Lecture 29
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
Prof. James P. Hambleton
Chapters 14: §14.1 Newton's Second Law, §14.2 Applications—Cartesian Coordinates and Straight-Line Motion[1]

Acknowledgements

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Newton's Second Law

Newton's second law for a particle[2] or the center of mass of an object[3]:

$$\sum \vec{F} = m\vec{a}$$

where $\sum \vec{F}$ is the sum of all external forces.

If $\sum \vec{F} \neq 0$, then object is accelerating. In other words, it is not in equilibrium using our previous terminology or, to use new terminology, it is now in dynamic equilibrium.

Newton's 2nd law cannot be used with an accelerating reference frame. A frame where Newton's 2nd law holds is called an inertial reference frame.

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.
[2] Assuming mass is constant.
[3] The book proves this for an arbitrary object, showing that the sum of external forces equals the product of the total mass (considered as a summation of particles) and the acceleration of the center of mass. In this course we only deal with translation, but of course object can rotate as well. The general theory involving rotation brings in none other than…moments!

Example: An elevator makes you feel heavier or lighter when it starts or slows down. A reference frame fixed to the elevator is therefore only an inertial reference frame when the elevator is moving at constant velocity (or stopped, a particular instance of constant velocity). A reference frame fixed to the earth is a valid reference frame, even when the elevator is accelerating[4].

Is a reference frame fixed at a point on the earth's surface an inertial reference frame?

Strictly speaking, no, but since the Earth turns relatively slowly relative to the motion of objects on its surface, this is a good approximation for many problems.

Cartesian Coordinates

In a Cartesian reference frame, Newton's second law can be expressed in terms of Cartesian components:

$$\begin{aligned}\sum \vec{F} &= \left(\sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}\right)\\ &= m\vec{a}\\ &= m\left(a_x \hat{i} + a_y \hat{j} + a_z \hat{k}\right)\end{aligned}$$

Equating the x-, y- and z-components, we have three scalar equations:

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

[4] See the elevator example in Section 14.1; we will work through a similar example momentarily.

Analysis steps:

1. Choose an appropriately aligned reference frame.
2. Construct the free-body diagram for the of object interest. If the object interacts with others (e.g., through pulleys or friction), you may need to construct the free-body diagrams for those as well.
3. Determine the motion using Newton's second law.

Example (familiar): Projectile motion.

Free-body diagram:

Force is $\vec{F} = (-W)\,\hat{j}$. In component form, $F_y = -W$ and $F_x = F_z = 0$.

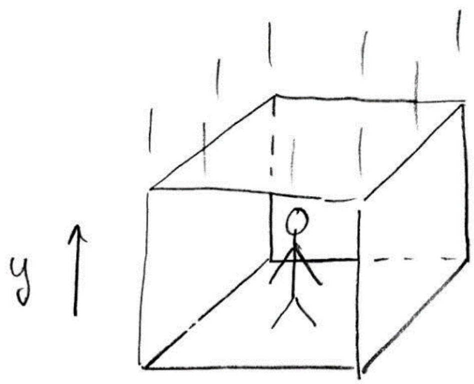
Using Newton's second law, we can write

$$F_x = ma_x = 0 \quad \Rightarrow \quad a_x = 0$$

$$F_y = -mg = ma_y \quad \Rightarrow \quad \boxed{a_y = -g}$$

$$F_z = ma_z = 0 \quad \Rightarrow \quad a_z = 0$$

<u>Example (elevator)</u>:



A 150 lb person stands on a scale in an elevator with acceleration $\vec{a}_e = a_e \hat{j}$.

(a) If the scale reads 155 lb, what is a_e?

(b) If $a_e = -2$ ft/s^2, what does the scale read?

Strategy: Since the elevator is accelerating, we will choose a reference frame that is fixed to the earth. This is a one-dimensional example (straight-line motion), so we will only worry about the y-direction. We will draw the FBD, sum forces, and consider the motion.

Draw FBD:

The normal force *N* corresponds to the scale reading.

$$\sum F_y = N - W = ma_e$$

Part (a): $N = 155 \text{ lb}$, $W = 150 \text{ lb}$

Work out mass *m* from the weight:

$$m = \frac{150 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}} = 4.658 \text{ slugs}$$

$$a_e = \frac{155 \text{ lb} - 150 \text{ lb}}{4.658 \text{ slugs}} = 1.073 \frac{\text{ft}}{\text{s}^2} \text{ (upward)}$$

$$\boxed{a_e = 1.07 \frac{\text{ft}}{\text{s}^2}}$$

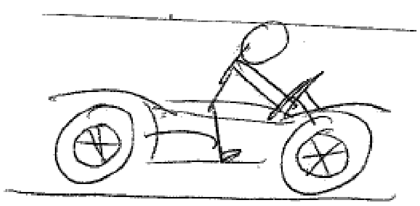
Note that the elevator can either be going up or going down!

Part (b):

$$
\begin{aligned}
N &= W + ma_e \\
&= 150 \text{ lb} + (4.658 \text{ slugs})\left(-2\frac{\text{ft}}{\text{s}^2}\right) \\
&= 140.7 \text{ lb}
\end{aligned}
$$

$$\boxed{N = 141 \text{ lb}}$$

<u>Example:</u> Problem 14.17 in textbook.



The combined weight of the motorcycle and rider is 360 lb. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.8$. The rider starts from rest, spinning the rear wheel. Neglect the horizontal force exerted on the front wheel by the road. In two seconds, the motorcycle moves 35 ft. What was the normal force between the rear wheel and the road?

Strategy: Draw FBD and consider the motion.

$$\sum F_x = ma_x : \qquad f = \mu_k N_r = ma_x \qquad (\#)$$

$$\sum F_y = ma_y = 0 : \qquad N_r + N_f = W \qquad (*)$$

In this case, the friction force f is the <u>thrust</u>.

Constant acceleration starting from rest, so $s = \frac{1}{2}a_x t^2$ and

$$
\begin{aligned}
a_x &= \frac{2s}{t^2} \\
&= \frac{2(35\,\text{ft})}{(2\,\text{s})^2} \\
&= 17.5\ \frac{\text{ft}}{\text{s}^2}
\end{aligned}
$$

Using Eq. (#), we find

$$
\begin{aligned}
N_r &= \frac{ma_x}{\mu_k} \\
&= \frac{\left(\dfrac{360\ \text{lb}}{32.2\ \text{ft/s}^2}\right)\left(17.5\ \dfrac{\text{ft}}{\text{s}^2}\right)}{0.8} \\
&= 244.6\,\text{lb}
\end{aligned}
$$

$$\boxed{N_r = 244\ \text{lb}}$$

Using Eq. (*), we conclude

$$
\begin{aligned}
N_f &= W - N_r \\
&= 360\ \text{lb} - 244.6\ \text{lb} \\
&= 115.4\ \text{lb}
\end{aligned}
$$

$$\boxed{N_f = 115\ \text{lb}}$$