Unifiable multi-commodity kinematic wave model

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Outline

• Introduction
• Fundamental diagrams
• Kinematic wave model
• Cell Transmission Model
• Examples
• Conclusion
Introduction
Multi-commodity traffic on SR-91
Motivation

- (Rey, Jin, and Ritchie, 2016) Trajectory estimation with Newell’s model
LWR vs non-FIFO

• LWR for multilane roads
  – empirical observation of FD
  – shock wave analysis
  – queue/delay estimation
  – incident management

• non-FIFO
  – different lanes
  – different aggressiveness
  – pricing schemes: HOT, tradable credits
Notations: Commodity and total variables

• commodity $m$ ($m=1,\cdots,M$):
  – density: $k_{\downarrow m}(x,t)$
  – speed: $v_{\downarrow m}(x,t)$
  – flow-rate: $q_{\downarrow m}(x,t)$
  – unit vector: $e_{\downarrow m}$

• vectors of commodity variables:
  – $k(x,t) = \sum_{m=1}^{M} k_{\downarrow m}(x,t) e_{\downarrow m}$
  – $v(x,t)$
  – $q(x,t)$

• total traffic:
  – density: $k(x,t)$
  – speed: $v(x,t)$
  – flow-rate: $q(x,t)$
Multi-commodity kinematic wave models

• (R1) Additive relations
  – \( k = \sum_{m=1}^{M} \kdown m \), \( q = \sum_{m=1}^{M} qdown m \)

• (R2) Commodity and total constitutive laws:
  – \( qdown m = kdown m \nudown m \)
  – \( q = kv \)

• (R3) Commodity speed-density relations:
  – \( vdown m = \eta down m (k) \)

• (R4) Commodity conservation equations:
  – \( \partial kdown m / \partial t + \partial qdown m / \partial x = 0 \)

• (R5) Weak solutions: discontinuous shock waves

• (R6) Entropy conditions: unique, physical laws

• Multi-commodity kinematic wave model: a system of \( M \) conservation equations:
  – \( \partial kdown m / \partial t + \partial kdown m \eta down m (k) / \partial x = 0 \)
FIFO vs Unifiable

- Total traffic speed: $v = \eta(k) \equiv \sum_{m=1}^{M} k^m \eta^m(k) \equiv \frac{\sum_{m=1}^{M} k^m \eta^m(k)}{\sum_{m=1}^{M} k^m}$

- FIFO: same commodity speeds
  - $v^1 = \ldots = v^M = v$
  - $\eta^1(k) = \ldots = \eta^M(k) = \eta(k)$
  - $q^m / q = k^m / k$
  - FIFO-MCKW: $M \times M$ Temple system of $(k, k^1, \ldots, k^{M-1})$

- Unifiable: $v = \eta(k) = \eta(k)$
  - total fundamental diagram: $q = \phi(k) \equiv k\eta(k)$
    - Greenshields vs triangular
  - LWR model: $\partial k / \partial t + \partial \phi(k) / \partial x = 0$

- FIFO vs unifiable:
  - multi-commodity traffic flows
  - commodity speed-density relations
  - kinematic wave models
Existing MCKW

- both unifiable and FIFO
  - CTM (Daganzo, 1995)
  - (Lebacque, 1996): $v_{\downarrow m} = \eta(k)$
- FIFO but not unifiable: same speed, different contributions
  - (Zhang and Jin, 2002): different free-flow speeds
  - (Jin, 2013): weaving vs non-weaving vehicles
- neither FIFO nor unifiable
  - (Daganzo, 1997): 1-/2-pipe regimes
  - (Benzoni-Gavage and Colombo, 2003): $v_{\downarrow m} = V_{\downarrow m} / V \eta(k)$
  - (Chanut and Buisson, 2003): passenger-car equivalents
- unifiable but not FIFO: no explicit speed-density relations
  - (Khoshyaran and Lebacque, 2012): multi-lane, implicit existence
  - (Rey et al., 2015): Newell’s model with FIFO violation
## Existing MCKW

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<td><strong>This study</strong></td>
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This study

• Explicit unifiable MCKW model
  – total speed-density relation, but may violate FIFO
  – commodity speed-density relations from generating functions
  – analytical solutions: Riemann problem
  – numerical simulations: unifiable CTM
  – examples: Riemann solutions, well-defined

• important and relevant
  – empirical evidences: total speed-density relation, FIFO violation
  – conceptual confusion: unifiable ⇒ FIFO
  – computational efficiency: total traffic by Hamilton-Jacobi and link transmission model
Unifiable commodity speed-density relations
More notations

- **Density proportions:**
  - $p_m = k_m / k$
  - $\sum m = 1 \uparrow M \equiv p_m = 1$, $p_m \in [0, 1]$
  - $p = \sum m = 1 \uparrow M \equiv p_m \ e \downarrow m$
  - $k = k \ p$

- **Commodity speed ratios:** $\eta_m (k, p) = \eta(k, p) \cdot \gamma_m (k, p)$
  - $\gamma_m (k, p) = v_m / v > 0$
  - FIFO iff $\gamma_m (k, p) = 1$ for all $m$
  - Commodity $m$ faster: $\gamma_m (k, p) > \gamma_m \uparrow'$ (k, p)

- **Commodity flow-rate proportions:**
  - $\xi_m (k, p) = q_m / q = p_m \cdot \gamma_m (k, p)$
  - $\sum m = 1 \uparrow M \equiv \xi_m (k, p) = 1$, $\xi_m (k, p) \in [0, 1]$
  - FIFO iff $\xi_m (k, p) = p_m$ for all $m$
  - $\sum m = 1 \uparrow M \equiv p_m \cdot \gamma_m (k, p) = 1$

- **$g_m (k, p) = \gamma_m (k, p) - 1$**
  - $g_m (k, p) > -1$
  - $\sum m = 1 \uparrow M \equiv p_m \cdot g_m (k, p) = 0$
Unifiable properties

• Unifiable speed-density relations
  \[ \sum_{m=1}^{M} k \downarrow m \cdot \eta \downarrow m (k) = k \cdot \eta(k) \]
  \[ \sum_{m=1}^{M} p \downarrow m \cdot \eta \downarrow m (k, p) = \eta(k) \]
  \[ \eta \downarrow m (k, p) = \eta(k) \cdot \gamma \downarrow m (k, p) \]

• one commodity \( m \): \( p = e \downarrow m \)
  \[ \eta \downarrow m (k, e \downarrow m) = \eta(k) \]

• unifiable and FIFO
  \[ \gamma \downarrow m (k, p) = 1, \eta \downarrow m (k, p) = \eta(k) \]
Unifiable trajectories
Construction

• \( g \downarrow m (k,p) = f \downarrow m (k,p) - f(k,p) \)
  – generating functions: \( f \downarrow m (k,p) \)
  – weighted average: \( f(k,p) = \sum_{m=1}^{M} p \downarrow m \cdot f \downarrow m (k, p) \)

• properties
  – \( g \downarrow m (k,p) > -1 \)
  – \( g \downarrow m (k,p) \leq b, g \downarrow m (k,p) \leq 1 + b \)
  – FIFO if all generating functions are the same
  – generating functions can depend on density or not
  – other requirements: commodity information propagation speed; concave commodity flux function
Example 1

- \( f_1 (p) = 1 - \frac{1}{4} \)
- \( f_2 (p) = 0 \)
Example 5

- $f_{\downarrow m}(p)H(k/K_{\downarrow c})$
  
  $- H(y) = 1/4 \max \{1-y,0\}$
Unifiable multi-commodity kinematic wave model and the Riemann problem
Model

- Model: \( m=1, \cdots, M-1 \)
  - \( \frac{\partial k}{\partial t} + \frac{\partial \phi(k)}{\partial x} = 0 \)
  - \( \frac{\partial p}{\partial t} + \frac{\partial \gamma}{\partial x} = 0 \)
- \( M=2, \)
  - \( p_1 = p, p_2 = 1-p \)
  - \( g(k,p), \gamma(k,p) = 1+g(k,p), \xi(k,p) = p\gamma(k,p) \)
  - \( \frac{\partial p}{\partial t} + \frac{\partial p\gamma}{\partial x} = 0 \)
  - \( \frac{\partial \rho}{\partial t} + \frac{\partial \xi}{\partial x} = 0 \)
- quasilinear form
  - \( [k\downarrow t \downarrow \rho \downarrow t ] + \Lambda[k\downarrow x \downarrow \rho \downarrow x] = 0 \)
  - two eigenvalues: \( \lambda_1(k) = \phi(k), \lambda_2(k,\rho/k) = 1/k \xi p(k,\rho/k) \)
- non-strictly hyperbolic: \( \lambda_1(k) = \lambda_2(k,\rho/k) \)
- linearly degenerate
  - the second Riemann invariant: \( z(k,\rho) = k \)
  - the first Riemann invariant: \( \omega(k,\rho) \)
The Riemann problem

• Initial condition:
  – \((k(x,0), p(x,0)) = \begin{cases} (k↓L, p↓L), & x<0 \\ (k↓R, p↓R), & x>0 \end{cases}\)

• Total traffic entropy conditions:
  – Lax entropy condition: deceleration shock; acceleration rarefaction
  – Hamilton-Jacobi: minimum principle, variational principle, Hopf-Lax formula
  – junction flux flux in Cell Transmission Model

• Total traffic wave (1-wave):
  – wave region: \(v↓1↑− t ≤ x ≤ v↓1↑+ x\)
  – \(k(x,t) = \begin{cases} k↓L, & x < v↓1↑− t \\ v↓1↑+ tk↓R, & x > v↓1↑+ t \end{cases}\)
Commodity (2-) waves
**FIFO case**

- $g(k,p)=0$, $\gamma(k,p)=1$, $\xi(k,p)=p$
- $\partial p/\partial t + \eta(k)\partial p/\partial x = 0$
  - Commodity density proportion propagates along with vehicles’ trajectories
- $\lambda \downarrow 2 (k, \rho/k) = \eta(k)$, $r \downarrow 1 = [1, \rho/k] \uparrow$, $w(k,\rho) = k$
- Solution of commodity density proportion:
  - $p(x,t) = \begin{cases} p \downarrow L, & x < v \downarrow R \, tp \downarrow R, & x > v \downarrow R \, t \end{cases}$
- Three constant states
  - 1-wave
  - 2-wave: contact wave
non-FIFO case

• Four constant states + three waves
  – one 1-wave
  – two 2-waves
• variables:
  – $v\downarrow 0 \uparrow -$ and $v\downarrow 1 \uparrow +$ solved
  – unknown: $v\downarrow 0 \uparrow -$ , $v\downarrow 0 \uparrow +$ , $v\downarrow 2 \uparrow -$ , $v\downarrow 2 \uparrow +$ ; $p\downarrow \ast$ , $p\uparrow \ast$
• across commodity waves:
  – upstream: $\partial p/\partial t + \partial \eta (k\downarrow L )\xi (k\downarrow L ,p)/\partial x = 0$
    • with $p(x,0) = \{\begin{array}{ccc} p\downarrow L & , x<0 p\downarrow \ast & , x>0 \end{array}$
  – downstream: $\partial p/\partial t + \partial \eta (k\downarrow R )\xi (k\downarrow R ,p)/\partial x = 0$
    • with $p(x,0) = \{\begin{array}{ccc} p\uparrow \ast & , x<0 p\downarrow R & , x>0 \end{array}$
non-FIFO case (cont’d)

- **1-wave**
  - shock wave:
    - \[ q_{\downarrow L} - q_{\downarrow R} / k_{\downarrow L} - k_{\downarrow R} = \xi(k_{\downarrow L}, p_{\downarrow \ast}) q_{\downarrow L} - \xi(k_{\downarrow R}, p_{\uparrow \ast}) q_{\downarrow R} / p_{\downarrow \ast} k_{\downarrow L} - p_{\uparrow \ast} k_{\downarrow R} \]
  - rarefaction wave:
    - \[ \frac{dk}{dy} = \frac{1}{\phi_{\downarrow}k(k)} \]
    - \[ \frac{dp}{dy} = \phi_{\downarrow}k(k)[p - \xi(k,p)] - \phi(k)\xi_{\downarrow}k(k,p)/[\phi(k)\xi_{\downarrow}p(k,p) - \phi_{\downarrow}k(k)k]\phi_{\downarrow}k(k) \]
    - \[ k(\lambda_{\downarrow}1(k_{\downarrow L})) = k_{\downarrow L}, p(\lambda_{\downarrow}1(k_{\downarrow L})) = p_{\downarrow \ast} , k(\lambda_{\downarrow}1(k_{\downarrow R})) = k_{\downarrow R}, p(\lambda_{\downarrow}1(k_{\downarrow R})) = p_{\uparrow \ast} \]

- **Another constraint**
  - \[ v_{\downarrow 0 \uparrow \ast} \leq v_{\downarrow 0 \uparrow \ast} \leq v_{\downarrow 1 \uparrow \ast} \leq v_{\downarrow 1 \uparrow \ast} \leq v_{\downarrow 2 \uparrow \ast} \leq v_{\downarrow 2 \uparrow \ast} \]

- **No general theory for existence and uniqueness**
  - examples in Section 5
  - numerical method based on a unifiable multi-commodity CTM
Unifiable multi-commodity Cell Transmission Model
Numerical method

• Godunov method
  – Riemann solutions for two-commodity flow
  – \( q(x=0,t>0), q \downarrow m (x=0,t>0) \)

• Cell Transmission Model
  – \( M \geq 2 \)
  – networks
  – simple calculations
Link model

- **demand/supply:**
  - \( \delta(k) = \phi(\min\{k, K\downarrow c\}) \)
  - \( \sigma(k) = \phi(\max\{k, K\downarrow c\}) \)
  - Lemma 4.1. \( \delta(k) \leq \min\{k, K\downarrow c\} \cdot \lambda \downarrow 1(0); \sigma(k) \leq -(K - \max\{k, K\downarrow c\}) \cdot \lambda \downarrow 1(k) \)

- **cell \( i \), time step \( j \):**
  - densities and density proportions: \( k\downarrow i\uparrow j, p\downarrow m, i\uparrow j, k\downarrow m, i\uparrow j \)
  - demand and supply: \( \delta\downarrow i\uparrow j = \delta(k\downarrow i\uparrow j), \sigma\downarrow i\uparrow j = \sigma(k\downarrow i\uparrow j) \)
  - commodity flow-rate proportions: \( \xi\downarrow m, i\uparrow j \)

- **boundary fluxes:**
  - \( q\downarrow i\uparrow j = \min\{\delta\downarrow i-1\uparrow j, \sigma\downarrow i\uparrow j\} \), vs \( q\downarrow m, i\uparrow j = q\downarrow i\uparrow j \cdot \xi\downarrow m, i-1\uparrow j \)

- **Conservation equation:**
  - \( k\downarrow i\uparrow j + 1 = k\downarrow i\uparrow j + \Delta t/\Delta x \cdot (q\downarrow i\uparrow j - q\downarrow i+1\uparrow j) \); \( k\downarrow m, i\uparrow j + 1 = k\downarrow i\uparrow j \cdot p\downarrow m, i\uparrow j + \Delta t/\Delta x \cdot (q\downarrow m, i\uparrow j - q\downarrow m, i+1\uparrow j); p\downarrow m, i\uparrow j + 1 = k\downarrow m, i\uparrow j+1/k\downarrow i\uparrow j+1 \)

- **\( \Delta x/\Delta t \geq \max_{k\in[0,K]} |\lambda \downarrow 1(k)| \cdot \max_{p, m=1, \ldots, M} \gamma\downarrow m(k, p) \):**
  - stronger than the traditional CFL condition for the total CTM: \( \Delta x/\Delta t \geq \max_{k\in[0,K]} |\lambda \downarrow 1(k)| \)
  - Theorem 4.2 The unifiable multi-commodity CTM is well-defined.
General junction model

• Notations:
  – $U$: set of upstream cells; $D$: set of downstream cells
  – $\Omega \downarrow a \ (a \in U \cup D)$: set of commodities
  – $\delta \downarrow a \uparrow j \ (a \in U)$: upstream demand; $\xi \downarrow a, m \uparrow j$: upstream commodity flow-rate proportions
  – $\sigma \downarrow b \uparrow j \ (b \in D)$: downstream supply
  – $q \downarrow a \uparrow j$, $q \downarrow a, m \uparrow j$: out-fluxes; $q \downarrow b \uparrow j$, $q \downarrow b, m \uparrow j$: in-fluxes
General junction model (cont’d)

• Turning proportions: \( \xi_{a \to b} = \sum_{m \in \Omega a \cap \Omega b} \xi_{a,m} \to b \)

• Demand levels:
  – cell \( a \in U \): \( \mu_{a \to j} = \delta_{a \to j} / C_a \)
  – average for non-empty \( U \cap U \) for \( b \in D \): \( \theta_{b} = \pi_{b} + \sum_{a \in U \cap U} \mu_{a \to j} C_{a \to b} / \sum_{a \in U \cap U} C_{a \to b} \)
  – maximum for \( b \in D \): \( \Theta_{b} = \max_{U \cap U \neq \emptyset, U \subseteq U} \theta_{b} \)
  – critical demand level: \( \Theta_{j} = \min_{b \in D} \{ \theta_{b} \} \)

• Out-fluxes of \( a \in U \)
  – total: \( q_{a \to j} = \min \{ \delta_{i \to j}, \Theta_{j} C_{a} \} \)
  – commodity: \( q_{a,m} = q_{a \to j} \xi_{a,m} \to j \)

• In-fluxes of \( b \in D \)
  – total: \( q_{b \to j} = \sum_{a \in U \cap U} q_{a \to j} \xi_{a \to b} \)
  – commodity: \( q_{b,m} = q_{a,m} \to j, m \in \Omega a \cap \Omega b \)

• unifiable junction model
  – FIFO unifiable junction model in (Jin, 2012c): a special case
Examples
Set-up for two-commodity flow

• Total Greenshields FD: \( \eta(k) = 1 - k \)
• non-FIFO, unifiable
  – commodity flow-rate proportion: \( \xi(k,p) = p(2 - 5/4 \ p + 1/4 \ p^{1/2}) \);
    concave
  – \( \gamma(k,p) = 2 - 5/4 \ p + 1/4 \ p^{1/2} \); increasing
• two characteristic wave speeds:
  – \( \lambda_{\downarrow 1}(k) = 1 - 2k \)
  – \( \lambda_{\downarrow 2}(k,p) = (1 - k)(2 - 5/2 \ p + 3/4 \ p^{1/2}) \in [1/4 \ (1 - k), 2(1 - k)] \)
  – non-strictly hyperbolic: \( \lambda_{\downarrow 1}(k) = \lambda_{\downarrow 2}(k,p) \)
• \( \Delta t = 1/2 \ \Delta x \) stronger than traditional CFL condition
  – \( \max_{k \in [0,K]} |\lambda_{\downarrow 1}(k)| = 1 \)
  – \( \max_{k \in [0,K], p \in [0,1]} \gamma(k,p) = 2 \)
Total shock wave

• Riemann problem
  – \( k \downarrow L = 0, \quad k \downarrow R = 1/8, \quad p \downarrow L = 1, \quad p \downarrow R = 0.8 \)

• Analytical solution
  – 1-wave: shock wave, \( v \downarrow 1 \uparrow – = v \downarrow 1 \uparrow + = v \downarrow R = 7/8 \)
  – upstream 2-wave: \( (k \downarrow L, p \downarrow L) \rightarrow (k \downarrow L, p \downarrow *) \), rarefaction wave \( p \downarrow * = 0.5363 \)
    - \( \lambda \downarrow 2 (k \downarrow L, p \downarrow L) = 1/4 < v \downarrow 1 \uparrow – \)
    - across 1-wave: \( p \uparrow * = 0 \)
    - downstream 2-wave: shock wave \( v \downarrow 2 \uparrow – = v \downarrow 2 \uparrow + = 1.16 \)

• Numerical solution
  – \( \Delta x = 1/8 \)
  – Neumann boundary condition
Total rarefaction wave

- Riemann problem
  - $k\downarrow L = 0.9$, $k\downarrow R = 0.6$, $p\downarrow L = 0.5$, $p\downarrow R = 0.7$

- Analytical solution
  - 1-wave: rarefaction wave, $v\downarrow 1 \uparrow - = -0.8$, $v\downarrow 1 \uparrow + = -0.2$
  - no upstream 2-wave: $\lambda\downarrow 2 (k\downarrow L, p\downarrow L) = 0.08125 > v\downarrow 1 \uparrow -$
  - across 1-wave: $p\uparrow * = 0.4438$
  - downstream 2-wave: shock wave $v\downarrow 2 \uparrow - = v\downarrow 2 \uparrow + = 0.3279$

- Numerical solution
  - $\Delta x = 1/8$
  - Neumann boundary condition
Convergence

- \(k(x,t=100), p(x,t=100)\)
  - \(e\downarrow n\): between \(\Delta x = 1/2 \uparrow n\) and \(\Delta x = 1/2 \uparrow n - 1\)
  - \(L^1\), \(L^2\), \(L^\infty\) norms
- Convergence rate: \(r\downarrow n = \log \downarrow 2 (e\downarrow n - 1 / e\downarrow n)\)
- Unifiable multi-commodity CTM: first-order convergent scheme

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Conclusion
Summary

• unifiable multi-commodity KW model
  – total traffic: LWR model
  – FIFO may be violated
• unifiable commodity speed-density relations
  – generating functions
• two-commodity model
  – properties
  – Riemann problem: total and commodity waves
• unifiable multi-commodity CTM
  – non-FIFO general junction model
  – CFL condition: well-defined
• Examples
  – consistency between analytical and numerical solutions
  – convergence of CTM
Contributions

• first explicit unifiable, non-FIFO model
• LWR model may violate FIFO
  – unifiable, non-FIFO commodity speed-density relations
• simpler analytically and numerically
  – than non-unifiable, non-FIFO models
• unifiable multi-commodity CTM
  – heterogeneous traffic: trucks, automated/connected/electrified vehicles
  – multi-lane roads
Future studies

• theoretical solutions
  – non-strictly hyperbolic
  – existence/uniqueness
  – two or more commodities

• unifiable multi-commodity CTM
  – non-FIFO general junction model: invariant?
  – new non-FIFO phenomenon?

• empirical calibration
  – generating functions, commodity speed-density relations
  – multi-lane fundamental diagrams

• Hamilton-Jacobi formulation
  – non-FIFO unifiable commodity flows?
  – linear order-changing model vs generating functions
Multilane FD (Yan and Jin, 2017. TRB)

Figure 3. A freeway segment on SR-91E

Figure 6. Comparison of near-steady and calibrated lane flow-rates in the congested regime.
Questions?