Multi-scale perimeter control approach in a connected-vehicle environment

Kaidi Yang\textsuperscript{a}, Nan Zheng\textsuperscript{a,b}, Monica Menendez\textsuperscript{a}

\textsuperscript{a}. IVT – Institute for Transport Planning and Systems, ETH Zurich, Switzerland
\textsuperscript{b}. School of Transportation Science and Engineering, Beihang University, China
Perimeter control of large-scale urban networks

- Through perimeter intersections
- Based on Macroscopic Fundamental Diagram (MFD)
- Goal is to achieve the maximum trip completion flow
Perimeter control of large-scale urban networks

- Through perimeter intersections
- Based on Macroscopic Fundamental Diagram (MFD)
- Goal is to achieve the maximum trip completion flow
- Idea is to properly restrict the inflow traffic to the network
Perimeter control of large-scale urban networks

Perimeter intersections are not explicitly considered

Why important?

- Perimeter intersections perform the control action
  - Total allowed inflow
  - Signal timings
  - Physical constraints at intersections
Perimeter control of large-scale urban networks

Perimeter intersections are not explicitly considered

Why important?

- Perimeter intersections perform the control action
- Importance recognized in the literature

“lower level local controllers must be properly designed to realize the optimal control inputs determined by the high-level schemes”

Hajiahmadi, 2015

“...further analysis is needed to identify signal parameters in the individual regions of a city to smoothly move traffic at the desired flows without concentrating a large number of vehicles at the boundaries of the regions. This is a challenging task.”

Geroliminis, et al., 2013
Perimeter control of large-scale urban networks

Perimeter intersections are not explicitly considered

Why important?

- Perimeter intersections perform the control action
- Importance recognized in the literature
- Complex problem
  - Large number of intersections, streams and phases
  - Local queue dynamics
  - Competing objective functions
Multi-scale perimeter control approach

Network and intersection level control integrated

- Designing signal timings of individual perimeter intersections
- Balancing the performance at both levels
Multi-scale perimeter control approach

Detailed information provided by connected vehicles

- Real-time and rich information
- Facilitating prediction
- Enabling feed-forward/feedback control
Multi-scale perimeter control approach

System noises

- Modelling noises (e.g. MFD, etc.)
- Measurement noises (e.g. network traffic accumulations, queue lengths, etc.)
- Prediction noises (e.g. predicted demand, etc.)

Widely used: robust control

- Performance guaranteed in the worst case
- Can be conservative if there are a variety of noises
Methodology

A multi-scale Model Predictive Control (MPC) approach

- Integration of network and intersection level
- Connected vehicles as only source of information

A stochastic Model Predictive Control (MPC) approach

- An alternative to robust control
- Trade off between efficiency and robustness
A multi-scale Model Predictive Control (MPC) approach
A multi-scale Model Predictive Control approach

**Inputs**
- Measured traffic accumulation and queue length from CVs
- Predicted demand in the next $L$ cycles at both network and local levels
A multi-scale Model Predictive Control approach

**Outputs**
- Optimal green time ratios for each phase at each intersection during current cycle
An MPC based controller
- Relies on the dynamic model of the traffic system
- Optimizes a finite time-horizon
- Implements only the current cycle

A multi-scale Model Predictive Control approach
A multi-scale Model Predictive Control approach

- Local dynamics specially treated
- Conflicting control objectives coupled
- Complex optimization problem solved by linearization
A multi-scale Model Predictive Control approach

Objective function: Total travel cost minimization

\[ \min J_D = C \sum_{l=1}^{L} (n_{11}(k + l|k) + n_{12}(k + l|k)) + C \sum_{l=1}^{L} \sum_{i \in I} \sum_{m \in M \setminus M_{\text{out}}} x_i^m(k + l|k) \]

- Summation of \( L \) cycles in the future
- Total travel time at the network level
- Total delay at the intersection level
A multi-scale Model Predictive Control approach

Network constraints

- Mass conservation at the network level
- MFD representation of the dynamics

\[
\begin{align*}
n_{11}(k + l + 1|k) &= n_{11}(k + l|k) + D_{11}(k + l|k)C + \beta_{21}(k + l|k)C \\
- &- \frac{n_{11}(k + l|k)}{n_{11}(k + l|k) + n_{12}(k + l|k)} G\left(n_{11}(k + l|k) + n_{12}(k + l|k)\right)C \\
n_{12}(k + l + 1|k) &= n_{12}(k + l|k) + D_{12}(k + l|k)C - \beta_{12}(k + l|k)C
\end{align*}
\]
A multi-scale Model Predictive Control approach

Intersection constraints

Arrival flow $\lambda$  Queue length $x$  Departing flow $\mu$
A multi-scale Model Predictive Control approach

Intersection constraints

- Demand/capacity constraints for departing flow

$$\mu^i_m(k + l|k) = \min \left\{ \frac{\alpha^i_m}{|I|} \frac{n_{12}(k + l|k)}{n_{11}(k + l|k) + n_{12}(k + l|k)} G(n_{11}(k + l|k) + n_{12}(k + l|k)), \sum_{p \in P_i} s^i_{mp} g^i_p(k + l) \right\}$$

$$\mu^i_m(k + l|k) = \min \{ x^i_m(k + l|k)/C + q^i_m(k + l|k), \sum_{p \in P_i} s^i_{mp} g^i_p(k + l) \}.$$ 

- Queue dynamics: conservation equation

$$x^i_m(k + l + 1|k) = x^i_m(k + l|k) + q^i_m(k + l|k) C - \mu^i_m(k + l|k) C$$

- Physical constraints for the green time ratio

$$\sum_{p \in P} g^i_p(k + l) \leq g^{i}_{max}$$

$$g^i_p(k + l) \geq g^{i}_{p,max}$$
A multi-scale Model Predictive Control approach

Coupling constraints

\[ \beta_{21}(k + l|k) = \sum_{i \in I} \sum_{m \in M_{in}^i} \mu_m^i(k + l|k) \]

\[ \beta_{12}(k + l|k) = \sum_{i \in I} \sum_{m \in M_{out}^i} \mu_m^i(k + l|k) \]

- Conservation of the inflow of the network
- Conservation of the outflow of the network
A multi-scale Model Predictive Control approach

Initial conditions

\[ n_{1j}(k|k) = \hat{n}_{1j}(k), \quad j = 1, 2 \]
\[ x_m^i(k|k) = \hat{x}_m^i(k), \quad \forall m \in M^i \setminus M_{out}^i, \quad \forall i \in I \]

- Current measurement of traffic accumulation inside the network
- Current measurement of the queue length at each intersection
- Both obtained from connected vehicles
Case study

- A case mimicking the city center of Zurich
- 20 controlled intersections
- Morning peak demand profile
Control performance: BB, PID and Multi-scale

![Graph showing control performance](image_url)
Control performance: BB, PID and Multi-scale

Side streams
(streams other than the inflow and outflow streams)

Inflow streams

Middle of the peak hour
Control performance: BB, PID and Multi-scale

<table>
<thead>
<tr>
<th></th>
<th>Moderate noises</th>
<th>Strong noises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modelling noises</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>Measurement noises</td>
<td>5%</td>
<td>15%</td>
</tr>
<tr>
<td>Prediction noises</td>
<td>10%</td>
<td>30%</td>
</tr>
</tbody>
</table>

No noises

Moderate noises

Strong noises
Methodology

A multi-scale Model Predictive Control (MPC) approach

- Integration of both network level and intersection level
- Connected vehicles as source of information

A stochastic Model Predictive Control (MPC) approach

- An alternative to robust control
- Trade off between efficiency and robustness
A stochastic Model Predictive Control approach
A stochastic Model Predictive Control approach

Traffic demand → Traffic network and intersection dynamics

Green time ratio → Traffic network and intersection dynamics

MPC controller

minimize network travel time + intersection delay
subject to
- Network constraints
- Intersection constraints
- Coupling constraints

Prediction (Feed-forward)

Measurement noises

Traffic accumulation/queue length

Modelling noises

Expected traffic demand at both levels

Explicitly considered

Prediction noises
A stochastic Model Predictive Control approach

- Stochastic optimization problem
- Objective: minimize mean total travel cost
- Noises as a sample set
- The mean total travel cost from the sample set
Multi-scale MPC vs. Stochastic MPC with noise

Multi-scale MPC

Stochastic MPC
Multi-scale MPC vs. Stochastic MPC with noise

**Total travel cost**

- **Multiscale MPC** vs. **Stochastic MPC**

**Total intersection delay**

- **Multiscale MPC** vs. **Stochastic MPC**
Conclusions

- Explicitly considering perimeter intersections are important.

- Multi-scale MPC approach
  - Successfully stabilizes the network
  - Reduces and balances the queue outside the network.
  - Reduces total travel costs

- Stochastic MPC approach
  - Successfully reduces the oscillations in the system.
Thanks for your attention!

- kaidi.yang@ivt.baug.ethz.ch
A stochastic MPC approach to handle system noises

- Noises are sampled as a sample set R.
- The stochastic MPC is formulated as a two stage stochastic optimization problem.

\[
\begin{align*}
\min & \quad E_r J_{D,r} \\
\text{s.t.} & \quad \sum_{p \in P} g^i_p(k) \leq g^i_{\text{max}}, \quad \forall i \in I \\
& \quad g^i_p(k) \geq g^i_{\text{min},p}, \quad \forall i \in I, p \in P^i
\end{align*}
\]

First stage problem

- Decision variables are the green ratios to be executed in the current cycle.
- Constraints are the physical constraints of the green ratios.
- Objective is to minimize the mean total travel cost across the sample set R.
- Travel cost for each sample is obtained from the second stage
A stochastic MPC approach to handle system noises

Second stage problem

\[
\min J_{D,r} = C \sum_{l=1}^{L} \left( n_{11,r}(k + l|k) + n_{12,r}(k + l|k) + \sum_{i \in I} \sum_{m \in M^i \setminus M^i_{out}} x_{m,r}^i(k + l|k) \right)
\]

Constraints are the same as the multiscale MPC with the decision variable at the first stage fixed, i.e. with fixed green time ratio in the current cycle.
Connected vehicles

Turning ratio

\[
\mu_m^i(k+l|k) = \min \left\{ \frac{\alpha_m^i}{|l|} \frac{n_{12}(k+l|k)}{n_{11}(k+l|k) + n_{12}(k+l|k)} G\left(n_{11}(k+l|k) + n_{12}(k+l|k)\right), \sum_{p \in P^i} s_{mp}^i g_p^i(k+l)\right\},
\]

\forall m \in M^i_{\text{out}}, \forall i \in I, \forall 0 \leq l \leq L

\[
\mu_m^i(k+l|k) = \min \{x_m^i(k+l|k)/C + q_m^i(k+l|k)\} \sum_{p \in P^i} s_{mp}^i g_p^i(k+l), \forall m \in M^i \setminus M^i_{\text{out}}, \forall i \in I, \forall 0 \leq l \leq L - 1
\]

Predicted demand

\[
n_{1j}^i(k|k) = \hat{n}_{1j}^i(k),
\]

\[
x_m^i(k|k) = \hat{x}_m^i(k),
\]

Measured queue length and accumulation
Sample size of stochastic MPC

\[
\min J_{D,r} = C \sum_{l=1}^{L} \left( n_{11,r}(k + l|k) + n_{12,r}(k + l|k) + \sum_{i \in I} \sum_{m \in M^i \setminus M^i_{out}} x_{m,r}^i(k + l|k) \right)
\]

Table. Mean and standard deviation of the second stage objective function

<table>
<thead>
<tr>
<th></th>
<th>Moderate noise</th>
<th>Large noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (min)</td>
<td>80120</td>
<td>79342</td>
</tr>
<tr>
<td>Standard deviation with sample size 1 (min)</td>
<td>3686</td>
<td>11981</td>
</tr>
<tr>
<td>Standard deviation with sample size 20 (min)</td>
<td>824</td>
<td>2679</td>
</tr>
<tr>
<td>Coefficient of variation with sample size 1</td>
<td>4.6%</td>
<td>15.4%</td>
</tr>
<tr>
<td>Coefficient of variation with sample size 20</td>
<td>1.0%</td>
<td>3.4%</td>
</tr>
</tbody>
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