j\} \), BN capacity \( \{\mu_i\} \)

Equilibrium commuting cost
\( \{\rho_{i,j}\} \)

Long-term Equilibrium
(location choice \( i \) and job choice \( j \))

Short-term Equilibrium
(arrival-time choice \( t \))

Travel demand
\( \{Q_{i,j}\} \)

Short-term policy:
Travel Demand Management (TDM)
Short-term TDM Policy: TNP Scheme

Definition

- A *tradable network permit* (TNP) is a right that allows its holder to pass through a pre-specified BN during a pre-specified time period.

  - Akamatsu, 2007; Wada & Akamatsu, 2013; Akamatsu & Wada, 2017

Assumptions

- A *competitive market* is assumed for trading permits.
- The number of issued permits is limited to be *less than or equal to BN capacity*.

  \[ \Rightarrow \text{Queues of all BNs are eliminated.} \]
Short-term Equilibrium under TDM Policy

Optimal-choice conditions in *short-term* equilibrium

\[
\begin{align*}
    \rho_{i,j} - C_{i,j}(t) &= 0 \quad \text{if } q_{i,j}(t) > 0 \\
    \rho_{i,j} - C_{i,j}(t) &\leq 0 \quad \text{if } q_{i,j}(t) = 0
\end{align*}
\]
\(\forall i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}\)

\[
C_{i,j}(t) \equiv \alpha_j \cdot \left[ s(t) + d_i \right] + \sum_{m \leq i} p_m(t)
\]

*Permit market* clearing conditions

\[
\begin{align*}
    \mu_i - \sum_{m \geq i} \sum_{j \in \mathcal{J}} q_{m,j}(t) &= 0 \quad \text{if } p_i(t) > 0 \\
    \mu_i - \sum_{m \geq i} \sum_{j \in \mathcal{J}} q_{m,j}(t) &\geq 0 \quad \text{if } p_i(t) = 0
\end{align*}
\]
**Long-term Equilibrium**

- **Optimal choice conditions in long-term equilibrium**

  \[
  \begin{cases}
  V - w_j - r_i - \rho_{i,j} = 0 & \text{if } Q_{i,j} > 0 \\
  V - w_j - r_i - \rho_{i,j} \geq 0 & \text{if } Q_{i,j} = 0
  \end{cases}
  \quad \forall i \in I, j \in J
  \]

- \[ Q_{i,j} = \int_{t \in T} q_{i,j}(t)dt \]

- **Land/labor market clearing conditions**

  - **Land market**

    \[
    \begin{cases}
    A_i - \sum_{j \in J} Q_{i,j} = 0 & \text{if } r_i > 0 \\
    A_i - \sum_{j \in J} Q_{i,j} \geq 0 & \text{if } r_i = 0
    \end{cases}
    \quad \forall i \in I
    \]

  - **Labor market**

    \[
    \begin{cases}
    L_j - \sum_{i \in I} Q_{i,j} = 0 & \text{if } w_j > 0 \\
    L_j - \sum_{i \in I} Q_{i,j} \leq 0 & \text{if } w_j = 0
    \end{cases}
    \quad \forall j \in J
    \]
**Proposition 1:** The integrated equilibrium is *socially optimal* in the sense that it minimizes the total transportation cost.

Equivalent linear programming problem (LP)

\[
\begin{align*}
\text{min} & \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \int_{t \in \mathcal{T}} c_{i,j}(t) \cdot q_{i,j}(t) \, dt \\
\text{s.t.} & \quad \sum_{m \geq i} \sum_{j \in \mathcal{J}} q_{m,j}(t) \leq \mu_i \quad (p_i(t)) \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \quad \sum_{j \in \mathcal{J}} \int_{t \in \mathcal{T}} q_{i,j}(t) \, dt \leq A_i \quad (r_i) \quad \forall i \in \mathcal{I} \\
& \quad \sum_{i \in \mathcal{I}} \int_{t \in \mathcal{T}} q_{i,j}(t) \, dt \geq L_j \quad (w_j) \quad \forall j \in \mathcal{J} \\
\end{align*}
\]

- \[c_{i,j}(t) \equiv \alpha_j \cdot [s(t) + d_i] \]
The long-term problem (master problem)

\[
\begin{align*}
\min_{Q \geq 0} & \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \alpha_j d_i Q_{i,j} + z_1 \\
\text{s.t.} & \sum_{j \in \mathcal{J}} Q_{i,j} \leq A_i & (r_i) \\
& \sum_{i \in \mathcal{I}} Q_{i,j} \geq L_j & (w_j)
\end{align*}
\]

The short-term problem (subproblem)

\[
\begin{align*}
z_1 \equiv \min_{q \geq 0} & \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \int_{t \in T} \alpha_j s(t) q_{i,j}(t) dt \\
\text{s.t.} & \sum_{m \geq i} \sum_{j \in \mathcal{J}} q_{m,j}(t) \leq \mu_i & (p_i(t)) \\
& \int_{t \in T} q_{i,j}(t) dt = Q_{i,j}^* & (\rho_{i,j})
\end{align*}
\]
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- Policy Implications

Concluding Remarks
Special Case I: Corridor without Heterogeneities

► Model settings
  - Single job class but multiple BNs

\[ A_I \quad A_i \quad A_2 \quad A_1 \quad L \]

\[ \mu_I \quad \mu_i \quad \mu_2 \quad \mu_1 \]

► Equivalent LP

\[
\min_{q \geq 0} \sum_{i \in \mathcal{I}} \int_{t \in \mathcal{T}} c_i(t)q_i(t)dt \\
\text{s.t.} \quad \sum_{m \geq i} q_m(t) \leq \mu_i \quad (p_i(t)) \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
\quad \int_{t \in \mathcal{T}} q_i(t)dt \leq A_i \quad (r_i) \quad \forall i \in \mathcal{I}
\]
**Proposition 2-A** (location-based temporal sorting pattern): Commuters residing closer to the CBD arrive at times in a time window closer to the desired arrival time.
Special Case II: Single BN with Heterogeneities

**Model settings**
- Single BN but multiple job classes

**Equivalent LP**

$$\min_{q \geq 0} \sum_{j \in J} \int_{t \in T} c_{1,j}(t)q_{1,j}(t)dt$$

s.t.  
$$\sum_{j \in J} q_{1,j}(t) \leq \mu_1 \quad (p_1(t)) \quad \forall t \in T$$

$$\int_{t \in T} q_{1,j}(t)dt \geq L_j \quad (w_j) \quad \forall j \in J$$
Proposition 2-B (job-based temporal sorting pattern): Commuters of higher value of time arrive at times closer to the desired arrival time.

Equilibrium arrival-flow rates at the CBD

Equilibrium commuting costs and permit prices (isocost curve)
Proposition 3 (temporal sorting pattern): Flows of commuters admit a combination of sorting patterns in Special Case I and II.

Equilibrium arrival-flow rates at the CBD

Equilibrium commuting costs and permit prices (isocost curve)
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The Long-term Problem

► Solving the short-term problem gives the equilibrium commuting cost $\rho^*(Q)$ for given travel demand $Q$.

$Q \rightarrow \text{Short-term Equilibrium} \rightarrow \rho^*$

► Equivalent convex optimization problem for the long-term equilibrium

\[ z_2 \equiv \min_{Q \geq 0} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \alpha_{ij} d_{ij} Q_{i,j} + \sum_{i \in \mathcal{I}} \int_{0 \to Q_i} \rho^*_i(\omega) d\omega \]

s.t. \[ \sum_{j \in \mathcal{J}} Q_{i,j} \leq A_i \quad (r_i) \quad \forall i \in \mathcal{I} \]

\[ \sum_{i \in \mathcal{I}} Q_{i,j} \geq L_j \quad (w_j) \quad \forall j \in \mathcal{J} \]

\text{ignored in a naive model}
### Integrated Model vs. Naive Model

**Integrated model**
- **Long-term** \(\{Q, r, w | \rho\}\)
  - \(\rho^*\) : equilibrium commuting cost
  - \(Q^*\) : travel demand
- **Short-term** \(\{q, p, \rho | Q\}\)

**Naive model**
- **Long-term** \(\{Q, r, w | \rho\}\)
  - \(C\) : commuting cost (exogenously given and fixed)
- **Short-term** \(\{q, p, \rho | Q\}\)
  - \(\rho^N\) : equilibrium commuting cost
  - \(Q^N\) : travel demand

**TNP scheme**

\(\rho^*, \rho^N\) : equilibrium commuting cost
\(Q^*, Q^N\) : travel demand
\(C\) : commuting cost
Spatial Sorting Patterns

► Integrated model vs. naive model

**Proposition 4:** The location-job distribution of commuters is more dispersed in the integrated model.
Long-term Policy: BN Capacity Expansions

- BN capacity expansions
  - A long-term policy of road construction

- Self-financing principle

**Lemma:** The optimal amount of investment on BN capacity expansions coincides with the revenue of permits.

- Wada & Akamatsu, 2013; Akamatsu, 2017
  - Optimal amount of investment in total:

\[
R(Q) \equiv \sum_{i \in I} \int_{t \in T_i} p_i^*(t \mid Q) dt
\]
Policy Implication for BN Capacity Expansions

- **Integrated model**
  - BN capacity expansions
  - Long-term \(\{Q, r, w \mid \rho\}\)
  - Short-term \(\{q, p, \rho \mid Q\}\)
  - TNP scheme

- **Naive model**
  - BN capacity expansions
  - Long-term \(\{Q, r, w \mid \rho\}\)
  - Short-term \(\{q, p, \rho \mid Q\}\)
  - TNP scheme

**Proposition 5**: The naive model overestimates optimal amount of total investment on BN capacity expansions, i.e. \(R(Q^N) \geq R(Q^*)\).
Concluding Remarks

- An *integrated model* effectively synthesizing the long term and short term is proposed.
- *Analytical solutions* for DSO problems in a corridor network are derived.
- *Qualitative properties* of solution in equilibrium are investigated.
- A model *considering short and long terms separately misleads long-term policies*, such as investment on BN capacity expansions.
Appendix: False BNs without Heterogeneities

► **Definition**

A *false bottleneck is a bottleneck whose permit price is always zero in equilibrium.*

► **False BNs do not affect essential properties, but only cause *irrelevant freeness* in the solution.**

► **Criterion** to detect false BNs

\[ \sum_{n \geq m} A_n \mu_m \geq \sum_{n \geq i} A_n \mu_i \]

for some \( m < i \).
Appendix: Comparisons of Land Rents and Wages

► Land rent

$\begin{align*}
r_i^*, r_i^N
\end{align*}$

Location index $i$

► Wage

$\begin{align*}
w_j^*, w_j^N
\end{align*}$

Job index $j$

- Integrated model
- Naïve model

Integrated model

Naïve model
Optimal Amount of Investment on Each BN

Examples

- Naive model
- Integrated model

Graphs showing investment vs. bottleneck index for both naive and integrated models.