Surge Pricing and Labor Supply in the Ride-Sourcing Market

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Ride Sourcing

• Ride sourcing refers to an emerging urban mobility service that private car owners drive their own vehicles to provide for-hire rides (Rayle et al. 2014).

• Companies like Uber and Lyft provide ride-hailing apps that intelligently match participating drivers to riders. These apps are free to use but usually a commission is charged for each transaction/ride (20-25% of the fare paid by the rider).
Surge Pricing

• Since their advent in 2009, ride-sourcing companies have enjoyed huge success, but have created many controversies as well.

• One of them centered on dynamic pricing (e.g., Uber’s surge pricing and Lyft’s prime time pricing) that price is algorithmically adjusted for different geographic areas and updated periodically.
Surge Pricing (Cont’d)

• Spatially differentiated dynamic pricing
• Surge multiplier times a base price
• Both customers and drivers are informed before a transaction occurs
• Falls within 1-2 (most of the cases) but can soar to 7 times or even higher without a cap
Surge Pricing (Cont’d)

• Advertised to be passively triggered to help balance supply and demand

• Unlimited price surge can be troublesome particularly when ride sourcing is one of the few viable options

• As platforms generally benefit from the surges, prices may surge unnecessarily higher or more frequently to exploit customers
Research Motivation

• It is thus necessary to investigate the impacts of surge pricing and derive insight on how to better manage or regulate the scheme.

• Surge pricing directly influences the spatial and temporal distribution of ride-sourcing labor supply. Previous studies, e.g., Chen et al. (2015) and Zha and Yin (2017), have investigated its spatial impacts.

• This paper focuses on investigating how surge pricing may temporally redistribute the supply of ride-sourcing vehicles.
Research Objective

• We propose a mathematical framework to describe the equilibrium of ride-sourcing market while explicitly considering ride-sourcing drivers’ work scheduling behaviors under surge pricing.

• The equilibrium framework is applied to investigate the impacts of surge pricing and then discuss potential management policies.
Outline

• Basic Premise
• Equilibrium Model
• Modeling and Analysis of Surge Pricing
Basic Premise
Two-Sided Market
Platform Matching

Matching Function (Yang and Yang, 2011)

\[ m^{c-t} = M(N^{vt}, N^c) \]

\( N^{vt} \): number of the vacant ride-sourcing vehicles at any moment

\( N^c \): number of the waiting customers at any moment.

It is assumed that, \( \partial M / \partial N^{vt} > 0 \), \( \partial M / \partial N^c > 0 \)

\[ N^c = w^c Q \text{ (Little’s Law)} \]

\[ m^{c-t} = Q \text{ (Steady State)} \]
Platform Matching (Cont’d)

Matching Function \( m^{c-t} = M(N^{vt}, N^c) \)

\[
N^{vt} = w^t T^{vt} \quad \quad N^c = w^c Q
\]

\( T^{vt} \) is the arrival rate of vacant vehicles per hour
\( w^t \) is the average searching time for a driver to find a customer
\( Q \) is the customers’ arrival rate per hour.
\( w^c \) is the average customer waiting time

\[
m^{c-t} = M\left(N^{vt}, N^c\right) = M\left(w^t T^{vt}, w^c Q\right)
\]

At the stationary state \( m^{c-t} = Q = T^{vt} \)

\[
Q = M\left(w^t Q, w^c Q\right) \quad \quad w^c = W(Q, w^t)
\]
Demand and Supply

Customer Demand

\[ Q = f(F, w^c, l) \]

where:

- \( w^c \): average waiting time of the riders
- \( l \): average trip time

Fleet Status Conservation

\[ u = N^{vt} + N^o = T^{vt} w^t + Ql = Qw^t + Ql \]
Basic Modeling System

- System of Equations for Ride-for-Hire System

\[ w^c = W(Q, w^t) \]
\[ Q = f(F, w^c, l) \]
\[ u = Qw^t + Ql \]

Given \( F \) and \( u \), we can solve the system to obtain \( Q \), \( w^t \) and \( w^c \).
System Adopted in This Paper

\[
\begin{align*}
\omega^c &= \frac{k}{2\nu \sqrt{\frac{w^t Q}{S}}} \\
Q &= \bar{Q} \exp\left( -\theta \left( F + \beta \omega^c + \tau l \right) \right) \\
u &= Qw^t + Ql
\end{align*}
\]

(Daganzo, 1978; Arnott, 1996)

\[
R(u) = \frac{(1-\eta)FQ}{u}
\]

The system implicitly defines a drivers’ revenue function \( R(u) \)
Revenue Function

- It is assumed that:

\[
\frac{\partial R(u)}{\partial u} < 0
\]

This assumption is generally valid. However, it is possible that more vehicle hours would increase the average hourly revenue for the drivers. Such a scenario occurs with an unrealistically small size of ride-sourcing fleet (Yang et al., 2005; Yang and Yang, 2011).
Equilibrium Model
Labor Supply

• Whether, when and how long does a driver work?
  – Ride-sourcing drivers enjoy flexibility in work hour scheduling
  – Competing theories exist on how drivers respond to hourly wage variation. For example, the number of work hours:

\[ \log hr = \beta_0 + \beta_1 \log wage + X\beta. \]

\( \beta_1 > 0 \) Drivers work longer as hourly wage increases: neo-classical hypothesis
\( \beta_1 < 0 \) Drivers work less as hourly wage increases: income-targeting hypothesis (or more general reference dependent hypothesis)
Labor Supply (Cont’d)

• Empirical evidences are mixed

\[ \beta_1 > 0 \quad \text{Farber (2004)} \quad \text{Farber (2005)} \quad \text{Farber (2008)} \]

\[ \beta_1 < 0 \quad \text{Camerer et.al (1997)} \quad \text{Chou (2000)} \]

\[ \text{Neo-classical} \]

\[ \text{Income-targeting} \]

Jonathan et.al (2015)
Farber (2015)
Chen et.al (2016)

Crawford et.al (2011)
Vincent (2016)
Time-Expanded Network

\[ w^e = \frac{k}{2v} \sqrt{\frac{w^e Q}{S}} \]
\[ Q = \overline{Q} \exp \left( -\theta \left( F + \beta w^e + \tau l \right) \right) \]
\[ u = Qw^e + Ql \]

\( R \downarrow a (u \downarrow a) \)
Cost Structure

\[
\bar{C}^{pm} = \sum_{a \in A} c_a^m \delta_a^p + c^{pm} = \sum_{a \in A} c_a^m \delta_a^p + \omega_1^m \left( h^p \right)^{\omega_2^m}, \forall p \in P, m \in M
\]

Disutility when traversing each link (additive)

Disutility of continuously working (path specific and non-additive)

where

\[
\begin{align*}
P; M & : \text{set of paths; set of driver classes} \\
\delta_a^p & : \text{link-path incidence} \\
\omega_1^m & : \text{cost associated with cumulative working hours} \\
\omega_2^m & : \text{captures the aversion to working long hours}
\end{align*}
\]
Utility Specification

**Neo-Classical:**

\[ U^{pm} = R^p \cdot \bar{C}^{pm}, \forall p \in P, \; m \in M \]

**Income Targeting (Farber, 2015):**

\[
U^{pm} = \begin{cases} 
(1 + \rho^m)(R^p - I^m) - \bar{C}^{pm} + U_0, & R^p < I^m \\
(1 - \rho^m)(R^p - I^m) - \bar{C}^{pm} + U_0, & R^p \geq I^m 
\end{cases}, \forall p \in P, \; m \in M
\]

where

- \( I^m \): target income level
- \( R^p \): average revenue of choosing path \( p \)
- \( \rho^m \): controls the degree of loss-aversion and is assumed to vary between \([0,1)\)
Market (Network) Equilibrium

- Drivers are assumed to choose work schedules to maximize their utilities
- At equilibrium, all chosen schedules would offer the same level of utility, which is higher than or equal to that of any unchosen schedule
- For each driver class, all paths that carry positive flows yield equal utility, which is no less than that of any unused path
Mathematical Definition

\[
\left(U^m - U^{pm}(f)\right)f^{pm} = 0, \ \forall m \in M, \ p \in P
\]

\[
U^m - U^{pm}(f) \geq 0, \ \forall m \in M, \ p \in P
\]

\[
f^{pm} \geq 0, \ \forall m \in M, \ p \in P
\]

\[
\sum_{p \in P} f^{pm} = N^m, \ \forall m \in M
\]

where

\(U^{pm}\) : the pay-off of drivers of class \(m\) choosing path \(p\); \(U^m = \max_{p \in P}(U^{pm})\)

\(f^{pm}\) : the number of drivers of class \(m\) choosing path \(p\)

\(N^m\) : the number of drivers of class \(m\)
Variational Inequality

Define $\Omega = \left\{ \sum_{p \in P} f_{pm} = \sum_{m \in M} N^m, f_{pm} \geq 0, \forall m \in M, \forall p \in P \right\}$

The equilibrium solution can be found by finding $f^* \in \Omega$ that satisfies:

$$\sum_p \sum_m \left( -U^{pm}(f^*) \right) \left( f_{pm} - f^{pm*} \right) \geq 0, \forall m \in M, p \in P, f \in \Omega$$
Neo-Classical Formulation

\[
\min_{f} Z = \sum_{a \in A_1} u_a \int_{0}^{u_a} -R_a(w) \, dw + \sum_{m \in M} \sum_{a \in A} c_a^m u_a^m + \sum_{p \in P} \sum_{m \in M} c^{pm} f^{pm}
\]

s.t.

\[
\sum_{p \in P} f^{pm} \leq N^m, \ \forall m \in M
\]

\[
f^{pm} \geq 0, \ \forall p \in P, \ m \in M
\]

\[
u_a = \sum_{m} \sum_{p} f^{pm} \delta^p_a, \ \forall a \in A
\]

*All ride-sourcing vehicles that provide service during a time period are assumed to be available for the following period.*
Solution Algorithm

• Both formulations have two distinctive features. Namely, the revenue function is implicitly defined and path costs are not link-additive.

• Various path-based solution algorithms can be applied to solve the formulations, in combination with a column generation scheme to avoid path enumeration.
Numerical Experiments

\[ Q_b = \bar{Q}_b \exp \left( -\theta_b \left( \bar{F}_b + \beta_b w^c_b + \tau_b l_b \right) \right), \ \forall b \in A_1 \]

We assume that 1) demand sensitivity of price (\( \theta_b \)) is lower in peak hours; 2) values of time (\( \beta_b, \tau_b \)) are higher in peak hours.
Numerical Experiments (Cont’d)

We consider 4 driver classes and each of them has a fleet size of 2,000 vehicles:

\[ m = 1, \text{ prefer to start early (7:00-10:00 a.m.) work long hours (} \omega_1^1 = 2 \) \]
\[ m = 2, \text{ prefer to start early (7:00-10:00 a.m.) work short hours (} \omega_1^2 = 3 \) \]
\[ m = 3, \text{ prefer to start early (15:00-19:00 p.m.) work long hours (} \omega_1^3 = 2 \) \]
\[ m = 4, \text{ prefer to start early (15:00-19:00 p.m.) work short hours (} \omega_1^4 = 3 \) \]

The income target for the corresponding driver class is assumed to be:

\[ I^m = \begin{cases} 
$200, & m = 2, 4 \\
$300, & m = 1, 3 
\end{cases} \]
## Neo-classical

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<th>End time</th>
<th>Work hours (hr)</th>
<th>Rest hours (hr)</th>
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## Income-Targeting

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Modeling and Analysis of Surge Pricing
Surge Multiplier

• Fare structure

\[ \bar{F}_b = F_0 + \gamma_b F_b, \quad \forall b \in A_1 \]

where

- \( F_0 \) : flag-drop fee
- \( F_b = \omega_b l_b \) : time-based charge
- \( \gamma_b \) : surge multiplier
Revenue-Maximizing Surge Pricing

\[
\begin{align*}
\max_{\pi, \lambda, f, \gamma > 0} \quad & J = \sum_{b \in A_1} \eta \left( F_0 + \gamma_b F_b \right) Q_b \\
\text{s.t.} \quad & G(\pi, \lambda, f \mid \gamma) = 0
\end{align*}
\]

where: \( G(\cdot) \) is a gap function used to characterize the equilibrium of drivers’ work scheduling
Numerical Example

![Graph showing surge multipliers over 24 hours]

- Surge Multiplier
- Hour
- X-axis: 1 to 24
- Y-axis: 0 to 3

The graph illustrates the surge multipliers throughout the day, with peaks occurring at certain hours.
Impact of Surge Pricing

![Graph showing impact of surge pricing on average waiting and searching times.](image-url)
Welfare Impacts

• Benchmark for comparison
  – Optimal static revenue-maximizing pricing where the surge multiplier is 2.02 for all study periods

• Metrics
  – Passengers: consumers’ surplus
  – Platform and drivers: joint revenue
Welfare Impacts (Cont’d)

Change of consumers' surplus
Change of joint revenue

$ x 10,000

Hour
Regulation Policy

• Commission cap regulation, if the platform exhibits evident market power
  – Fixed percentage -> fixed cap (e.g., 20%-> $4)
  – Practically implemented as mile-based or time-based charge (i.e., $/mile, $/hour)

• With a properly chosen cap, a revenue-maximizing platform will have incentive to maximize the number of transactions (realized demand), which are positively related to consumers’ surplus; the additional revenue from price surge completely goes to the drivers
Impact of Regulation

$$\max_{\pi, \lambda, f, \gamma > 0, \bar{\eta} \geq 0, N \geq 0} \hat{J} = \sum_{b \in A_1} \hat{\eta}_b Q_b$$

s.t.

$$G(\pi, \lambda, f \mid \gamma, \hat{\eta}, N) = 0$$

$$\lambda \geq \pi^R$$

$$\hat{\eta}_b \leq \bar{\eta}, \ \forall b \in A_1$$

where

$$\pi^R : \text{is the vector of the reservation profit levels (per work session);}$$

$$\bar{\eta} : \text{commission cap.}$$
A proper choice of commission cap (if necessary) should seek the balance between the profit margin of the ride-sourcing company and the market efficiency.
Summary

• We have proposed a modeling framework to capture the temporal effects of surge pricing on the ride-sourcing labor supply

• Drivers and platforms are better off under revenue-maximizing surge pricing while customers are worse off in highly surged periods

• Surge pricing can create a win-win situation in certain periods as compared with its static counterpart

• Capping the commission may enhance market efficiency
Thank You!

Questions?