Lecture 26
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
Prof. James P. Hambleton
Review of Midterm Exam; Chapters 13: §13.4 Curvilinear Motion—Cartesian Coordinates[1]

Acknowledgements

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Curvilinear Motion

Recall that we described a general motion along a curved path (curvilinear motion) through space through the position vector $\vec{r}(t)$.

In terms of a reference frame with an associated Cartesian coordinate system[2], we have

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

As compared to straight-line motion, we now have up to three scalar quantities to consider (2 for 2D).

A key feature is that (like equilibrium) the <u>motion along one axis is independent of the motion along the other axes</u>.

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.
[2] In Section 13.7, the book covers polar and cylindrical coordinates. We will not cover this.

Velocity and acceleration are again obtained by differentiating, where differentiating a vector simply means differentiating each scalar component:

$$
\begin{aligned}
\vec{v}(t) &= \frac{d\vec{r}(t)}{dt} \\
&= \frac{dx(t)}{dt}\hat{i} + \frac{dy(t)}{dt}\hat{j} + \frac{dz(t)}{dt}\hat{k}
\end{aligned}
$$

We denote the velocity components by

$$
v_x = \frac{dx}{dt}
$$

$$
v_y = \frac{dy}{dt}
$$

$$
v_z = \frac{dz}{dt}
$$

Then,

$$
\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}
$$

Similarly,

$$
\begin{aligned}
\vec{a}(t) &= \frac{d\vec{v}(t)}{dt} \\
&= \frac{dv_x(t)}{dt}\hat{i} + \frac{dv_y(t)}{dt}\hat{j} + \frac{dv_z(t)}{dt}\hat{k} \\
&= a_x(t)\hat{i} + a_y(t)\hat{j} + a_z(t)\hat{k}
\end{aligned}
$$

The <u>acceleration components</u> are

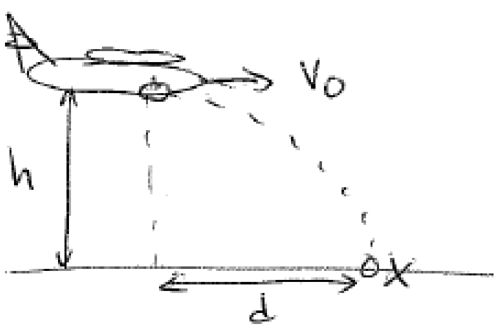
$$a_x = \frac{dv_x}{dt}$$

$$a_y = \frac{dv_y}{dt}$$

$$a_z = \frac{dv_z}{dt}$$

We will only look at 2D problems, so we only worry about two components (*x* and *y*) of position, velocity, and acceleration.

<u>Example:</u> Problem 13.75 from the textbook.



To drop an object onto a target from a height of h = 30 m with a velocity of $v_0 = 40\,\text{m/s}$, what is the distance d at which the object should be dropped?

Strategy: We need to know how long it will take for the object to fall (*y*-direction) and then, in that time, how far it will move horizontally (*x*-direction). In the absence of other information, we will neglect drag and assume acceleration in *y*-direction is given by acceleration due to gravity.

Select reference frame as usual (*x*-direction horizontal and *y*-direction vertical), with the origin on the <u>ground</u> and $x = 0$ corresponding to the position of the <u>plane</u> at $t = 0$.

First, consider the y-component of the motion:

$$a_y = -g$$

$$\begin{aligned} v_y(t) &= \int a_y dt \\ &= -gt + C_1 \end{aligned}$$

$$v_y(0) = 0 \quad \Rightarrow \quad C_1 = 0$$

$$\begin{aligned} y(t) &= \int v_y \, dt \\ &= \int (-gt) \, dt \\ &= -\frac{1}{2} g t^2 + C_2 \end{aligned}$$

$$y(0) = h \quad \Rightarrow \quad C_2 = h = 30\,\text{m}$$

Determine drop time using $y\left(t_{drop}\right) = 0$:

$$y\left(t_{drop}\right) = -\frac{1}{2} g t_{drop}^2 + h = 0$$

$$\begin{aligned} t_{drop} &= \sqrt{\frac{2h}{g}} \\ &= \sqrt{\frac{2(30\text{ m})}{\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)}} \\ &= 2.473\text{s} \end{aligned}$$

Now, consider the *x*-component of the motion:

$$x(t) = v_0 t$$

$$\begin{aligned} x\left(t_{drop}\right) &= v_0 t_{drop} \\ &= \left(40\ \frac{\text{m}}{\text{s}}\right)(2.473\ \text{s}) \\ &= 98.9\ \text{m} \end{aligned}$$

$$\boxed{d = 98.9\,\text{m}}$$