The Intelligent-Driver Model with Stochasticity: New Insights into Traffic Flow Oscillations

Martin Treiber
TU Dresden
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Overview

➢ Traffic Flow Oscillations: Empirical Facts
  - Stationary detector data
  - Trajectory data
  - Floating-car data
  - Platoon and ring experiments

➢ A General Model Framework
  - Instability
  - Noise
  - Action points

➢ Results
Empirical Facts: Oscillations from Stationary Detector Data (A9-South near Munich)

- **Downstream front:** Either fixed or moving upstream with velocity $c$
- **Upstream front:** Determined by supply and demand
- **Internal structures:** Moving with velocity $c$ as well
- **Amplitude** of internal structures grows when moving upstream
- **Frequency** grows with severity of bottleneck

(a) A9 Süd, 29.10.1998

German A9 South, Oct 29, 1998
... from Trajectory Data
(NGSIM I=80, reconstructed)
... from Floating-Car Data
(TomTom, A99 South near Munich)

- TOMTOM: approx
  700,000 vehicles X 1
  data telegram/min
= $10^9$ datasets/day
Platoon behind a leading car driving at 30 km/h: Speed as a function of time and vehicle number.

Speed standard deviation once stationary...
... and in Bicycle Experiments (Jiang et al., 2016)
II. General Framework

Time-continuous car-following model

\[ \frac{dv_n}{dt} = f(s_n, v_n, v_l) + \xi_n(t) \]

Relative distance from instability threshold

\[ \epsilon = 1 - \frac{a}{a_c} \]

Acceleration noise

\[ \langle \xi_n(t) \rangle = 0, \]
\[ \langle \xi_n(t)\xi_m(t') \rangle = Q\delta_{nm}\delta(t-t') \]

=> Mechanisms I (flow instability) and II (noise)

Action points:

Change acceleration only if

\[ |f(s_i(t), v_i(t), v_l(t)) - f(s_i(t'), v_i(t'), v_l(t'))| > \Delta a, \]
\[ \Delta a \sim U(0, \Delta a_{\text{max}}) \]

=> Mechanism III: indifference zone
Mechanism II: White Noise Creates Correlated Fluctuations!
Mechanism III: Action Points in Action

[Graph showing acceleration over time for different vehicles (Veh 1, Veh 5, Veh 10, Veh 15, Veh 25).]
Subcritical Noise-Induced Oscillations: Analytics

Linearisation of the stochastic equation w/o action points:

\[
\frac{dy_n}{dt} = u_{n-1} - u_n,
\]

\[
\frac{du_n}{dt} = f_s y_n + f_v u_n + f_l u_{n-1} + \xi_n(t)
\]

\[y_n = \text{gap deviation}\]
\[u_n = \text{speed deviation}\]

Partial derivatives of the acceleration function at the steady-state point:

\[f_s = \frac{\partial f}{\partial s}\]
\[f_v = \frac{\partial f}{\partial v}\]
\[f_l = \frac{\partial f}{\partial v_l}\]
Fourier Ansatz

modal expansion

\[
\begin{pmatrix}
    y_n(t) \\
    u_n(t)
\end{pmatrix} = \sum_k \begin{pmatrix}
    \hat{y}_k(t) \\
    \hat{u}_k(t)
\end{pmatrix} e^{ink}
\]

resulting stochastic linear 2X2 system

\[
\frac{d}{dt} \vec{X}_k = -L_k \vec{X}_k + \vec{\xi}
\]

\[
\vec{X}_k = \begin{pmatrix}
    \hat{y}_k \\
    \hat{u}_k
\end{pmatrix}
\]

modal noise term

\[
\langle \vec{\xi}(t) \rangle = 0, \quad \langle \vec{\xi}(t) \vec{\xi}'(t') \rangle = D \delta(t - t')
\]

\[
D = \frac{Q}{N} \begin{pmatrix}
    0 & 0 \\
    0 & 1
\end{pmatrix}
\]

\[
L_k = \begin{pmatrix}
    0 & 1 - e^{-ik} \\
    -f_s & -(f_v + f_le^{-ik})
\end{pmatrix}
\]
Solution: Fluctuation-Dissipation Theorem

$$S_k(\omega) = \frac{Q}{N |\text{Det}(L + i\omega \mathbf{1})|^2} \begin{pmatrix} 2(1 - \cos k) & i\omega(1 - e^{-ik}) \\ -i\omega(1 - e^{ik}) & \omega^2 \end{pmatrix}$$
Total Fluctuation Intensity

modal fluctuation intensity

\[ S_{uu_k} = \int S_{uuk}(\omega) \, d\omega \]

spectral intensity

\[ S_{uu}(\omega) = \sum_k S_{uuk}(\omega) \]
Total Fluctuation Intensity

\[ S_{uu} = \sum_k \int S_{uuk}(\omega) \, d\omega \]
Reproducing the Platoon Experiment
(Leader's Speed=30 km/h)

Mechanism I: Instability, alone

deterministic followers (IDM, $\varepsilon=0.6$, $Q=0$, $\Delta a_{\text{max}}=0$)
with noisy leader
Reproducing the Platoon Experiment

(Leader's Speed=30 km/h)

Mechanism II: noise at marginal stability

speed time series

speed standard deviation

stochastic followers (IDM, \( \varepsilon=0 \), \( Q=0.32 \text{ m}^2/\text{s}^3 \), \( \Delta a_{\text{max}}=0 \)), same for the leader
Reproducing the Platoon Experiment
(Leader's Speed=30 km/h)

Mechanism III: action points at marginal stability

speed time series

speed standard deviation

stochastic followers (IDM, $\varepsilon=0$, $Q=0$, $\Delta a_{\text{max}}=1$ m/s$^2$),
same for the leader
Applying the Mechanisms to Other Models

(a) Speed [km/h] vs Time [s]

(b) Speed standard deviation [m/s]

(c) Speed [km/h] vs Time [s]

(d) Speed standard deviation [m/s]

(e) Speed [km/h] vs Time [s]

(f) Speed standard deviation [m/s]
Reproducing the Bicycle Ring Experiment

Jiang et al., 2016

subcritical IDM with noise (v₀=4 m/s)
IDM @ Low Desired Speeds: even Subcritical Regime ($\varepsilon=-0.2$) leads to Oscillations

IDM with noise

Deterministic IDM