Fair Dynamic Resource Allocation in Transit-based Evacuation Planning

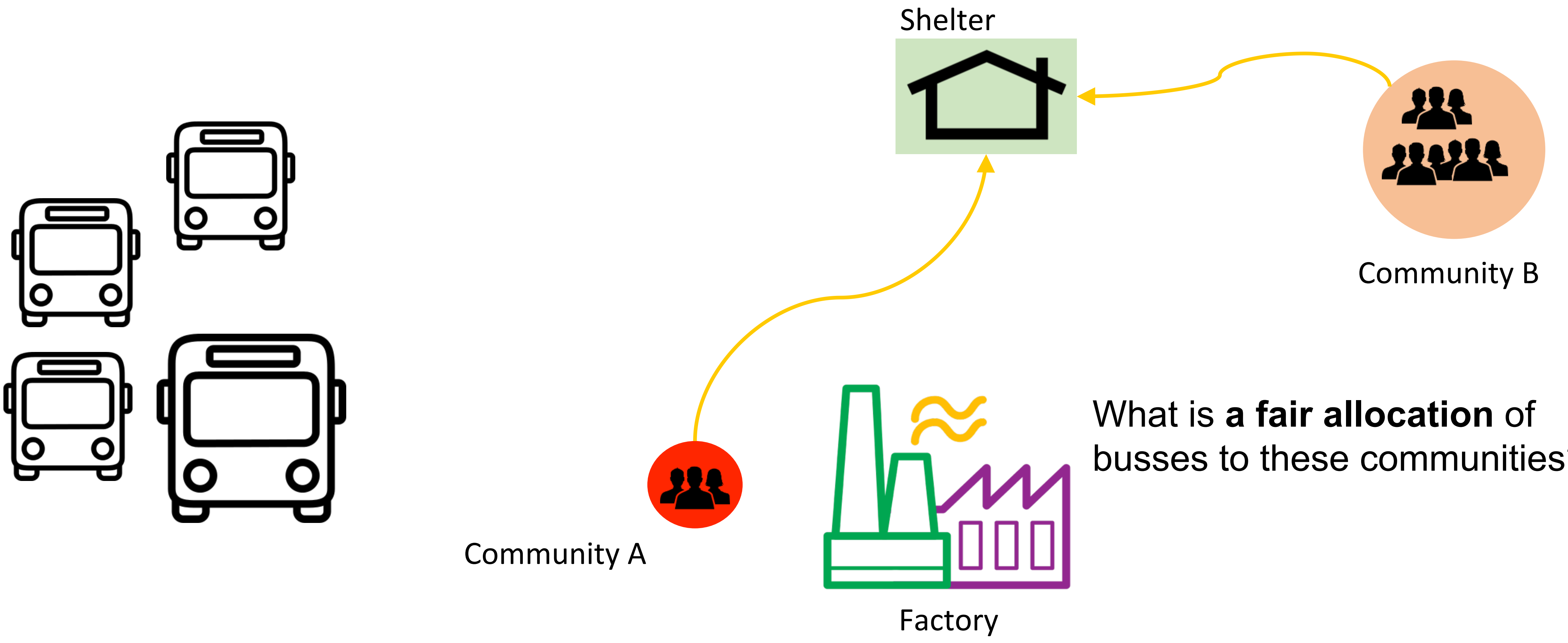
Soheila Aalami
PhD Candidate,
&
Lina Kattan , PEng, PhD
Professor, Urban Alliance Professor in Transportation Systems Optimization

Department of Civil Engineering
Schulich School of Engineering, University of Calgary
Calgary, Canada

Outline

- Introduction
- Resource Allocation and Utility
- Weighted Proportional Fairness
- Assumptions and Problem Formulation
- Distributed Solution Algorithm
- Numerical Example and Discussion of Results
- Conclusion

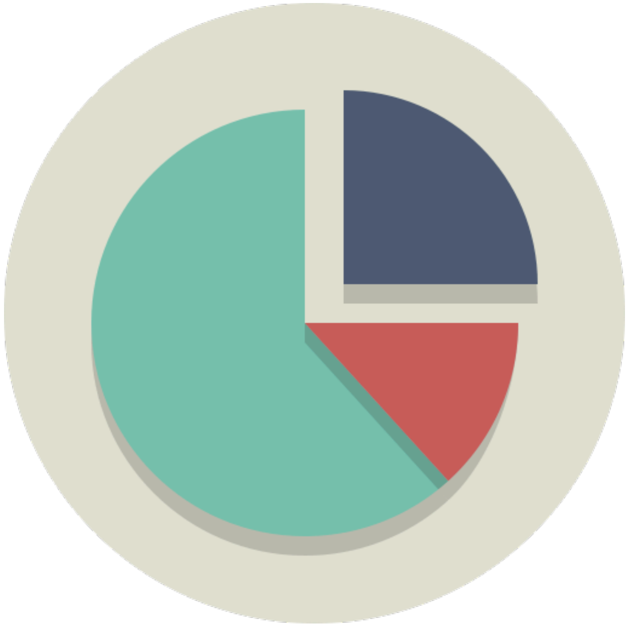
Introduction



What is **a fair allocation** of busses to these communities?

Model Application: short notice evacuation such as forest fire, chemical leaks from a factory, volcanic eruption, etc.

Resource Allocation and Utility



$u_1(r_1)$

$u_2(r_2)$

…

$u_n(r_n)$

$$\max F(u_1(r_1), \ldots, u_n(r_n))$$

$$\Sigma\, r_i \leq |\mathcal{R}|$$

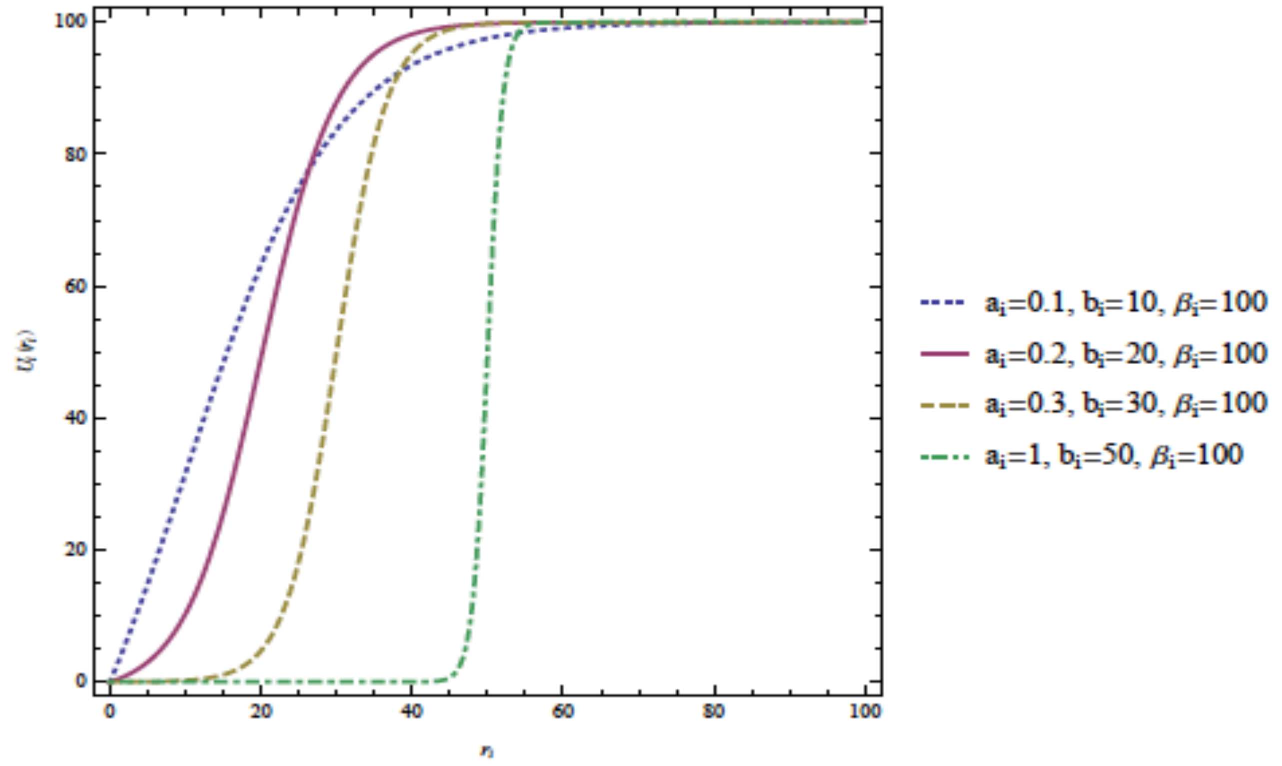
Utility Function's Properties:

1. $\forall$ r, $U_i(r_i) \geq 0$, i.e., utility is always non-negative.
2. $U_i(0) = 0$, i.e., in the case no evacuee is moved to a shelter, the utility is zero.
3. $U_i(\cdot)$ is a non-decreasing function of r_i, i.e., allocating more resources to a pickup location does not reduce the utility of the pickup location.
4. $\exists\, r_i^M < \infty$, s.t. $\forall\, r \geq r_i^M$, $U_i(r) = U_i(\infty) < \infty$, i.e., there is a optimum rate r_i^M of evacuation for which all the evacuees can be safely moved to shelters before the deadline. Having a higher evacuation rate than r_i^M is not helpful and does not increase the utility.

Sigmoidal utility function

$$U_i(r_i) = c_i \beta_i \left(\frac{1}{1 + e^{(-a_i(r_i - b_i))}} - d_i \right)$$

where a_i denotes the steepness of the curve, b_i is the inflection point of the utility function, β_i is the maximum value of the utility function, and $c_i = \frac{1+e^{a_i b_i}}{e^{a_i b_i}}$ and $d_i = \frac{1}{1+e^{a_i b_i}}$ are constants.



Different Objectives

- Minimum network clearance time
- Maximum social welfare
- **Fair resource allocation**
- ...

Fairness

- Fairness is a debatable topic

- There are many different definitions for fairness
  - Equal allocation
  - Max-Min fairness
  - **Proportional fairness**

Weighted Proportional Fairness

Proportional Fairness [Frank Kelly 1998]

Given a collection $\{\mathcal{U}_1(x_1), \cdots, \mathcal{U}_n(x_n)\}$ *of utility functions and a set of weights* $\{w_1, \cdots, w_n\}$, $w_i > 0$, *a feasible resource allocation* $\mathcal{A}^* = \{x_1^*, \cdots, x_n^*\}$ *is called a weighted proportional fair allocation if it satisfies:*

$$\sum_{i=1}^{n} w_i \frac{\mathcal{U}_i(x_i) - \mathcal{U}_i(x_i^*)}{\mathcal{U}_i(x_i^*)} \leq 0$$

for any feasible allocation $\mathcal{A} = \{x_1, \cdots, x_n\}$.

Severity level $w_i > 0$ for each pickup location P_i

Weighted Proportional Fairness: Equivalent Formulation

$$\sum_{i=1}^{n} w_i \frac{\mathcal{U}_i(x_i) - \mathcal{U}_i(x_i^*)}{\mathcal{U}_i(x_i^*)} \leq 0$$

for any feasible allocation $\mathcal{A} = \{x_1, \cdots, x_n\}$.

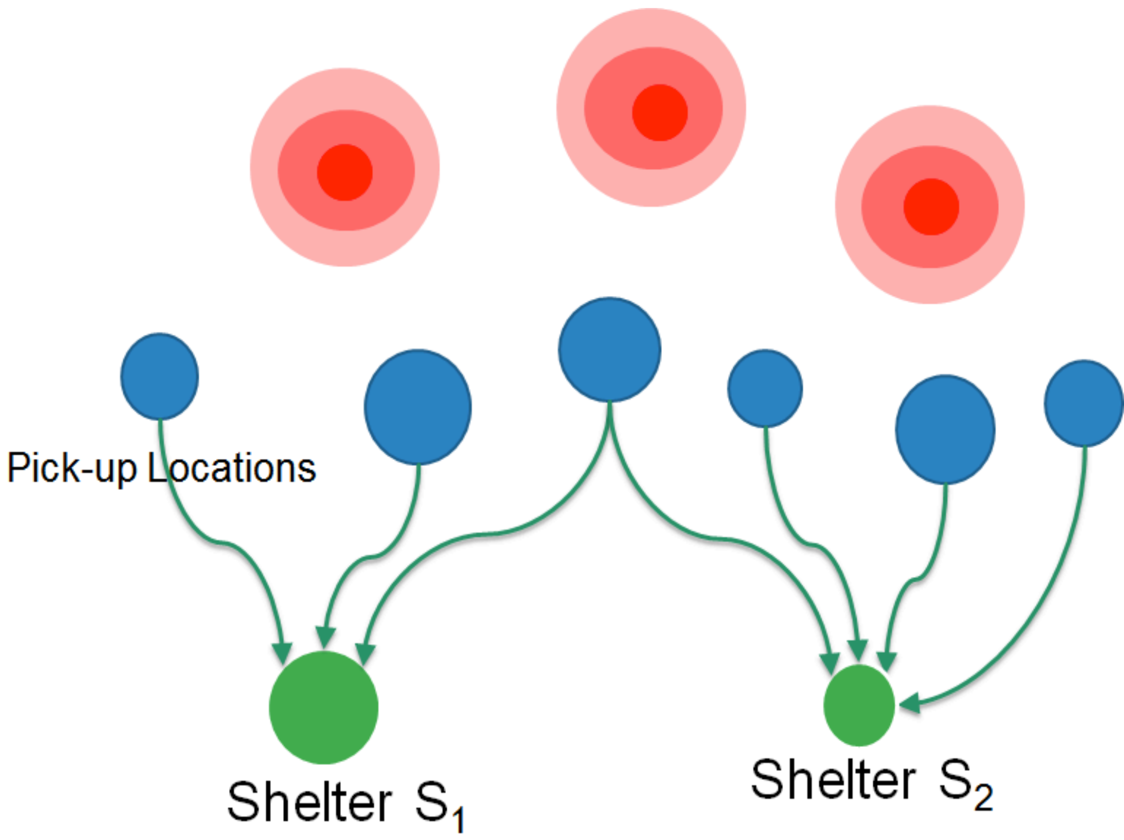
Is equivalent to:

$$\texttt{Maximize} \sum_{i=1}^{n} w_i \ln(\mathcal{U}_i(x_i))$$

Severity level $w_i > 0$ for each pickup location P_i

Assumptions / Formulation

- Severity level $w_i > 0$ for each pickup location P_i
- A fleet F of public transit vehicles with limited capacity is given
- Capacity of each shelter is limited
- Total capacity of shelters can accommodate the whole population
- Round trip travel times can change over time



	Shelter 1	Shelter2	Shelter 3	
Pickup P_1	f_{11}	f_{12}	f_{13}	Fleet size Constraint
Pickup P_2	f_{21}	f_{22}	f_{23}	
Pickup P_3	f_{31}	f_{32}	f_{33}	
Pickup P_4	f_{41}	f_{42}	f_{43}	
Pickup P_5	f_{51}	f_{52}	f_{53}	
	|S1|	|S1|	|S1|	

?

Problem Formulation:

SCHULICH
School of Engineering
UNIVERSITY OF CALGARY

$$\text{Maximize} \quad \sum_{i} w_i \ln\left(U_i(\vec{f_i})\right)$$

$$\sum_{i=1}^{n} \frac{\Gamma_i f_{ij}}{\tau_{ij}} \leq |S_j| \qquad \forall j \in \{1, \cdots, m\}$$

First constraint ensures that the number of evacuees moved to each shelter should be less than the capacity of the shelter.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij} \leq |F|$$

Second constraint ensures that capacity of all transit vehicles allocated to move evacuees from pickup locations to shelters does not exceed the size of the fleet

$$f_{ij} \geq 0 \qquad \forall i \in \{1, \cdots, n\}, \ \forall j \in \{1, \cdots, m\}$$

Lagrangian Dual of the Problem

$$L(\mathcal{A}, \boldsymbol{\mu}) = \sum_{i} w_i \ln U_i(\vec{f}_i) + \mu_0 \left(|F| - \sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij} \right) + \sum_{j=1}^{m} \mu_j \left(|S_j| - \sum_{i=1}^{n} \frac{\Gamma_i f_{ij}}{\tau_{ij}} \right)$$

8.1. *The Dual Problem*

The dual of (19) can be written as

$$L(\mathcal{A},\boldsymbol{\mu}) = \sum_{i} w_i \ln U_i(\vec{f_i}) + \mu_0 \left(|F| - \sum_{i=1}^{n}\sum_{j=1}^{m} f_{ij} \right) + \sum_{j=1}^{m} \mu_j \left(|S_j| - \sum_{i=1}^{n} \frac{\Gamma_i f_{ij}}{\tau_{ij}} \right) \tag{20}$$

where $\mathcal{A} = \langle \vec{f_1}, \cdots, \vec{f_n} \rangle$ denotes the allocations and $\boldsymbol{\mu} = \langle \mu_0, \cdots, \mu_m \rangle$ are the dual coefficients.

The answer to the PF-RA problem can be obtained by

$$\max_{\mathcal{A}} \min_{\boldsymbol{\mu}>0} L(\mathcal{A},\boldsymbol{\mu}) \tag{21}$$

The interesting point is that $L(\mathcal{A},\boldsymbol{\mu})$ is separable in $\vec{f_i}$. Define

$$L_i(\vec{f_i},\boldsymbol{\mu}) = w_i \ln U_i(\vec{f_i}) - \mu_0 \sum_{j=1}^{m} f_{ij} - \sum_{j=1}^{m} \frac{\mu_j \Gamma_i f_{ij}}{\tau_{ij}} = w_i \ln U_i(\vec{f_i}) - \sum_{j=1}^{m} \left(\mu_0 + \frac{\mu_j \Gamma_i}{\tau_{ij}} \right) f_{ij} \tag{22}$$

Then (20) can be rewritten as

$$L(\mathcal{A},\boldsymbol{\mu}) = \sum_{i=1}^{n} L_i(\vec{f_i},\boldsymbol{\mu}) + \mu_0 |F| + \sum_{j=1}^{m} \mu_j |S_j| \tag{23}$$

SCHULICH School of Engineering
UNIVERSITY OF CALGARY

Cost associated with allocating f_{ij} between i and j

Net benefit of P_i

$$L_i(\vec{f}_i, \boldsymbol{\mu}) = w_i \ln U_i(\vec{f}_i) - \mu_0 \sum_{j=1}^{m} f_{ij} - \sum_{j=1}^{m} \frac{\mu_j \Gamma_i f_{ij}}{\tau_{ij}}$$

Weighted Utility (**Benefit/Gain**) associated with pick up location P_i

Cost associated with the use of Shelter j

$$L(\mathcal{A}, \boldsymbol{\mu}) = \sum_{i=1}^{n} L_i(\vec{f}_i, \boldsymbol{\mu}) + \mu_0 |F| + \sum_{j=1}^{m} \mu_j |S_j|$$

We can split the dual problem into **three smaller sub-problems and propose one algorithm for each part**

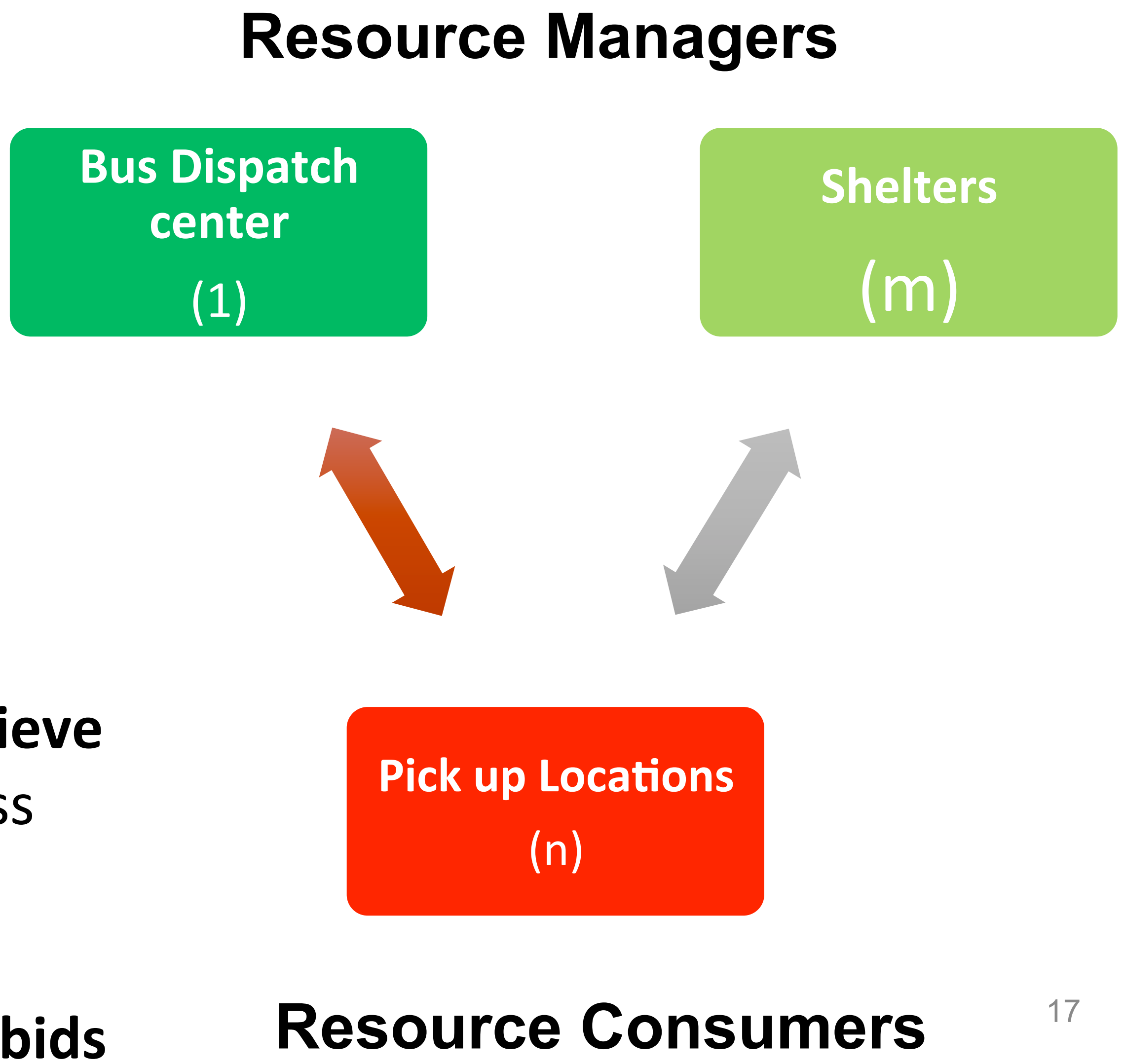
Solution Algorithm:
Proportionally Fair Dynamic Distributed Algorithm (PFD^{2}A)

Three independent agents:

1. The shelters
2. The transit vehicle dispatching center
3. The pickup locations

The central idea of the suggested algorithm is to interpret the Lagrange multipliers μ as the **virtual unit prices** associated with resource consumption

- Total number of agents: **n+m+1**
- **Agents** interact by working **cooperatively to achieve the single goal** of optimizing proportional fairness during the evacuation process
- Agents communicate through **publishing prices/bids**



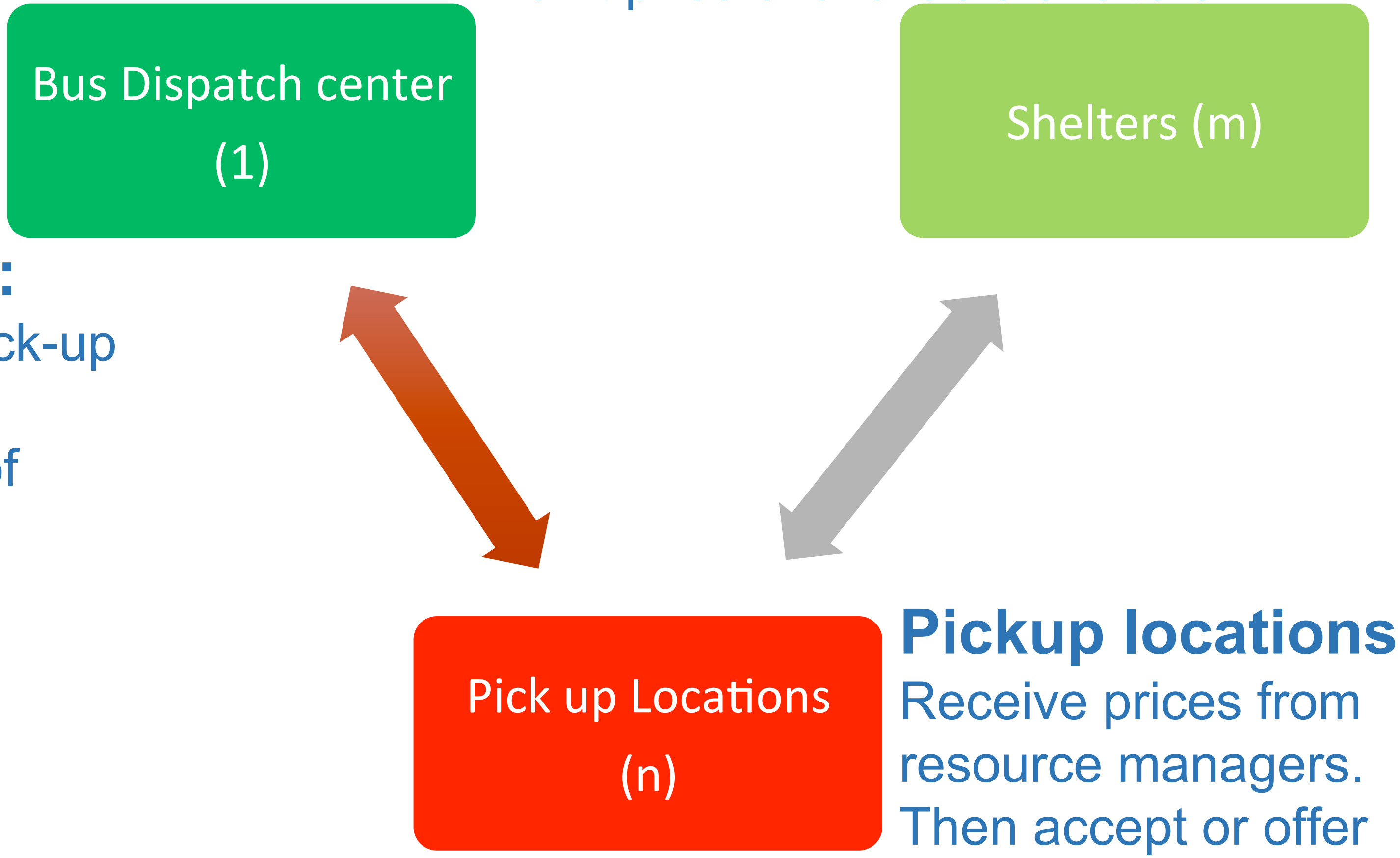
Solution Algorithm (Cntd.) PFD²A

SCHULICH School of Engineering
UNIVERSITY OF CALGARY

Iterative Algorithm – Market Clearance Price:

Shelters: Receive bids from pick-up locations and decide about the unit price of available shelters

Resource Managers:



Dispatch Center:
Receive bids from pick-up locations and decide about the unit price of available Resource

Resource Consumers:

Pickup locations:
Receive prices from resource managers. Then accept or offer new Bids

Given the price vector $\boldsymbol{\mu}$ for the resources, P_i submits the following bid vector:

$$\boldsymbol{\nu}_i = \langle \nu_{i0}, \nu_{i1}, \cdots, \nu_{in} \rangle = \langle \mu_0 \sum_{j=1}^{n} f_{ij}^*, \frac{\mu_1 \Gamma_i f_{i1}^*}{\tau_{i1}}, \cdots, \frac{\mu_n \Gamma_i f_{in}^*}{\tau_{in}} \rangle \tag{30}$$

where ν_{i0} denotes the bid of P_i for the transit vehicles, ν_{ij}, $j = \{1, \cdots, n\}$ denote the bid of P_i for shelter S_j, and $\vec{f}_i^* = \langle f_{i1}^*, \cdots, f_{in}^* \rangle = \text{argmax}_{\vec{f}} L_i(\vec{f}, \boldsymbol{\mu}(t))$. Note that Lemma 8.1 denotes that $\vec{f}_i^*$ can be efficiently found.

To find the optimal prices, we compute the gradient of L:

$$\nabla_{\boldsymbol{\mu}} L = \langle |F| - \sum_{i=1}^{n} \sum_{j=1}^{m} f_{i1}, |S_1| - \sum_{i=1}^{n} \frac{\Gamma_i f_{i1}}{\tau_{i1}}, \cdots, |S_m| - \sum_{i=1}^{n} \frac{\Gamma_i f_{im}}{\tau_{im}} \rangle \tag{31}$$

At the optimal point we have:

$$|F| - \sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij} = 0 \quad \text{and} \quad |S_j| - \sum_{i=1}^{n} \frac{\Gamma_i f_{ij}}{\tau_{ij}} = 0, \text{ for } j \in \{1, \cdots, m\} \tag{32}$$

Since $\mu_0 \sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij}^* = \sum_{i=1}^{n} \nu_{i0}$ and $\sum_{i=1}^{n} \nu_{ij} = \sum_{i=1}^{n} \frac{\mu_j \Gamma_i f_{ij}^*}{\tau_{ij}}$ for $j \in \{1, \cdots, m\}$, we have:

$$\mu_0^* = \frac{\sum_{i=1}^{n} \nu_{i0}}{|F|} \quad \text{and} \quad \mu_j^* = \frac{\sum_{i=1}^{n} \nu_{ij}}{|S_j|}, \text{ for } j \in \{1, \cdots, m\} \tag{33}$$

Resource Consumers

SCHULICH School of Engineering UNIVERSITY OF CALGARY

Algorithm 1 Executed by pickup location P_i

procedure ADJUST $\vec{f_i}$

  Initialize $t = 0$

  loop

    Receive $\boldsymbol{\mu}(t)$ ▷ Blocks until the next μ is published by Algorithm 2 and Algorithm 3

    $\vec{f_i^*}(t+1) = \underset{\vec{f}}{\operatorname{argmax}}\, L_i(\vec{f}, \boldsymbol{\mu}(t))$

    Compute $\boldsymbol{\nu}_i(t+1) = \langle \mu_0 \sum_{j=1}^{n} f_{ij}^*, \frac{\mu_1 \Gamma_i f_{i1}^*}{\tau_{i1}^{(t)}}, \cdots, \frac{\mu_n \Gamma_i f_{in}^*}{\tau_{in}^{(t)}} \rangle$

    Publish $\nu_i(t+1)$ ▷ The value will be received by resource managers

    $t = t + 1$

  end loop

end procedure

Resource Managers

SCHULICH
School of Engineering
UNIVERSITY OF CALGARY

Algorithm 2 Executed by transit vehicle dispatch center

procedure ADJUST μ_0
  Initialize $\mu_0(0)$ to a positive number, $t = 0$
  loop
    Publish $\mu_0(t)$ ▹ The value will be received by pickup locations
    For all i, receive $\nu_{i0}(t+1)$ from pickup location P_i
    $\mu_0(t+1) = \frac{\sum_{i=1}^{n} \nu_{i0}}{|F|}$
    $t = t + 1$
  end loop
end procedure

Algorithm 3 Executed by shelter S_j

procedure ADJUST μ_j
  Initialize $\mu_j(0)$ to a positive number, $t = 0$
  loop
    Publish $\mu_j(t)$ ▹ The value will be received by pickup locations
    For all i, receive $\nu_{ij}(t+1)$ from pickup location P_i
    $\mu_j(t+1) = \frac{\sum_{i=1}^{n} \nu_{ij}}{|S_j|}$
    $t = t + 1$
  end loop
end procedure

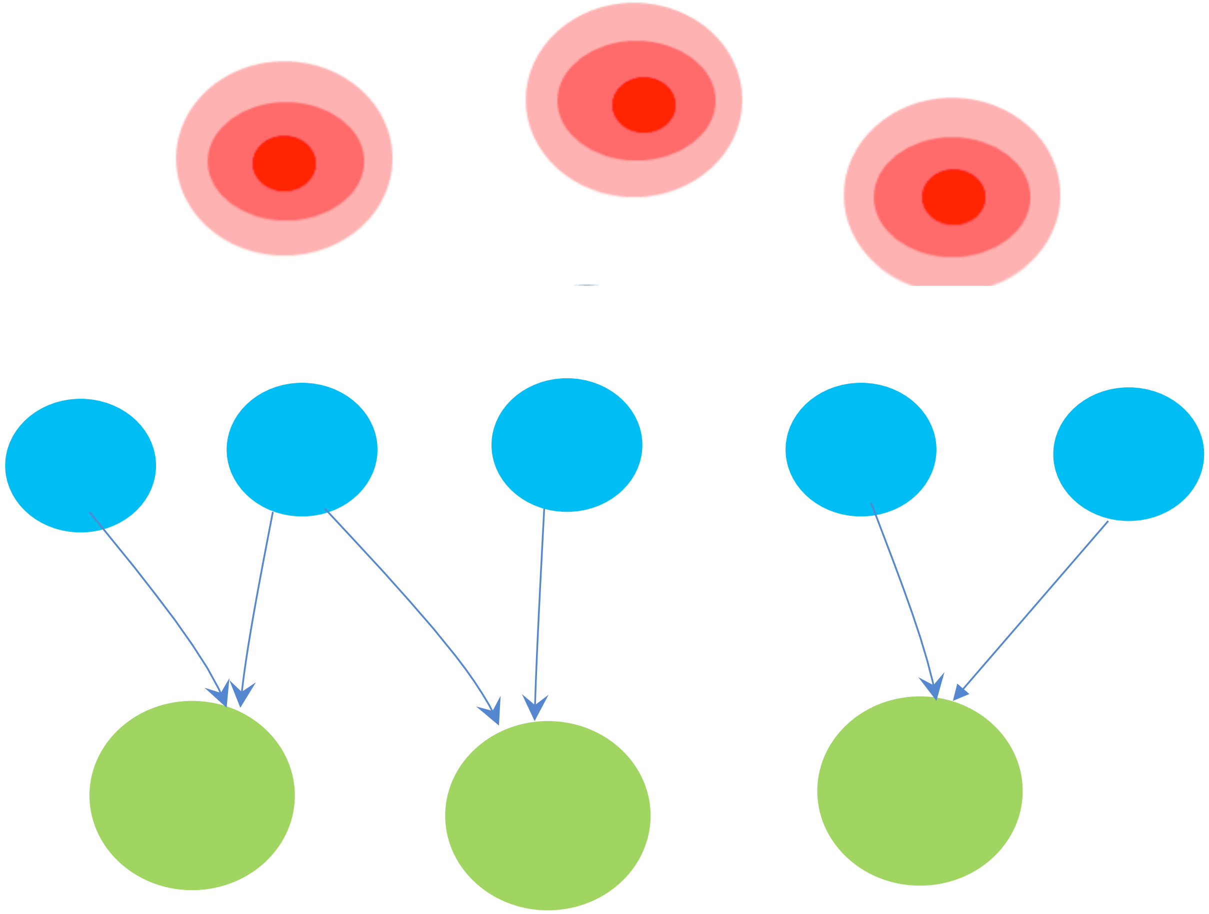
Numerical Example

5 Pick Up Locations and 3 shelters

$$\langle |P_1|, \cdots, |P_5| \rangle = \langle 1000, 2000, 3000, 4000, 5000 \rangle$$
$$\langle |S_1|, |S_2|, |S_3| \rangle = \langle 4500, 5500, 6500 \rangle$$

The fleet size is 500. The round trip times are supposed to be

$$[\tau_{ij}] = \begin{bmatrix} 10 & 20 & 30 \\ 15 & 15 & 15 \\ 30 & 25 & 20 \\ 30 & 25 & 20 \\ 40 & 30 & 20 \end{bmatrix}$$



Different Variations of the Resource Allocation Problem

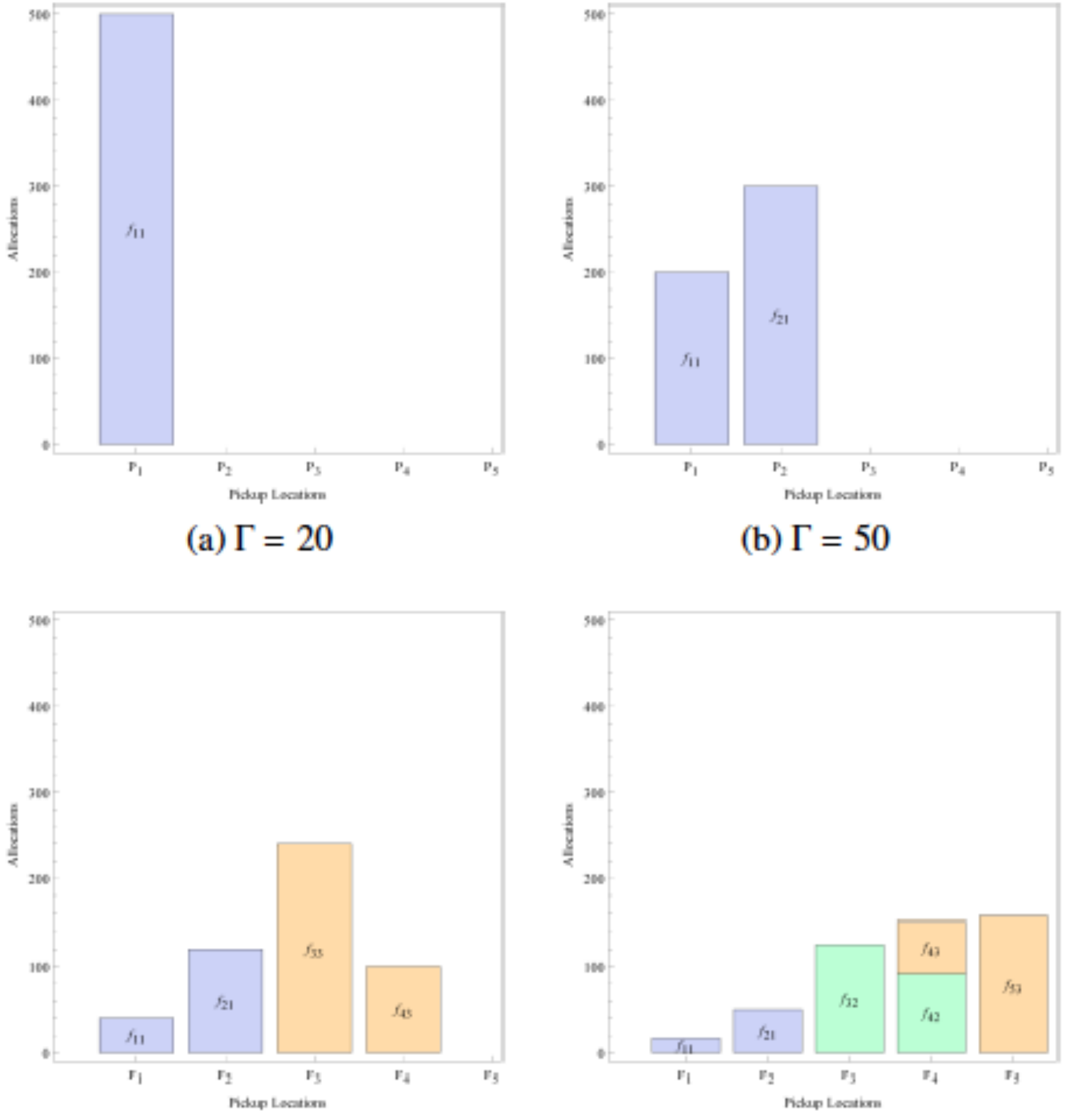
- **Maximum evacuation rate resource allocation (MR-RA):** The objective is to maximize the number of evacuees who reach safety by a given evacuation deadline.

- **Maximum social welfare resource allocation (MSW-RA):** maximizing the summation of the weighted utility functions of the pickup locations while the severity of the disaster in each pick-up location and evacuation deadlines are considered.

- **Proportionally fair resource allocation (**PF-RA): The objective is to allocate the resources among different pick-up locations according to the criterion of proportional fairness

Results of MR-RA: Maximum evacuation rate resource allocation

$$[\tau_{ij}] = \begin{bmatrix} 10 & 20 & 30 \\ 15 & 15 & 15 \\ 30 & 25 & 20 \\ 30 & 25 & 20 \\ 40 & 30 & 20 \end{bmatrix}$$



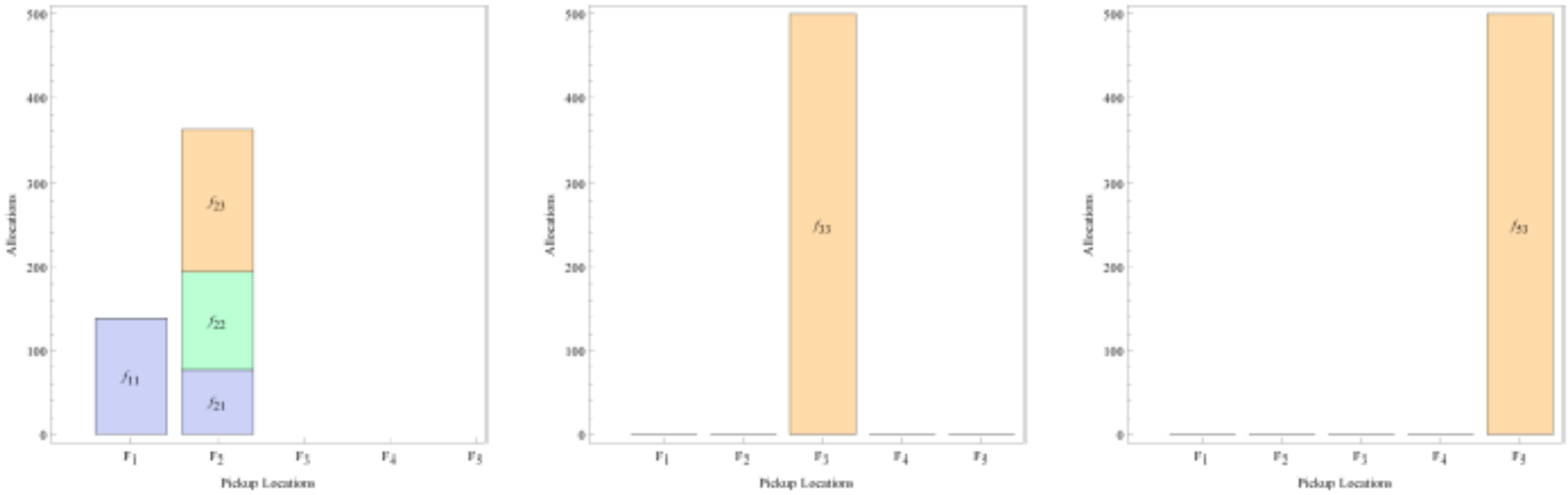
(a) Γ = 20

(b) Γ = 50

(c) Γ = 250

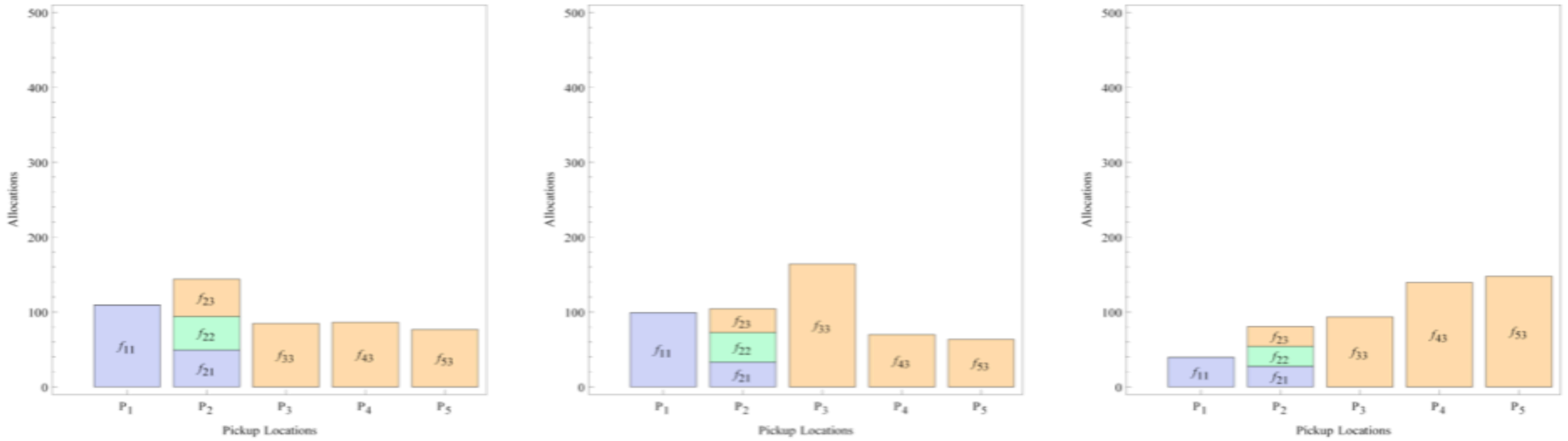
(d) Γ = 600

Results of MSW-RA: **Maximum social welfare**
(i.e. summation of the weighted utility functions of the pickup locations)



(a) weights=⟨1, 1, 1, 1, 1⟩ (b) weights=⟨1, 1, 2, 1, 1⟩ (c) weights=⟨1, 2, 3, 4, 5⟩

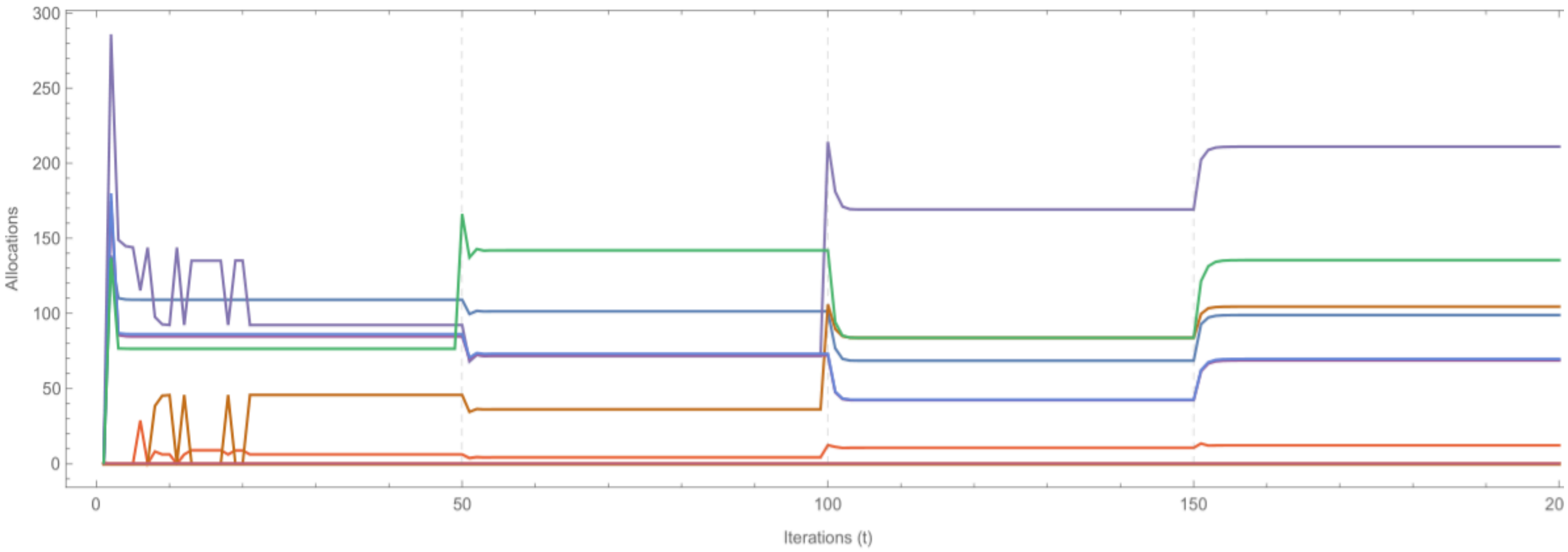
Results of PF-RA: Weighted Proportional Fairness: **assigns a non-zero share to each pickup location**



(a) weights=⟨1, 1, 1, 1, 1⟩ (b) weights=⟨1, 1, 2, 1, 1⟩ (c) weights=⟨1, 2, 3, 4, 5⟩

Clearance time of pickup locations for different weights w_i (*i.e.*, severity) with PF-RA.

SCHULICH School of Engineering UNIVERSITY OF CALGARY



Introducing changes over time

- At iteration 50, the **population** of P_5 is reduced by 10% and its **severity** level is doubled.
- At iteration 100, the **population** of P_2 is increased by 20% and its **severity** is tripled.
- At iteration 150, the **fleet size** is changed from 500 to 700.

PFD^{2}A is able to converge quickly and adapt to the changes in the parameters.

Summary of the Results:

- **MR-RA** *is biased and favors the pickup locations closer to the shelters* in order to maximize the number of evacuees reaching safety.

- **MSW-RA** is extremely *unfair in some cases and assigns no resource* to some of the pickup locations.

- **PF-RA** was shown to have the following properties:
  1. Is fair, while it tries not to sacrifice efficiency for fairness.
  2. Can handle different severity levels and deadlines.
  3. Can adapt to changes in the evacuation parameters (population, deadlines, severity and travel times).
  4. Can be efficiently solved.

Contribution of the paper

SCHULICH School of Engineering
UNIVERSITY OF CALGARY

- Introducing the semantic of "proportional fairness " to the emergency evacuation problem: Can be applicable to many other transportation problems where the focus is achieving Fairness

- Developing a **dynamic and distributed** algorithm (PFD^2 A) based on the Lagrangian dual method to find a proportional fair allocation of resources to respond to the dynamic changes in the emergency situation

- Developing a unified method to analyze/compare different variation of the problem (Max. evacuation rate, Maximizing social welfare, etc..)

Limitations and Possible Extensions

- This paper focuses proportional fairness for the population that relies solely on transit for evacuation.
- In real life situation, depending on the type of disaster, some people may choose to evacuate on foot, others would take public transit, while the rest of the evacuees would take personal vehicles
- Considering personal vehicles as part of the evacuation may increase transit travel time. Mass panic may result in multiple accident or extremely over utilized routes, possibly blocking some emergency evacuation routes and thus high unreliability in the estimates of travel time.

Thank You!