A car sharing auction with temporal-spatial OD connection conditions

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A core problem of Transportation and Traffic Theory

The Negative Externality of people’s trips is an important problem for transportation system.

1. Bottleneck model (Vickrey, 1969)
   - travel behavior: departure time choice
   - policy: congestion charging

2. Traffic Assignment
   - UE (Beckmann, 1956), SUE (Daganzo and Sheffi, 1977), Hyperpath (Spiess and Florian, 1989; Bell, 2009)
   - travel behavior: route choice
   - policy: pricing of links for system optimum (Henderson, 1985)

3. Tradable permit scheme
   - travel behavior: route choice and departure time choice
   - policy: market mechanism for optimal allocation
What is the cause of the negative externality?

1. Capacity of transportation system
2. Users use the same transport facility simultaneously

Link capacity of each road link is fixed.

Service capacity of each OD and time slot depends on vehicle paths.

Vehicle capacity (disaggregate capacity)

Link capacity (aggregate capacity)
What is the cause of the negative externality?

1. Capacity of transportation system
2. Users use the same transport facility simultaneously
3. Flexible OD of mobility services generates negative/positive externalities.

The important point of mobility sharing

“Sharing a vehicle by more than one person” is not essential.
“Connecting trips temporally and spatially” is essential.
Our Concept

**bottleneck** → **network** → service with “knots”

- link capacity
- aggregate capacity
- multi aggregate OD
- disaggregate capacity
- multi disaggregate OD
- temporal-spatial connection condition

**Research Objective**

Designing mobility sharing auction mechanism considering temporal-spatial OD connection condition.
Our approach

Fundamental Theory for mobility services with “knots”

Service Implementation in real world and Travel Behavior Analysis for tradable permit

Today’s presentation

Hara and Hato (Transportation, 2017)
Setting1: the Assumption of Transportation Services

We assume the following statements.
1. Mobility sharing means car sharing in this study.
2. A trip can be done in a time slot.
3. There is no delay.
4. The number of vehicle is $\mu$ and each vehicle can move independently.

We don’t consider the delay effect.
Setting 2: Supplier, Users and the Capacity limit

**the setting of service supplier**

We assume that **service supplier** is the player to **maximize social welfare** by operating vehicle effectively. The supplier tackles an unbalanced demand problem between ports by allocating users to permits satisfying temporal-spatial OD connected condition at all time slot.

**the setting of users and users’ value of permit**

Users have the **value of permits** and the value of each permit is different by **time slot** and OD pair. Users determine the value by their OD demand, their desired usage timeslot, their value of time and so on.

**vehicle capacity limit $\mu$**

Vehicle capacity limit means the **number of vehicles** service supplier has. Service supplier can issue permits depending on the vehicle capacity limit.

$$\sum_{i \in I} \sum_{pq \in L} x^i_{pq}(t) = \mu \quad \forall t \in T$$

$x^i_{pq}(t) \in \{0, 1\}$ is a discrete variable. If user $i$ is allocated to a permit to use vehicle between OD $pq$ at time slot $t$, this variable is 1. Otherwise this variable is 0.
Meeting temporal-spatial OD connection condition is satisfying the following equation for any node \( q \).

\[
\sum_{i \in I} \sum_{p \in N} x_{pq}^i (t - 1) = \sum_{i \in I} \sum_{p \in N} x_{qp}^i (t) \quad t = 2, \ldots, T, \forall q \in N
\]
Example of the Setting

Example network

Users’ demand and value of permit

<table>
<thead>
<tr>
<th>User ID</th>
<th>OD</th>
<th>value (t = 1)</th>
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A trip-connected path allocation \{A, D, E\}

- t=1: A
- t=2: D
- t=3: E

Sum of values (Social Welfare) is 220.

A trip-connected path allocation \{A, C, B\}

- t=1: A
- t=2: C
- t=3: B

Sum of values (Social Welfare) is 250.
Permits Allocation of Social Optimum

- optimization problem to maximize the sum of users’ values [SO]

\[
SS = \max_{\{x^i\}} \sum_{i \in I} \sum_{t \in T} \sum_{pq \in L} v^i_{pq}(t) x^i_{pq}(t)
\]

subject to

\[
\sum_{t \in T} \sum_{pq \in L} x^i_{pq}(t) = 1 \quad \forall i \in I
\]

single demand condition

\[
\sum_{i \in I} \sum_{pq \in L} x^i_{pq}(t) = \mu \quad \forall t \in T
\]

capacity limit

\[
\sum_{i \in I} \sum_{pq \in L} x^i_{pq}(t - 1) = \sum_{i \in I} \sum_{pq \in L} x^i_{pq}(t) \quad t = 2, \ldots, T, \forall q \in N
\]

temporal-spatial OD connection

\[
x^i_{pq}(t) \in \{0, 1\} \quad \forall pq \in L, \forall i \in I, \forall t \in T
\]

each user’s permits allocation

If users bid their true values, service supplier only has to solve this optimization problem [SO] and social optimum is achieved. However, if users bid their false values strategically, it is difficult to maximize social welfare. Therefore, we need a mechanism to make users bid true values.
VCG Mechanism for Mobility Sharing

Vickrey–Clarke–Groves Mechanism satisfy efficiency and strategy-proofness. We extend VCG mechanism for mobility sharing permits.

VCG mechanism for mobility sharing permit

1. All users bid on the permits, which becomes the users’ demand.
2. Auctioneers decide the allocation of permits in order to maximize the sum of bidding values under temporal-spatial OD connection conditions and capacity limit conditions.
3. Winners must pay for the permits and the price is the winner’s externality, which is the decrease in optimal social welfare when she is included in the auction. The price is called “Vickrey payments”.

Under this mechanism, users have an incentive to bid their true values because they cannot get the larger utilities by telling a lie than the utilities by telling truth.

Therefore, supplier can solve the optimization problem and the social optimum is achieved.
The example of Vickrey payments

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The payment of user $i$

$$P^{\uparrow i}(v) = W(0, v^{\uparrow - i}) - W^{\uparrow - i}(v)$$

Maximum Social Welfare without $i$ in the bidders set

$$P^{\uparrow A}(v) = 210 - 160 = 50$$

The Vickrey payment is the negative externality of user A’s usage.
The Solution Method of Single-Minded Bid Case

- The winner determination problem [SO] is included in combinatorial optimization problems. And these problems are NP-hard in general.
- First, we assume that users bid for only 1 timeslot permit.
- Users will bid for the most variable timeslot permit for them.
- This assumption is single-minded bids in combinatorial auction.
- Computational effort is small.
- In this case, we can interpret the price by the dual problem.
Linear relaxation is identical to LP

- In single-minded bidder setting, the social optimization problem is redefined by the following equations:

\[
\max_x \sum_{i \in I} \sum_{t \in T} \sum_{pq \in L} v^{i}_{pq} (t^i) \cdot x^{i}_{pq} (t^i)
\]

subject to

\[
\sum_{i \in I} \sum_{pq \in L} x^{i}_{pq} (1) = \mu
\]

\[
\sum_{i \in I} \sum_{p \in N} x^{i}_{p} (t - 1) = \sum_{i \in I} \sum_{p \in N} x^{i}_{pq} (t) \quad t = 2, \ldots, T, \forall q \in N
\]

\[
0 \leq x^{i}_{pq} (t^i) \leq 1 \quad \text{linear relaxation} \quad \forall i \in I, \forall pq \in L, \forall t \in T
\]

Since the constraint coefficient matrix is totally unimodular, the solution of the linear relaxation problem [SO-SMB] is identical to that of the LP problem [SO-SMB-LP] (see Hoffman and Kruskan (1956))

Solving LP problem is much easier than IP problem!
Vickrey payments decomposition

- From the dual problem of [SO-SMB],
  \[ P_{pq}^{t}(t) = a_{p}(t) - a_{q}(t+1) \quad \forall t \]

1) Vickrey payments are decomposed into the payment for the previous user and the income from the following user.
2) Vickrey payments between the same OD pairs and time slots are the same price.
3) Users leave their origin as consumers and arrive at their destination as suppliers.
4) What they consume (or supply) is the opportunity to use mobility sharing.
Negative price

- In this mechanism, there is negative price.

The payment of each user

\[ P^1 = (82 + 20 + 92) - (20 + 92) = 82 \]
\[ P^5 = (95) - (95 + 92) = -92 \]
\[ P^4 = (95 + 20 + 72) - (95 + 20) = 72 \]

- This mechanism generates the negative price and efficient allocation naturally.
The extension for activity chain

- If some users want round trips, our model framework can be extended easily.

\[
\max_x \sum_{i \in I} \sum_{t \in T} x_{pq}(t) - \sum_{i \in I} \sum_{p \in N} x_{pq}(t) - \sum_{i \in T} \sum_{pq \in L} x_{pq}(t) - x_{pq}(t) 
\]

subject to

- \( x_{pq}(t) \in \{0, 1\} \)
- \( x_{pq}(t) \in l \)
- \( t \in \{1, 2, \ldots, T - k_i\} \)
- \( q \in N \)
The solution method in multiple bidding case

- In the case of multiple bidding case, the LP relaxation is not guaranteed.
- We propose the new solution method by combining the primal-dual algorithm and a branch and bound algorithm.

- Finally, we compare the exact solution (multiple bidding), approximate solution (single-minded bidding), and First come, first serve rule.
Efficiency of mobility sharing auction and first come, first serve rule

The number of time slots is 38.

The choice probability follows the logit model as follows: 

\[ P_i = \frac{\exp(\theta \cdot v_i)}{\sum_j \exp(\theta \cdot v_j)} \]

where \( P_i \) is the maximum value in the user's value vector. The second case is that each user attempt to stochastically select the bidding case is 833. However, the social welfares of all first come, first serve rules do not compare to that of two.

Table 1. As mentioned above, the social welfare of the multiple bidding case is 837 and that of the single-minded bidding case is 833.

We assume that the scale parameter is 0. The probability of each time slot is 1 when the value is 0.

We show the result of the efficiency of mobility sharing auction and first come, first serve rule.

Fig. 7. Efficiency of mobility sharing auction and first come, first serve rule. The number of users = 2000.

The number of time slots = 400.

The efficiency is considerably higher than that of all the first come, first serve rules. The result indicates that the mobility sharing auction is efficient than the other allocation rules. The results indicated that the mobility sharing auction was easy to implement in practice, and it is feasible and practical even if the number of time slots is high. When compared with the efficiency of the first come, first served rule, the findings indicated that the mobility sharing auction possessed a considerably higher efficiency than the other allocation rules. The results indicated that the mobility sharing auction was easy to implement in practice, and it is feasible and practical even if the number of time slots is high.

The extended framework, it was shown that the model framework focused on mobility sharing that allows the price to be the same between the OD pair and do not only value one-way trips. In the extended framework, it was shown that the model framework focused on.
Conclusions and Future work

• The essence of mobility sharing is the service with “knots” of trips/paths.
• For satisfying temporal-spatial OD connection condition, we proposed the mobility sharing auction.
• From the LP relaxation, Vickrey payment is decomposed into usage fee and income.
• It means mobility sharing is the transaction on “knots” to exchange the opportunity of vehicles usage.

• For future work, we need to study
  – Dynamic mechanism (online auction setting)
  – Preference elicitation mechanism for more easier bidding system because bidding all items is high cognitive cost for users
  – Comparison with other reservation systems and other mobility services (for example, ride sharing)

Τηανκ ψου φορ ψουρ αττεντιον!
Appendix
Fig. 6. Computational Times of numerical examples.

Fig. 7. Efficiency of mobility sharing auction and first come, first serve rule.

- The number of users = 2000
- The number of users = 5000

The number of vehicles: 10
The number of sharing ports: 100
## Numerical Example (capacity $\mu = 2$)

<table>
<thead>
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<th>User ID</th>
<th>OD</th>
<th>value ($t = 1$)</th>
<th>value ($t = 2$)</th>
<th>value ($t = 3$)</th>
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### 1st car’s allocation

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Sum of values (Social Surplus) is 270.

### 2nd car’s allocation

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</table>

Sum of values (Social Surplus) is 255.
The Externality of Vehicle Moving in Mobility Sharing

1) spatial dispersion of mobility

2) mismatch between previous users and next users

The important point of mobility sharing

“sharing a vehicle by more than one person” is not essential.
“Connecting trips temporally and spatially” is essential.
Our Concept: Mechanism Design for transportation system

- A unified framework for transportation policy

![Diagram of temporal-spatial connected transportation services and network with a bottleneck]
What is the cause of the negative externality?

• We need a unified framework
  – various travel behaviors such as mode, route, departure time, destination choice
  – various transportation system such as road network, public transport, on-demand mobility service
Research Objective

- Research Objective
  - Designing tradable permits system for mobility sharing

- Sharing System has constrained supply capacity

- Early Studies about bottleneck tradable permits/credits

- Technical issues
  - Early studies focus on road network.
  - Mobility sharing doesn’t have constrained road capacity, but have constrained vehicle capacity.
Our Concept: Mechanism Design for transportation system

- A unified framework for transportation policy

Bottleneck network

Temporal-spatial connected transportation services

- Hara and Hato (2014, 2017a, 2017b)
- Akamatsu (2007)
- Yang and Wang (2011)
- Beckmann (1956), Daganzo and Sheffi (1977)
- Kuwahara (2007)
- Vickrey (1969)
Why is “a service” important?

• Many innovative mobility service
  – bicycle sharing, car sharing, ride sharing
• The driving force of these services is ICT.
  – In reality, there is still inefficient because of the characteristics of travel behavior