Lecture 23
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
Prof. James P. Hambleton
Chapters 9: §9.1 Theory of Dry Friction[1]

Acknowledgements

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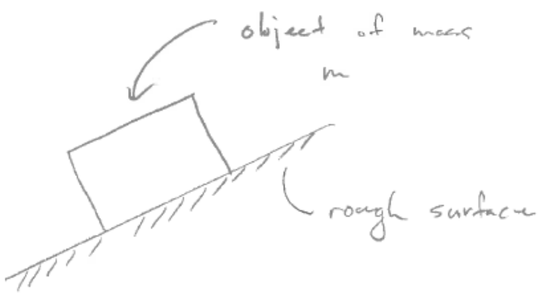
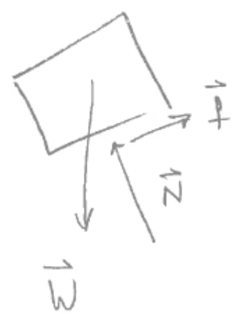
Theory of Dry Friction

Recall the following problem, where the friction force $\vec{f}$ is determined simply by equilibrium:

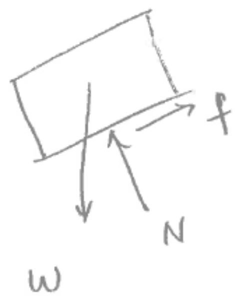




At what inclination will the box slide, and what will be the friction force $\vec{f}$?

Like trusses and frames, we will now consider only 2D problems, and will work in terms of force components. We can therefore drop the vector symbol $\overrightarrow{(\ )}$ and consider simply f and N.

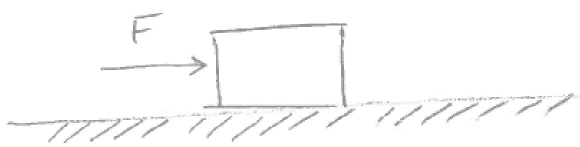
[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

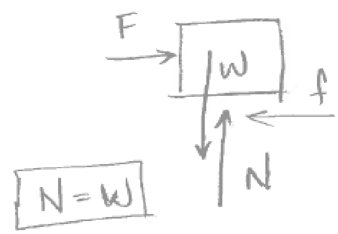


- *f* is the friction force that acts parallel to the interface between two objects.
- *N* is the normal force that acts perpendicular to the interface.

Reconsider this problem with a horizontal surface and a horizontal (tangential) force *F*.

FBD





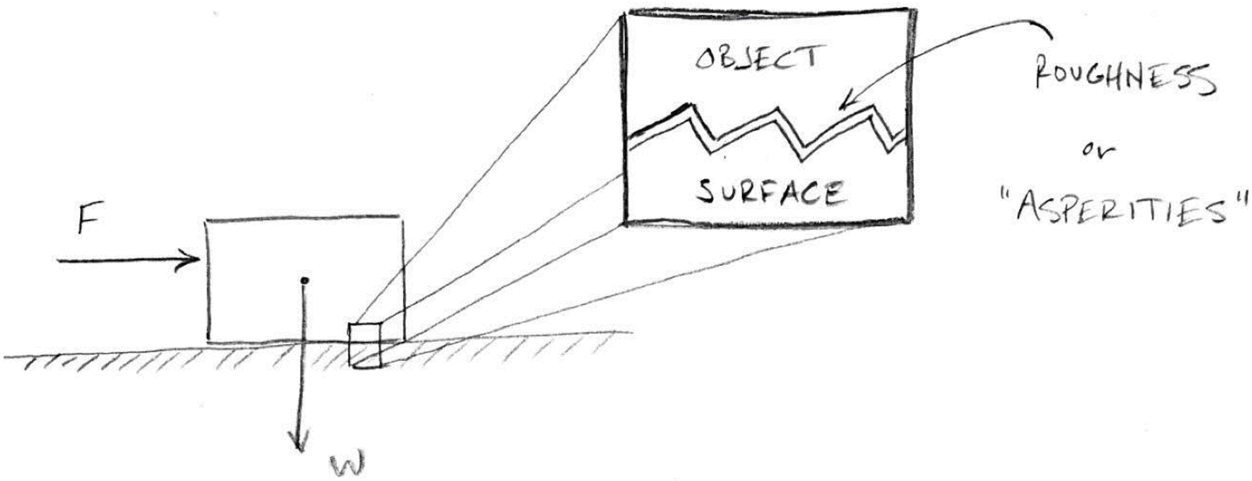
Now, to sharpen our questions[2]:

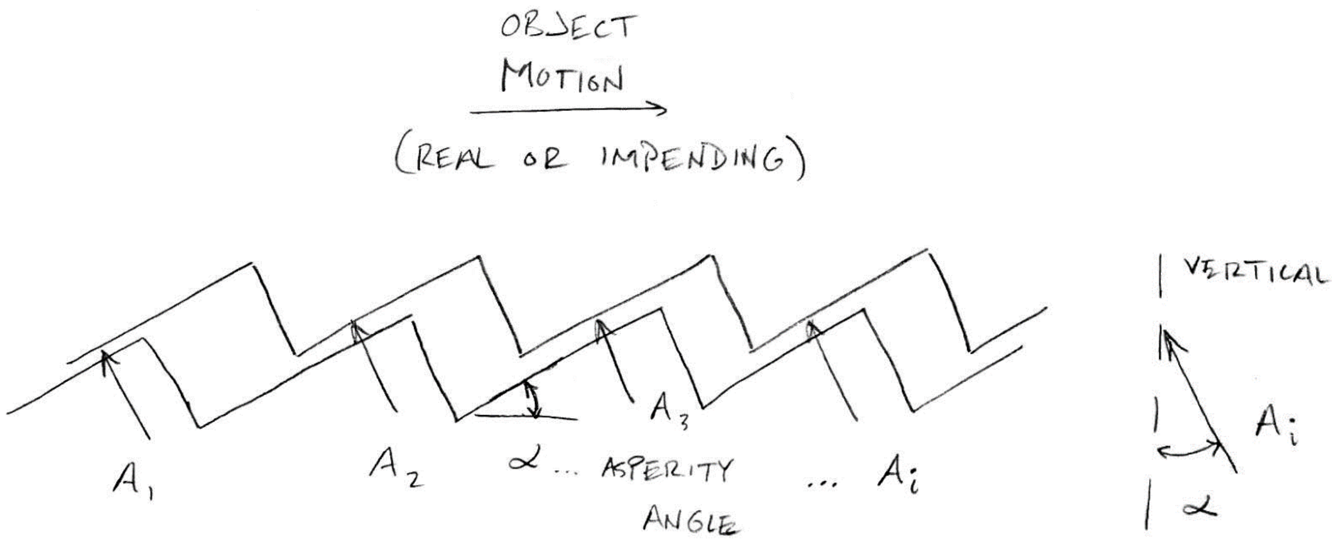
- At what value of *F* will the object start to slide?
- Once the box starts to slide, what is the value of *F* required to keep the object sliding at a constant rate[3]?

Dry friction arises from microscopic roughness of the surfaces in contact, and is fundamentally different from wet, or lubricated, friction (which we will not consider).

[2] We can relate this example to the physical problem of an eraser resting on a desktop.
[3] If the object is not accelerating, the net forces must again be zero. That is, in a particular *inertial reference frame*, the object is in equilibrium (see Section 3.1 for details).

A simple (and incomplete) dry friction model





Equilbrium of object, assuming smooth asperities:

$$\sum F_x = 0: F - \sum_i A_i \sin\alpha = 0$$

$$\sum F_y = 0: \sum_i A_i \cos\alpha - W = 0$$

Eliminate $\sum_i A_i$ from these equations to find

$$F = (\tan\alpha)W \quad \text{or} \quad \boxed{F = (\tan\alpha)N}$$

This is the force required to cause the object to start to slide. Have you seen an equation like this before?

The coefficient on N is the coefficient of static friction μ_s, and it relates to the angle α, a measure of roughness.

$$\boxed{\mu_s = \tan\alpha}$$

Note that the contact area does not enter this law of dry friction.

From experience (and in particular measurements), we know that the value of F required to initiate sliding is greater than the value required to keep the object sliding. This simple model cannot predict this. The field dedicated to the study of friction is tribology.

Coefficient of Static Friction

The maximum friction force[4] exerted by dry contacting surfaces that are not in relative motion is

$$f_{\max} = \mu_s N$$

[4] The book writes simply as $f = \mu_s N$, which may be confusing.

where μ_s is the coefficient of static friction. Note that μ_s does not have units.

The magnitude of friction force $|f|$ can in fact take on any value between 0 and $f_{\max}$. Any friction forces in this range are admissible.

$$0 \le |f| \le f_{\max}$$

Slip is impending if $|f| = f_{\max}$. In this case, the two surfaces are on the verge of slipping relative to each other.

Coefficient of Kinetic Friction

The magnitude of the friction force exerted by dry contacting surfaces that are in motion (slipping or sliding) relative to each other is

$$|f| = \mu_k N$$

where μ_k is the coefficient of kinetic friction. It is does not have units, and is generally smaller than μ_s, i.e., $\mu_k < \mu_s$.

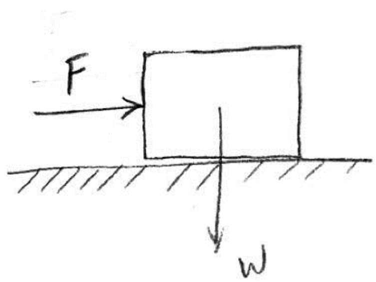
Note that the friction force is determined uniquely once sliding commences.

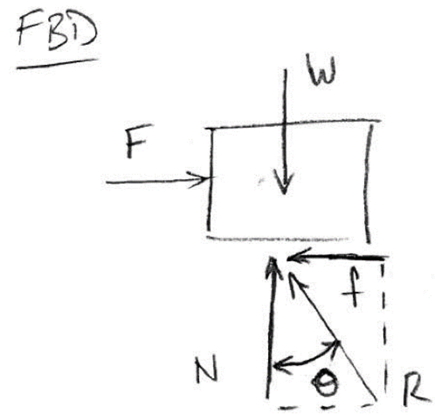
Angles of friction

We can alternatively write $f_{\max} = (\tan\theta_s)N$ and $f = (\tan\theta_k)N$, where

$\boxed{\mu_s = \tan\theta_s}$ and $\boxed{\mu_k = \tan\theta_k}$.

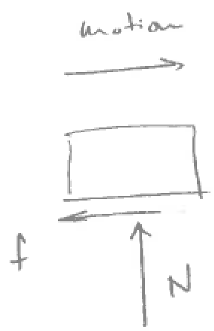
These angles are useful when we want to express the reaction between the contact surfaces in terms of the magnitude *R*.

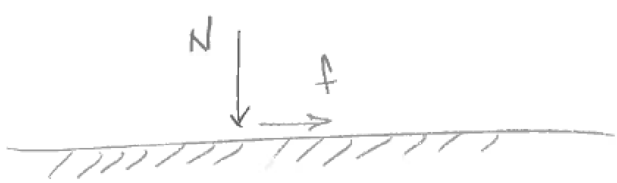




Direction of the friction force

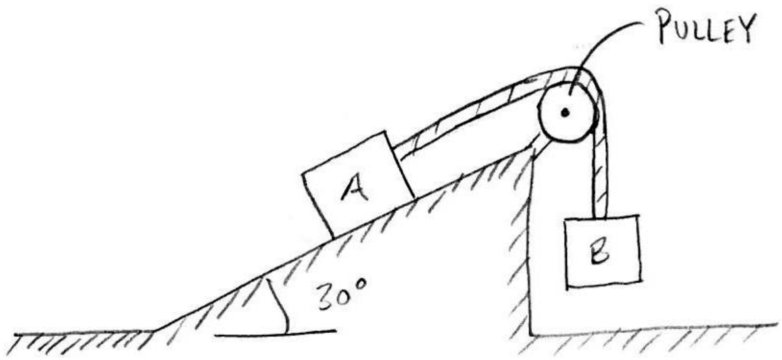
The friction force on the movable object is always in a direction that opposes the motion, or the impending motion. The friction force on the surface is equal and opposite.





Object Surface

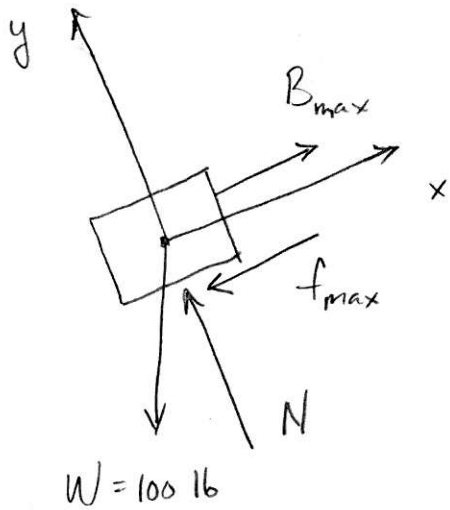
<u>Example</u>: Problem 9.11 from the textbook.



Box *A* weighs 100 lb, and the coefficients of friction between box *A* and the ramp are $\mu_s = 0.30$ and $\mu_k = 0.28$. For what range of weights of the box *B* will the system remain stationary?

<u>Strategy</u>: We are interested in the case of <u>impending sliding</u>, and will therefore use the coefficient of static friction. We need to consider two possibilities: (1) box *B* is too heavy, such that box *A* is about to slide up, and (2) box *B* is too light, such that box *A* about to slide down. Draw the FBD for each case and solve for the tension in cable, which will equal the weight of box *B*. In this case, it is useful to rotate axes, aligning the *x*-axis with the surface.

<u>Case 1:</u> (Box *B* is too heavy.)



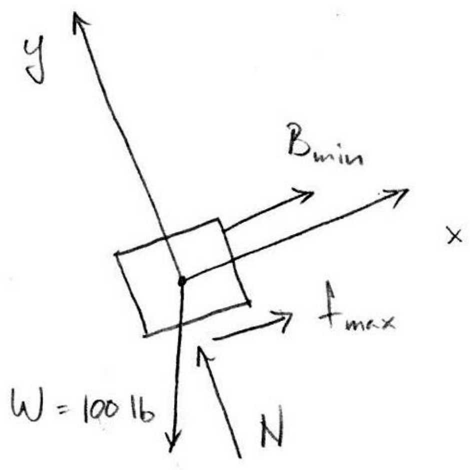
$$\sum F_x = 0: \; B_{\text{max}} - f_{\text{max}} - (100\,\text{lb})\sin 30^\circ = 0$$

$$B_{\text{max}} - 0.3N - (100\,\text{lb})\sin 30^\circ = 0 \quad (\#)$$

$$\sum F_y = 0: N - 100\cos 30^\circ = 0 \;\; \Rightarrow \;\; \boxed{N = 86.6\,\text{lb}}$$

Substituting N = 86.6 lb into Eq. (#), we find $\boxed{B_{\text{max}} = 76.0\,\text{lb}}$.

Case 2: (Box *B* is too light.)



We note that N = 86.6 lb does not change from Case 1.

$$\sum F_x = 0: \quad B_{\text{min}} + f_{\text{max}} - (100\,\text{lb})\sin 30^\circ = 0$$
$$B_{\text{min}} + 0.3N - (100\,\text{lb})\sin 30^\circ = 0 \quad (*)$$

Substituting N = 86.6 lb into Eq. (*), we find $\boxed{B_{\text{min}} = 24.0\,\text{lb}}$.

Since $B_{\text{min}} > 0$ we know that box *A* will slide by itself, if it were not for the support of box B.

The stable range for the weight of box *B* is $\boxed{24.0\text{ lb} < B < 76.0\text{ lb}}$

Go through Examples 9.1, 9.2, and 9.3 in the textbook on your own.