Effect of Information Availability on Stability of Traffic Flow: Percolation Theory Approach

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Motivation

• Connected and automated vehicles (CAVs) will soon join the current vehicle fleet.
• Their introduction will re-shape the transportation industry:
  • Safety
  • Congestion
  • Emissions and energy consumption
  • Freight
  • etc.
• Understanding CAVs’ full impact on traffic performance requires accurate modeling of their dynamics/movement.

Source: http://planningforreality.org/
Motivation

- Safety and efficiency are the main motivations behind bringing these technologies to market.
- Developing string stable connected and automated systems is the key to achieve the desired safety and efficiency.

**String Stable Platoon**

**String Unstable Platoon**
Many studies investigated string stability in connected, automated systems (mostly CACC systems):

- The benefits of CACC over ACC systems were identified (Zhang and Orosz, 2015; Lu et al., 2002).

- The immediate leader and platoon head were found to be the key to ensuring string stability (Swaroop, 1997).

- The possibility of developing decentralized CACC systems was shown (Naus et al., 2010).
Problem Statement

Telecommunications affect Traffic Pattern
Traffic Pattern affects connectivity (Telecommunications)
Problem Statement

• Communications dynamics and information loss was not captured in these studies.
The main objective of this study is to develop a methodology to incorporate signal interference and information loss into analytical investigations of string stability. This study introduces a methodology based on Percolation Theory.
Percolation Theory

• The propagation of information within the vehicular network is similar to the
  • Transport of a fluid through porous media
  • Spread of a disease among people

Percolation Theory

• **Continuum Percolation**: There exists a critical density ($P_c$), above which there is certainly a cluster with an infinite size in the system.

• Two types of Continuum Percolation:
  • **Boolean model**
  • Random Connection model

• Boolean model: Each point $(x)$ in the space is a center of a circle with radius $r$ ($C(x,r)$).

• Combination of these circles divide the space into two regions: occupied region and vacant region.

• A pair of nodes is connected if both nodes belong to the same occupied/vacant region.
Definitions

• Communicating Vehicles

\[ h(x_i, x_j) = \begin{cases} 1 & \text{if } |x_i - x_j| < \min\{R_i, R_j\} \\ 0 & \text{Otherwise} \end{cases} \]

• Homogeneous Poisson Point Process

\[ P(X_\lambda(L) = k) = \frac{(\lambda S L)^k}{k!} e^{-\lambda SL} \]

• Communication Path

[Diagram of communicating vehicles and communication path]
Definitions

• **Connected k-component** \( (CC_k) \): A set of \( k \) communicating vehicles that is not a subset of another set of communicating vehicles.
  
  • Distance between two \( CC_k \)s is the minimum distance between pairs of the vehicles in the \( CC_k \)s.
Percolation of Vehicular Ad-Hoc Networks

Connected Components: Length Estimation

\[ P(N_L = k) = \text{prob}\{ R_i \text{ contains a } CC_K \mid (R_{II} \cup R_{III}) \text{ is empty} \} \]

\[
P(N_L = k) = \frac{\left( \int_{X-R}^{X+L+R} \lambda(x)dx \right)^k}{k!} e^{-\int_{X-R}^{X+L+R} \lambda(x)dx} - \left( \int_{X+L}^{X+L+R} \lambda(x)dx \right) = \frac{\left[ \lambda(L + 2R) \right]^k}{k!} e^{-\lambda(L+2R)} \]

\[
\int_{X+L}^{X+2R} \lambda(x)dx \cdot e^{-\lambda R} \cdot e^{-\lambda R} = \frac{\left[ \lambda(L + 2R) \right]^k}{k!} e^{-\lambda L}
\]
Percolation of Vehicular Ad-Hoc Networks

Critical Density of Connected Components:

At the percolation point, the density of connected k-components within a circle with radius $R$ is given by,

$$\lambda_c(k) = \lambda_c \left( \frac{\lambda_c(3R)^k}{k!} \right) e^{-\lambda_c R}$$

Combining all the above equations:

$$f(\lambda_c, R, k) = \left[ 1 - (\mu + 1)e^{-\mu} \right] \left[ \left( \frac{3\mu}{k!} \right)^k e^{-\mu} \right] - 1 = 0$$

Where

$$\mu = -\log \left( 1 - A_c(R) \right) \quad A_c(R) = 1 - e^{-\lambda_c R}$$
Percolation of Vehicular Ad-Hoc Networks

- Results indicate that percolation first occurs at $k=3$ and $A_c(R)=0.785$. 

![Graph showing percolation of vehicular ad-hoc networks](image-url)
Communication Probability

- For $A_c(R) \geq 0.785$: all the vehicles receive the information.
- For $A_c(R) < 0.785$: the probability that there is only one point in any circle with radius $R$ is:

$$P(N_{2R} = 1) = 2R\lambda e^{-2R\lambda}$$
Stability Analysis Approach

In line of the definition of string stability, the following criteria guarantees the string instability of a heterogeneous traffic flow (Ward, 2009):

\[ \sum_n \left[ \frac{f_v^n}{2} - f_{\Delta v}^n f_v^n - f_s^n \right] \left[ \prod_{m \neq n} f_s^m \right]^2 < 0 \]

where

\[ f_s^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial s_n} \right|_{(s^*, 0, V(s^*)}} \]

\[ f_v^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial s_v} \right|_{(s^*, 0, V(s^*)}} \]

\[ f_{\Delta v}^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial \Delta v_n} \right|_{(s^*, 0, V(s^*)}} \]
### Traffic Simulation Framework

<table>
<thead>
<tr>
<th>No Automation Not Connected</th>
<th>No Automation Connected</th>
<th>Self-Driving Not Connected</th>
</tr>
</thead>
</table>

- **Acceleration Behavior:** Probabilistic
- **Perception of Surrounding Traffic Condition:** Subjective
- **Reaction Time:** High
- **Safe Spacing:** High
- **High-Risk maneuvers:** Possible

- The car-following model of Talebpour, Hamdar, and Mahmassani (2011) is used.
  - Probabilistic
  - Recognizes two different driving regimes:
    - Congested
    - Uncongested
  - Consider crashes endogenously
Traffic Simulation Framework

- No Automation, Not Connected
- No Automation, Connected
- Self-Driving, Not Connected

- Acceleration Behavior: Deterministic
- Perception of Surrounding Traffic Condition: Accurate
- Reaction Time: Low
- Safe Spacing: Low
- High-Risk maneuvers: Very Unlikely

- The Intelligent Driver Model (Treiber, Hennecke, and Helbing, 2000) is used.
Traffic Simulation Framework

- Speed should be low enough so that the vehicle can react to any event outside of the sensor range ($v_{\text{max}}$) (Reece and Shafer, 1993 and Arem, Driel, Visser, 2006).

$$v_{\text{max}} = \sqrt{-2a_n^{\text{acc}} \Delta x}$$

$$a_n(t) = \min(a_n^d(t), k(v_{\text{max}} - v_n(t)))$$

$$a_n^d(t) = k_a a_{n-1}(t - \tau) + k_v (v_{n-1}(t - \tau) - v_n(t - \tau)) + k_d (s_n(t - \tau) - s_{\text{ref}})$$
Stability Analysis Results

\[ \bar{a} = 1.4 m/s^2 \quad \bar{b} = -2.0 m/s^2 \]

Full Connectivity

\[ a = 1.4 m/s^2 \]
\[ b = -2.0 m/s^2 \]
Stability Analysis Results

\[
\bar{a} = 3.0 \text{m/s}^2 \quad \bar{b} = -4.0 \text{m/s}^2
\]
Conclusion

• This paper presents a Percolation Theory based approach to incorporate partial communication and information loss into analytical investigation of string stability.

• The analytical studies reveal that as communication range increases, the system becomes more stable.

• At communication ranges above 130m, the system performs very similar to the system with full connectivity assumption.
Questions and Comments
Model Parameters

- Automated vehicle parameters are
  \( k_a = 1.0, k_v = 0.58, \) and \( k_d = 0.1 \)
- Regular and connected vehicles parameters are shown in the tables.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Typical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity Exponents of the Generalized Utility</td>
<td>( \gamma = 0.2 )</td>
</tr>
<tr>
<td>Asymmetry Factor for Negative Utilities</td>
<td>( w_m = 2.0 )</td>
</tr>
<tr>
<td>Velocity Uncertainty Variation Coefficient</td>
<td>( \alpha = 0.08 )</td>
</tr>
<tr>
<td>Weighing Factor for Accidents</td>
<td>( w_c = 10000.0 )</td>
</tr>
<tr>
<td>Maximum Anticipation Time Horizon</td>
<td>( \tau_{\text{max}} = 4.0\text{s} )</td>
</tr>
<tr>
<td>Logit Uncertainty Parameter (Intra-Driver Variability)</td>
<td>( \beta = 5.0 )</td>
</tr>
<tr>
<td>Maximum Acceleration</td>
<td>( a_{\text{max}} = 4\text{m/s}^2 )</td>
</tr>
<tr>
<td>Minimum Acceleration</td>
<td>( a_{\text{min}} = -8\text{m/s}^2 )</td>
</tr>
</tbody>
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<th>Typical Value</th>
</tr>
</thead>
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<tr>
<td>Free Acceleration Exponent</td>
<td>( \delta = 4.0 )</td>
</tr>
<tr>
<td>Desired Time Gap</td>
<td>( T = 4.5\text{s} )</td>
</tr>
<tr>
<td>Jam Distance</td>
<td>( s_o = 2.0\text{m} )</td>
</tr>
<tr>
<td>Maximum Acceleration</td>
<td>( \bar{a} = 1.4\text{m/s}^2 )</td>
</tr>
<tr>
<td>Desired Deceleration</td>
<td>( \bar{b} = -2.0\text{m/s}^2 )</td>
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