Integrated Public Transport Timetable Synchronization and Vehicle Scheduling with Demand Assignment: A Bi-objective Bi-level Model Using Deficit Function Approach

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Outline:
1. Motivations and Previous Studies
2. Background of the Deficit Function
3. Objectives
4. Optimization Framework
5. Mathematical Model
6. Solution Method and Numerical Studies
7. Conclusions
8. Future Perspectives

What is a perfect Public-Transport transfer connection? How this can be materialized?

Cartoons courtesy of Avi Ceder
Motivation

Why Integrated PT Timetable Synchronization and Vehicle Scheduling is Important?

- **Synchronized timetables** in public transport (PT) networks are used to reduce the inter-route or inter-modal passenger transfer waiting time and provide a well-connected service.

- Design of **intelligent PT timetable synchronization** is one way to improve the integration and service quality of a PT system with increased connectivity, synchronization, and attractiveness towards a far more user-oriented, system-optimal, smart and sustainable travel.

- Most previous studies on the **PT timetable synchronization problem** have treated the problem separately, rather than coupled with, or isolated from, other operation planning activities, such as vehicle scheduling, transit assignment and network design.
Previous Studies

Main Approaches used in Previous Studies:

- **Graphical Optimization Approach**
  - Computer-aided interactive graphic system, e.g., (Rapp and Gehner, 1976)
  - Graphical person-computer interactive, e.g., (Liu et al., 2017)
  - Timed transfer system (TTS), e.g., (Vuchic, 1981; 2005)

- **Continuum Approximation-based Analytical Approach**
  - Continuum approximate analytical models, e.g., (Wirasinghe et al., 1977) and (Wirasinghe, 1980)
  - Optimal design parameters, e.g., (Lee and Schonfeld, 1991)
  - Analytical formulations for idealized PT systems

- **Mathematical Programming Approach**
  - 0-1 integer programming model, e.g., (Voß, 1992)
  - Heuristic/metaheuristic algorithms, e.g., regret methods, optimization-based heuristic, simulated annealing, tabu search, e.g., (Wong et al., 2008)
  - Setting the headways and offset times of routes e.g., (Ceder et al., 2001)

- **Control Theory-based Approach**
  - Operational control strategies/tactics, e.g., holding, speed-changing, skip-stop
  - Predictive control, adaptive control, e.g., (Dessouky et al., 2003); (Liu et al., 2015)
Deficit Function at $k = d(k, t)$:

The total number of trip departures at $k$ less the total number of arrivals at $k$-up to and including time $t$

The fleet size Theorem

For a given set of terminals $T$ and a fixed schedule of trips $S$, the minimum number of vehicles required to service all trips in $S$ is:

$$N(s) = \sum_{k \in T} D(k) = \sum_k \max d(k, t)$$
DEFICIT FUNCTION THEORY
(+1 every departure, -1 every arrival)

Five chains:
1-3
2-6
5-9
7-4
8-DH-10

Fixed Schedule

3 veh = min at "a"

Arrival (-1)

d(a,t)

d(b,t)

Deadheading trip (DH)

Time
Shifting consideration of departure times within given tolerances

What tools will help to minimize the number of vehicles?
Objectives

- To provide a bi-objective, bi-level decision-making framework for the integrated PT timetable synchronization and vehicle scheduling problem considering passenger demand assignment.

- To formulate comprehensive objective function components that take both PT users and operators interests into account including five system-performance measures, i.e., total passenger in-vehicle travel time, total initial waiting time, total transfer waiting time, total passenger load discrepancy or overcrowding hours, and fleet size.

- To develop a novel deficit function (DF)-based sequential search method combined with network flow and shifting vehicle departure time techniques for solving the proposed mathematical model.
The upper level model from the perspective of PT operators aims at: (i) minimize total operation costs, which are related to fleet size, and (ii) minimize total passenger-hour cost, which include the total passenger in-vehicle travel time, total passenger initial waiting time, total passenger transfer waiting time, and total passenger load discrepancy or overcrowding hours.

The lower level problem from the perspective of PT passengers is a standard schedule-based transit assignment problem with capacity constraints.

ITSVS=Integrated Timetable Synchronization and Vehicle Scheduling

TA=Transit Assignment

Schematic representation of the bi-objective, bi-level optimization framework
Mathematical Model (a)

The Upper Level Integrated Timetable Synchronization and Vehicle Scheduling Problem

Objective Functions:

- to minimize the total passenger-hour cost in the system (PT users perspective)
- to minimize the number of vehicles required, i.e., fleet size (PT operators perspective)

\[ Z_1 = \alpha_1 \sum_{i, j \in N} PH(i, j) + \alpha_2 \sum_{i, j \in N} IWT(i, j) + \alpha_3 \sum_{i, j \in N} TWT(i, j) + \alpha_4 \sum_{r \in R} LD_r \]

\[ Z_2 = FS \]

\[ \text{PH}(i, j) = \text{Passenger hours between nodes } i \text{ and } j, \quad i, j \in N \text{ (defined as passenger riding time in a PT vehicle on an hourly basis; it measures the time spent by passengers in vehicles between the two nodes);} \]

\[ \text{IWT}(i, j) = \text{Initial waiting time between nodes } i \text{ and } j, \quad i, j \in N \text{ (defined as the amount of time passengers spend at the boarding stops between the two nodes);} \]

\[ \text{TWT}(i, j) = \text{Transfer waiting time between nodes } i \text{ and } j, \quad i, j \in N \text{ (defined as the amount of time passengers spend at the transfer stops between the two nodes);} \]

\[ \text{LD}_r = \text{Passenger load discrepancy cost on route } r \text{ (defined as the difference between the expected load and the desired occupancy multiplied by the corresponding route segment travel time; passenger load discrepancy cost measures the overcrowding on vehicles);} \]

\[ \text{FS} = \text{Fleet size (defined as the number of PT vehicles needed to provide all trips along a chosen set of routes);} \]

\[ \alpha_k = \text{Monetary or other weights, } k=1, 2, 3, 4. \]
Mathematical Model (b)

**Constraints**

**Bundle departure constraints**
\[ \sum_{m=L_r}^{U_r} x_r^m = 1, \quad \forall r \in R \]

**Fleet size constraint**
\[ \sum_{u \in U} D(u) \leq N_0 = FS \]
\[ D(u) \in \mathbb{N}^0, \quad \forall u \in U \]

**Deficit function (DF) bounds constraints**
\[ d(u,t) \leq D(u), \quad t \in [T_1, T_2], u \in U \]

**Decision variable constraints**
\[ x_r^m = \begin{cases} 1, & \text{if } m \text{ even-headway departures are selected for route } r \smallskip \text{otherwise.} \\ 0, & \end{cases} \quad \forall m \in M, r \in R \]

**Offset times**

Deficit function-based shifting departure time (SDT) procedure

**Remark**: The optimal solution set \( \{x_r^m\} \) indicates the optimal number of departures. From \( \{x_r^m\} \) an optimal timetable can be constructed with proper route offset times. Given the generated timetable, a vehicle schedule can be easily derived using the first-in-first-out (FIFO) rule [see Ceder and Stern, (1981) and Ceder, (2016) for details] or the chain extraction (CE) procedure [see Gertsbach and Gurevich, (1977) for details].
Mathematical Model (c)

The Lower Level Transit Assignment (TA) Problem

An illustration of transition from a physical network to a diachronic graph

Algorithm 1 Capacity restrained incremental assignment

Step 0 (Preliminaries): Calculate initial passenger O-D demand $d_{i1}$ and number of passengers $L_{i1}$ on vehicle at stop $i$. Set $n := 1$.

Step 1 (Incremental loading): If $d_{in} \leq \lambda C_v - L_{in}$, then the packet of $d_{jn}$ passenger demand is loaded onto the $n$-shortest path, i.e., $L_{in} := L_{in} + d_{jn}$, and stop; otherwise, the packet of $(\lambda C_v - L_{in})$ demand is loaded onto the $n$-shortest path, i.e., $L_{in} := L_{in} + (\lambda C_v - L_{in})$.

Step 2 (Update): Set $n := n + 1$, $d_{in} = d_{i(n-1)} - (\lambda C_v - L_{i(n-1)})$, and go to Step 1.
Mathematical Model (d)

Model Integration

\[
\begin{align*}
\min_{\mathbf{s}} Z_1(\mathbf{S}, \mathbf{L}(\mathbf{S})) &= \alpha_1 \sum_{i,j \in N} PH(i, j) + \alpha_2 \sum_{i,j \in N} IWT(i, j) + \alpha_3 \sum_{i,j \in N} TWT(i, j) + \alpha_4 \sum_{r \in R} LD_r \\
\min_{\mathbf{s}} Z_2(\mathbf{S}, \mathbf{L}(\mathbf{S})) &= FS \\
n\text{s.t.} \quad \text{Timetable synchronization and vehicle scheduling constraints: (20)-(24)} \\
\text{where the passenger load on vehicles } \mathbf{L}(\mathbf{S}) \text{ is obtained by solving the following TA problem:} \\
\min_{\mathbf{L}} f(\mathbf{S}, \mathbf{L}) \\
n\text{s.t.} \quad \text{vehicle capacity constraint: } L_i \leq \lambda C_v, \forall i \in N \\
\text{passenger vehicle run choice and flow propagation constraints defined in Algorithm 1}
\end{align*}
\]

Complexity Analysis

The nature of the upper level ITSVS model is **non-linear, bi-objective integer programming with linear constraints**, which is a special case of the **integer quadratic programming problem** that is known to be a **NP-hard problem**. In addition, solving the upper level ITSVS problem requires solving the lower level TA problem that is in effect a **non-linear constraint** of the upper level model, which makes the whole problem **non-convex**. Due to its intrinsic, **non-linear and non-convex complexity**, the bi-objective, bi-level ITSVS-TA problem is extremely difficult, to solve mathematically, especially for large scale networks, for an **optimum global solution**.
**Solution Method-1**

**Network Flow Technique for Vehicle Scheduling with Fixed Schedule**

Network flow-based model for estimating the minimum fleet size

\[
\begin{align*}
\text{Max } N_1 &= \sum_{g \in G} \sum_{g' \in G} x_{gg'} \\
\text{s.t. } \sum_{g' \in G} x_{gg'} &\leq 1, \quad g \in G \\
\sum_{g \in G} x_{gg'} &\leq 1, \quad g' \in G \\
x_{gg'} &\in \{0,1\}, \quad \text{all } (g,g') \text{ admissible} \\
x_{gg'} &= 0, \quad \text{all } (g,g') \text{ inadmissible}
\end{align*}
\]

A solution with \( x_{gg'} = 1 \) indicates that trips \( g \) and \( g' \) are joined. The objective function maximizes the number of such joinings.

**The Max-Flow Fleet Size Theorem**

**Theorem 2** (The max-flow fleet size theorem). Let \( N_{MF}(S) \) and \( n \) denote the number of chains and trips of schedule \( S \), respectively. Then,

\[
\text{Min } N_{MF}(S) = n - \text{Max } N_1
\]

**Proof.** Given a set of \( G = \{g : g = 1, \ldots, n\} \) required trips. Assigning each trip separately to an individual vehicle results in a fleet size of \( n \) vehicles. If \( x_{gg'} = 1 \), then trip \( g' \) can be performed after trip \( g \) by the same vehicle \( v_g \). Thus, the vehicle \( v_{g'} \) assigned to trip \( g' \) can be saved. The required fleet size thus can be reduced from \( n \) to \( n - 1 \). Similarly, the value of max-flow \( \text{Max } N_1 \) means \( \text{Max } N_1 \) vehicles can be saved by linking trips together. Thus, the minimum number of vehicles required to perform all trips in \( G \) is \( n - \text{Max } N_1 \). This completes the proof. \( \square \)
An example vehicle scheduling problem using max-flow technique and its equivalent DF solution.
Solution Method-3

Deficit Function-based Sequential Search Method

A Computerized Procedure for the ITSVS-TA:
decompose the original bi-objective model to a series of one-objective models

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**Algorithm 2** Deficit Function-based Sequential Search (DF-SS) method

**Step 0** *(Lower bound and upper bound calculation):* Apply the network flow technique to obtain an initial lower bound of the required fleet size \( N_L' \). Construct the DF for each route \( r \) with \( m = L_r, \ r \in R, \) and calculate the lower bound of the fleet size as \( N_L = \min \left\{ N_L', \sum_{u \in U} D(u) \right\} \); construct the DF for each route \( r \) with \( m = U_r, \ r \in R, \) and calculate the upper bound of the fleet size as
\[
N_U = \min \left\{ n, \sum_{u \in U} D(u) \right\}.
\]

**Step 1** *(Initialization):* Set \( v = 1, \ Z_2^1 = 1, \ Z = [0]_{N_U - N_L + 1} \times 2, \ X = [0]_{N_U - N_L + 1} \times |R| \).

**Step 2** *(\( Z_1^v \) Calculation):* Decompose the original bi-objective model to a single objective model with \( Z_2^v \) known. Solve the resulting single objective integer programming model. This yields the value of \( Z_1^v \) and a solution set of \( [x_{n_1}^{m,v}, x_{n_2}^{m,v}, \ldots, x_{n|l|}^{m,v}] \). Replace the \( v \)-th row of \( Z \) with \( [Z_1^v, Z_2^v] \); replace the \( v \)-th row of \( X \) with \( [x_{n_1}^{m,v}, x_{n_2}^{m,v}, \ldots, x_{n|l|}^{m,v}] \).

**Step 3** *(Move):* Set \( Z_2^{v+1} = Z_2^v + 1 \).

**Step 4** *(Stopping rule):* If \( Z_2^{v+1} > N_U \), go to Step 5; otherwise, set \( v := v + 1 \) and go to Step 3.

**Step 5** *(Solution output):* Generate Pareto-efficient solutions from matrix \( X \) with associated objective function values \( (Z_1, Z_2) \) generated by matrix \( Z \).
## Solution Method-4

### Overall Solution Procedure

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 0</td>
<td>Estimate possible route frequencies based on O-D demand, vehicle capacity, policy frequency requirement and other parameter values.</td>
</tr>
<tr>
<td>Step 1</td>
<td>Generate Pareto-efficient solutions using Algorithm 1 and Algorithm 2.</td>
</tr>
<tr>
<td>Step 2</td>
<td>Modify route offset times using the SDT procedure, based on schedulers’ DF observations, personal preferences or other practical considerations, so as to generate more possible Pareto-efficient solutions.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Display the combined Pareto-efficient solutions obtained in Step 1 and Step 2 in a fleet cost two-dimensional (2D) space.</td>
</tr>
</tbody>
</table>
Numerical Studies-1

A Small Numerical Example

Example network, its demand data, and the construction of load profiles

Example network with average travel time in minutes (given)

Origin-destination (O-D) matrix in passenger per hour (given)

The load profile of route $r_{a \to b}$ (derived)

The load profile of route $r_{b \to a}$ (derived)
Numerical Studies-1: Results

Trade-off between passenger-hour cost and vehicle fleet size
The analysis begins with setting a set of possible number of departures (frequencies) for each route. To this end, a network synthesis process (see Ceder 2016 for details) is utilized to generate a set of initial frequencies based on passenger O-D demand.
Numerical Studies-2: Analysis and Results

Assignment result on the diachronic graph applied to the Spiess-Florian network

Pareto-efficient solutions of the Spiess-Florian network case study
Conclusions

- Create a methodology utilizing a multi-criteria decision-making framework for the integrated PT timetable synchronization and vehicle scheduling problem with passenger demand assignment taking into consideration both the PT users and operators interests.
- Propose a useful deficit function (DF)-based sequential search (DF-SS) method, combined with a network-flow technique and a shifting departure time (route offset time) procedure for solving the problem to obtain a set of Pareto-efficient solutions.
- Facilitate the decision-making process of PT schedulers using graphical features of the DF and the two-dimensional (2D) fleet-cost space in finding a desired solution.
- Demonstrate that the numerical results of the proposed model and solution method are effective and have a potential for being applied to large-scale realistic PT networks.
Future Perspective: Moving Platform
Food for Thought
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End of the Presentation
Thank you!