Dynamic Modeling and Control of Taxi Services in Large-Scale Urban Networks: A Macroscopic Approach

Mohsen Ramezani
Mehdi Nourinejad
Motivation

- **Prominent:**
  Accessible, Convenient, Fast, Shared

- **Critical:**
  Contributing to VKT, Thin Market, Credence good

- **Needs:**
  Modeling (equilibrium, dynamic)
  Management (faring, fleet size, zoning, dispatching)

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Contributions

• Proposing a macroscopic and city-scale taxi service model
  ▪ Multi region, Multi-firm taxi model

• Addressing the interrelated effects of taxis on other traffic modes

• Proposing a taxi operations control strategy
  ▪ network objective function
Outline

• Introduction
• Taxi Dynamics Modeling
• Large-scale Traffic Modeling
• Mixed Traffic Modeling
• Optimal Problem Formulation
• Dispatching Solution: A Model Predictive Approach
• Case Study
Taxi Dynamics

- **Occupied taxis:**
  - modeled as normal traffic (personal vehicles)

- **Vacant taxis:**
  - bilateral passenger-taxi search
  - simultaneous co-existence of vacant taxis and unserved passengers (search friction)
  - emergent meeting (boarding) relationship

\[ b = B(c^V, p) \]
\[ c^V \geq 0; p \geq 0 \]
\[ \frac{\partial B}{\partial c^V} > 0; \frac{\partial B}{\partial p} > 0 \]
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\[ c^v \geq 0; p \geq 0; v \geq 0 \]
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Taxi Dynamics

- Microscopic
- Event-based

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Taxi Dynamics

Impact of Network Speed on the Bilateral Meeting Function

\[ b = B(c^v, p, v) \]

\[ B(t) = Ac^v(t)^{\gamma_1} p(t)^{\gamma_2} v(t)^{\gamma_3} \]

\[ c^v \geq 0; p \geq 0; v \geq 0 \]

\[ \frac{\partial B}{\partial c^v} > 0; \frac{\partial B}{\partial p} > 0; \frac{\partial B}{\partial v} > 0 \]
Traffic Dynamics

- Large-scale urban traffic modeling
  - Macroscopic Fundamental Diagram (MFD)
  - MFD is a unimodal and low-scatter relation between region vehicle density (accumulation) and space-mean flow (production)
  - Urban regions with small variance of link densities have well-defined MFDs
- Typical urban areas (heterogeneous networks) can be partitioned to homogeneous regions (Ji and Geroliminis, 2012, TR-part B)
- Carefully-devised signal control strategies can result in well-defined MFDs (Ramezani et al., 2015, TR-part B)
Formulation

\[
\begin{align*}
\text{no. vacant taxis} & \quad \text{no. occupied taxis} & \quad \text{no. dispatched taxis} & \quad \text{total no. taxis}
\end{align*}
\]

\[
c_i^v(t) + c_i^o(t) + c_i^d(t) = c_i(t) \quad \forall i \in \{1, 2\}
\]

\[
c_1(t) + c_2(t) = C
\]
Modelling (I)

\[
\frac{dn_{11}(t)}{dt} = q_{11}(t) + M_{21}^n(t) - M_{11}^n(t)
\]
\[
\frac{dn_{22}(t)}{dt} = q_{22}(t) + M_{12}^n(t) - M_{22}^n(t)
\]
\[
\frac{dn_{12}(t)}{dt} = q_{12}(t) - M_{12}^n(t)
\]
\[
\frac{dn_{21}(t)}{dt} = q_{21}(t) - M_{21}^n(t)
\]
Modelling (I)

\[
\begin{align*}
\frac{dn_{11}(t)}{dt} &= q_{11}^n(t) + M_{21}^n(t) - M_{11}^n(t) \\
\frac{dn_{22}(t)}{dt} &= q_{22}^n(t) + M_{12}^n(t) - M_{22}^n(t) \\
\frac{dn_{12}(t)}{dt} &= q_{12}^n(t) - M_{12}^n(t) \\
\frac{dn_{21}(t)}{dt} &= q_{21}^n(t) - M_{21}^n(t)
\end{align*}
\]

\[
M_{12}^n(t) = \frac{n_{12}(t)}{n_1(t) + c_1(t)} \frac{P_1(n_1(t) + c_1(t))}{l_1^n(t)}
\]

\[
M_{21}^n(t) = \frac{n_{21}(t)}{n_2(t) + c_2(t)} \frac{P_2(n_2(t) + c_2(t))}{l_2^n(t)}
\]

\[
M_{11}^n(t) = \frac{n_{11}(t)}{n_1(t) + c_1(t)} \frac{P_1(n_1(t) + c_1(t))}{l_1^n(t)}
\]

\[
M_{22}^n(t) = \frac{n_{22}(t)}{n_2(t) + c_2(t)} \frac{P_2(n_2(t) + c_2(t))}{l_2^n(t)}
\]

normal vehicle dynamics
Modelling (II)

\[
\begin{align*}
\frac{dc_{11}^o(t)}{dt} &= b_{11}(t) + M_{21}^o(t) - M_{11}^o(t) \\
\frac{dc_{22}^o(t)}{dt} &= b_{22}(t) + M_{12}^o(t) - M_{22}^o(t) \\
\frac{dc_{12}^o(t)}{dt} &= b_{12}(t) - M_{12}^o(t) \\
\frac{dc_{21}^o(t)}{dt} &= b_{21}(t) - M_{21}^o(t)
\end{align*}
\]

\[
\begin{align*}
M_{12}^o(t) &= \frac{c_{12}^o(t)}{n_1(t) + c_1(t)} \frac{P_1(n_1(t) + c_1(t))}{l_1^o(t)} \\
M_{21}^o(t) &= \frac{c_{21}^o(t)}{n_2(t) + c_2(t)} \frac{P_2(n_2(t) + c_2(t))}{l_2^o(t)} \\
M_{11}^o(t) &= \frac{c_{11}^o(t)}{n_1(t) + c_1(t)} \frac{P_1(n_1(t) + c_1(t))}{l_1^o(t)} \\
M_{22}^o(t) &= \frac{c_{22}^o(t)}{n_2(t) + c_2(t)} \frac{P_2(n_2(t) + c_2(t))}{l_2^o(t)}
\end{align*}
\]

\[
b_{ij}(t) = B(c_i^y(t), p_{ij}(t), v_i(t))
\]

\[
vli(t) = pli(nli(t) + cli(t))/nli(t) + cli(t)
\]
Modelling (III)

\[
\begin{align*}
\frac{dp_{11}(t)}{dt} &= q_{11}(t) - b_{11}(t) \\
\frac{dp_{22}(t)}{dt} &= q_{22}(t) - b_{22}(t) \\
\frac{dp_{12}(t)}{dt} &= q_{12}(t) - b_{12}(t) \\
\frac{dp_{21}(t)}{dt} &= q_{21}(t) - b_{21}(t)
\end{align*}
\]

passenger dynamics

\[
\begin{align*}
\frac{dc_{11}^v(t)}{dt} &= M_{11}^o(t) + M_{21}^d(t) - b_{11}(t) - w_{12}(t) \\
\frac{dc_{22}^v(t)}{dt} &= M_{22}^o(t) + M_{12}^d(t) - b_{22}(t) - w_{21}(t)
\end{align*}
\]

vacant taxi dynamics

\[
\begin{align*}
\frac{dc_{12}^d(t)}{dt} &= w_{12}(t) - M_{12}^d(t) \\
\frac{dc_{21}^d(t)}{dt} &= w_{21}(t) - M_{21}^d(t)
\end{align*}
\]

dispatched taxi dynamics

\[
\begin{align*}
b_{ij}(t) &= B\left(c_i^v(t), p_{ij}(t), v_i(t)\right) \\
M_{12}^d(t) &= \frac{c_{12}^d(t)}{n_1(t) + c_1(t)} \frac{P_1(n_1(t) + c_1(t))}{l_1^d(t)} \\
M_{21}^d(t) &= \frac{c_{21}^d(t)}{n_2(t) + c_2(t)} \frac{P_2(n_2(t) + c_2(t))}{l_2^d(t)}
\end{align*}
\]
Optimal Problem

\[
\min_{w_{12}(t), w_{21}(t)} J = \int_{t_0}^{t_f} \left( \alpha_1 [n_1(t) + n_2(t)] + \alpha_2 [c_1^o(t) + c_2^o(t)] + \alpha_3 [c_1^v(t) + c_2^v(t)] + \alpha_4 [c_{12}^d(t) + c_{21}^d(t)] + \alpha_5 [p_1(t) + p_2(t)] \right) dt
\]

\text{TTS of occupied taxis} \quad \text{TTS of vacant taxis} \quad \text{TTS of dispatched taxis}

\text{TTS of personal vehicles} \quad \text{Waiting time of passengers}

s.t.

\[
0 \leq n_{11}(t), n_{22}(t), n_{12}(t), n_{21}(t),
\]

\[
c_1^o(t), c_2^o(t), c_{12}^o(t), c_{21}^o(t),
\]

\[
c_1^v(t), c_2^v(t), c_{12}^d(t), c_{21}^d(t)
\]

\[
p_{11}(t), p_{22}(t), p_{12}(t), p_{21}(t)
\]

\[
\forall t \in [t_0, t_f]
\]

while

\[
n_i(t) + c_i(t) \leq n_i^{\text{jam}} \quad i \in 1, 2; \forall t \in [t_0, t_f]
\]

\[
0 \leq w_{12}(t), w_{21}(t) \quad \forall t \in [t_0, t_f]
\]

All the traffic states all the time are positive

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Model Predictive Control

\[
J = \min_{w_{12}(t), w_{21}(t)} \int_{t_0}^{t_f} \left[ \alpha_1 [n_1(t) + n_2(t)] + \alpha_2 [c_{1}^o(t) + c_{2}^o(t)] + \alpha_3 [c_{1}^V(t) + c_{2}^V(t)] + \alpha_4 [c_{12}^d(t) + c_{21}^d(t)] + \alpha_5 [p_1(t) + p_2(t)] \right] dt
\]

\[\text{s.t.}\]
\[
0 \leq n_{11}(t), n_{22}(t), n_{12}(t), n_{21}(t), c_{11}^o(t), c_{22}^o(t), c_{12}^o(t), c_{21}^o(t), c_{1}^V(t), c_{2}^V(t), c_{12}^d(t), c_{21}^d(t), p_{11}(t), p_{22}(t), p_{12}(t), p_{21}(t) \quad \forall t \in [t_0, t_f]
\]

\[\text{while}\]
\[
n_i(t) + c_i(t) \leq n_i^{\text{jam}} \quad i \in 1, 2; \forall t \in [t_0, t_f]
\]
\[
0 \leq w_{12}(t), w_{21}(t) \quad \forall t \in [t_0, t_f]
\]
No-Dispatch

Dispatch

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Discussion

mohsen.ramezani@sydney.edu.au
mehdi.nourinejad@mail.utoronto.ca
Assessment of the prediction horizon

![Graph showing improvement percentage (%)]

- $N_c=1$
- $N_c=2$
- $N_c=3$
- $N_c=4$
- $N_c=5$

Improvement = $\frac{\text{Prediction Horizon}}{\text{Actual Horizon}} - 1$ [\%]