The morning commute in urban areas with heterogeneous trip lengths

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In a nutshell

- How to extend it to a city?
In a nutshell

• The speed MFD or production MFD (NFD) [Geroliminis and Daganzo, 2008]

\[ \Omega(n) = \frac{n v(n)}{L} \]
In a nutshell

• The speed MFD or production MFD (NFD) [Geroliminis and Daganzo, 2008]

\[
\int_{t_1}^{t_2} t \sigma_n(n(t)) dt = l
\]
LITERATURE REVIEW
# Review of literature with MFD-like dynamics

## Homogeneous users

- Geroliminis and Levinson (2009)
- Fosgerau and Small (2013)

(Stochastic identical trip length)

- Arnott (2013)
- Arnott, Kokoza and Naji (2016)

## Heterogeneous trip length

- Fosgerau (2015)
- Daganzo and Lehe (2015)
Review of literature with MFD-like dynamics

Homogeneous users

\[ t(t) = \frac{L}{v(t)} \] (2003)

Geroliminis and Levinson (2009)

Fosgerau and Small (2013)

(Stochastic identical trip length)

Arnott (2013)

Heterogeneous trip length

Fosgerau (2015)

Dagan and Lehe (2015)
Scheduling preferences

- Based on marginal utility rates:

\[ U = \int_{0}^{\uparrow t \downarrow d} h(t) \, dt + \int_{t \downarrow a}^{\uparrow 0} d \, dt + \int_{t \downarrow a}^{\uparrow 0} w(t) \, dt \]

\[ \text{Marginal utility rate} \]

\[ \text{Time} \]
Scheduling preferences

• Based on marginal utility rates
• No strong consensus on the shape despite several RP and SP studies [Small, 1982; Hendrickson and Planck, 1984; Tseng and Verhoef, 2008; Hjorth et al., 2015]
• Several functional forms proposed
Scheduling preferences

• Based on marginal utility rates
• No strong consensus on the shape despite several RP and SP studies [Small, 1982; Hendrickson and Planck, 1984; Tseng and Verhoef, 2008; Hjorth et al., 2015]
• Several functional forms proposed

![Graph showing marginal utility over time]
If Users differ only by trip length \((l)\),
\[
\frac{d}{dt} \left( \frac{h}{v} \right)(t) < 0 \text{ and } \frac{d}{dt} \left( \frac{w}{v} \right)(t) > 0 \text{ for all times } t,
\]
\(t \downarrow d(l) \) and \(t \downarrow a(l) \) continuous and differentiable, 
[some mild technical conditions],
Then UE exhibits the regular sorting property [Fosgerau, 2015]:
ANALYTICAL RESULTS
Continuity (proposition)

Assumptions:

- Continuum of users
- At home: \( h(t,\theta) \) [positive and continuous w.r.t. \( t \)]
- At work: \( w(t,\theta,T^{\uparrow*}) \) [continuous w.r.t. \( t \), except possibly at \( t=T^{\uparrow*} \)]
- \( l, T^{\uparrow*} \) and \( l^T T^{\uparrow*} \) are continuously distributed

\[ \Rightarrow n(t) \text{ and } v(t) \text{ continuous.} \]

------------------------------------ Attention -------------------------------------------

- Continuity result valid for a large set of \( h \) and \( w \) functions, including \( \alpha-\beta-\gamma \) preferences.

The next results require the same assumptions, plus \( \alpha-\beta-\gamma \) preferences.
Fundamental relationships

\[ u(t \downarrow a, t \uparrow*, l) = -\alpha \tau(t \downarrow a, l) - sp(t \downarrow a, t \uparrow*, l) \]

**1st order condition of User Equilibrium:**

\[
\frac{\partial u}{\partial t_a}(t_a, t^*, l) = 0 \iff -\alpha \frac{\partial \tau}{\partial t_a}(t_a, l) - \frac{\partial sp}{\partial t_a}(t_a, t^*, l) = 0
\]

\[
\int_{t_a - \tau(t_a, l)}^{t_a} v(t) dt = l \implies \frac{\partial \tau}{\partial t_a}(t_a, l) = \frac{v(t_d) - v(t_a)}{v(t_d)}
\]

⇒ For early users: \[ v(t_a(t_d, l)) = \frac{\alpha - \beta}{\alpha} v(t_d) \]

⇒ For late users: \[ v(t_a(t_d, l)) = \frac{\alpha + \gamma}{\alpha} v(t_d) \]

⇒ For on-time users: \[ \frac{\alpha - \beta}{\alpha} v(t_d) \leq v(t_a(t_d, l)) \leq \frac{\alpha + \gamma}{\alpha} v(t_d) \]
Fundamental relationships

• 2\textsuperscript{nd} order condition: if $t \downarrow a \neq t \uparrow \ast$ and $\nu(t)$ is differentiable\textsuperscript{1}, then at equilibrium

$$\frac{\partial^2 \tau}{\partial t_a^2}(t_a, l) \geq 0$$

Or, equivalently: $\nu^\uparrow (t \downarrow d)/\nu(t \downarrow d) \uparrow 2 \geq \nu^\uparrow (t \downarrow a)/\nu(t \downarrow a) \uparrow 2$

\textsuperscript{1} The exogenous differentiability assumption is avoided in the paper.
Intuition in the \((x, \ln(V))\) space...

- **Bijection:** \(x = f(t) = \int_0^t v(u) du\)
- \(V(x) = v(f^{-1}(x))\)
- **1\textsuperscript{st} order condition (FOC) for early users:**
  \[
  \ln(V(x|d)) - \ln(V(x|a)) = \ln(\alpha/\alpha-\beta)
  \]
- **2\textsuperscript{nd} order condition (SOC) for early users:**
  \[
  d\ln V / d x (x|a) \leq d\ln V / d x (x|d)
  \]
Intuition in the \((x, \ln(V))\) space...

SOC imposes a “point-to-point concavity” to \(\ln(V)\) for early users.

\[
\ln \left( \frac{\alpha}{\alpha - \beta} \right)
\]

\(L' > L\)

Locally FIFO

Trip-length serves as sorting criterion
Analytical results

• Single peak assumption (A2): $n(t)$ strictly increasing before $t \downarrow p$, strictly decreasing after.

• $A2+ (\alpha - \beta - \gamma) \Rightarrow$ Early users travel before $t \downarrow p$, late users after (Corollary of FOC + SOC).

• Proposition: $A1+A2+ (\alpha - \beta - \gamma) \Rightarrow$ FIFO sorting among early (resp. Late) users having identical $\beta/\alpha$ (resp. $\gamma/\alpha$). Longer trips start and finish earlier (resp. later).

See also: Alternative approach based on exogenous assumptions
Adjustment mechanism

Simulation

SIMULATIONS
Simulation description

• Agent-based event-based dynamics
  – 4000 agents,
  – 10 families of $\alpha$– $\beta$– $\gamma$ preferences
  – $l$, $t^*$ uniformly distributed.

• MSA:
  – full knowledge of last iteration,
  – 2% update their decision at every iteration.
Lessons from simulations

- Confirms theoretical findings (continuity, FIFO, sorting).
Influence of demand-to-capacity ratio

Vickrey (1969) (FIFO)

Present work (FIFO)

LIFO

- Smoothness
- Cost composition
- Instability and heavy congestion
FUTURE RESEARCH DIRECTIONS
General scheduling preferences

- Transition FIFO/LIFO

- Impact of congestion
Social optimum

- Applicability of envelope theorems?
- An example with hypercongestion
Further research directions

• Stability:
  – Depends on the adaptation mechanism,
  – Easy to observe with simulations,
  – Difficult to characterize analytically.

• Spatially heterogeneous settings.
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References