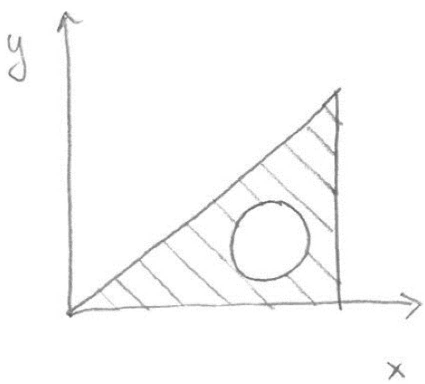
Lecture 22
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
Prof. James P. Hambleton
Ch. 7: §7.2 Composite areas; §7.3 Distributed loads[1]

Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Composite Areas (continued)

"Negative" areas:



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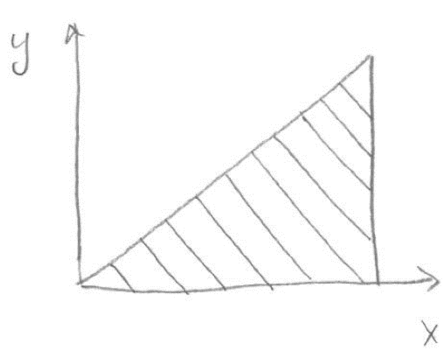

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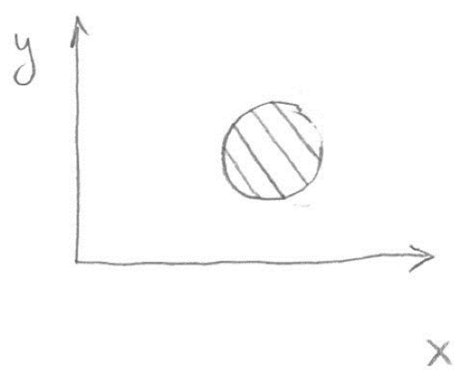


[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

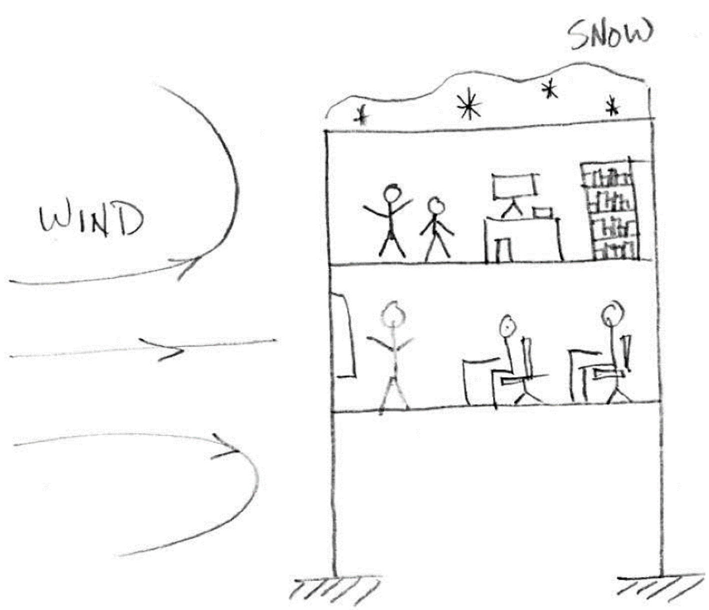
We simply need to subtract, rather than add, in our formulas:

$$\bar{x} = \frac{\int_{A_1} x dA - \int_{A_2} x dA}{\int_{A_1} dA - \int_{A_2} dA} = \frac{\bar{x}_1 A_1 - \bar{x}_2 A_2}{A_1 - A_2}$$

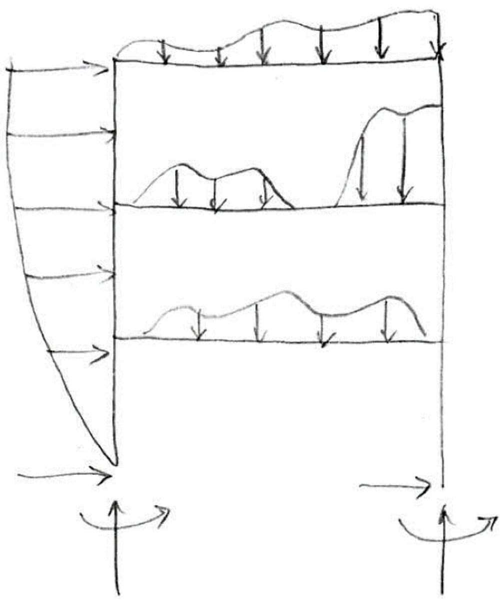
Go through Example 7.4 in the textbook to work through such a problem on your own.

Distributed Loads

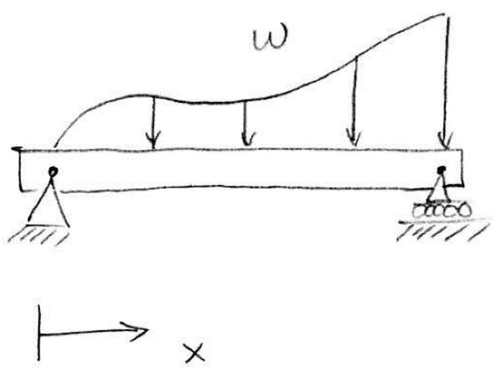
Thus far we have dealt with concentrated forces and couples. Many loads are in fact distributed.



Free-body diagram showing distributed loads:



Consider a distributed load acting on a *simply supported*[2] beam:

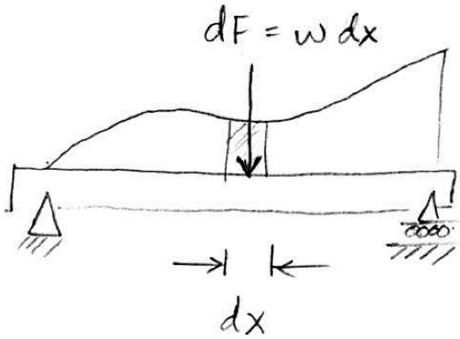


The variable w denotes the distributed load, or <u>load intensity</u>, which has units of $[w]$ = force/length (e.g., N/m or lb/ft).

Note that w is a function of the position, x! In general, we can write $w = w(x)$.

[2] Define "simply supported" if not done previously in the class.

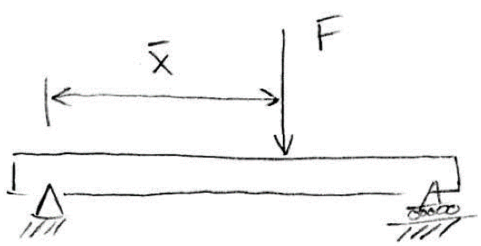
Physically, w is force acting over a small area $\Delta x \to dx$:



The total force F is

$$F = \int_L dF = \int_L w dx$$

To be an equivalent system, where does the force act?

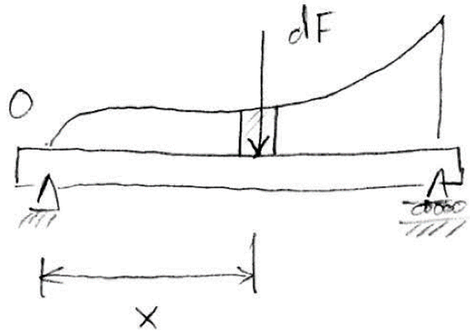


Consider for the time being that clockwise moments are positive.

$$M = \bar{x} F$$

What is $\bar{x}$?

Consider the moment of the infinitesimal force *dF* about the origin *x* = 0:



Assume for convenience that clockwise forces are positive (just for this derivation).

$$dM = xdF$$
$$= x(wdx)$$

$$M = \int_L dM$$
$$= \int_L xdF$$
$$= \int_L xwdx$$

Equate this to $M = \bar{x}F$:

$$\bar{x}F = \int_L xwdx$$

Since $F = \int_L wdx$, we can write

$$\bar{x}\int_L wdx = \int_L xwdx$$

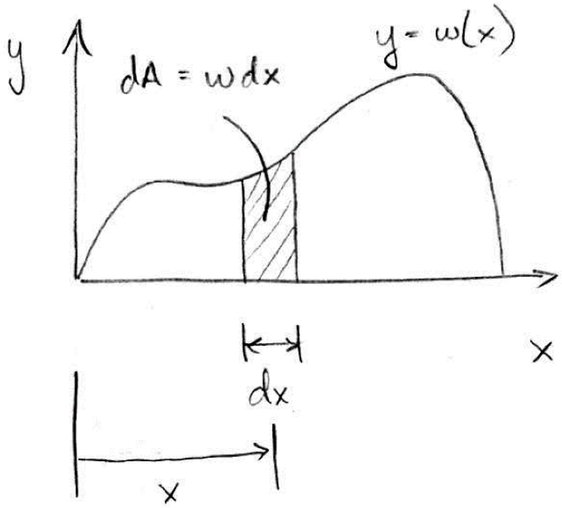
or

$$\bar{x} = \frac{\int_L xw\,dx}{\int_L w\,dx}$$

This resembles something we have just seen…

<u>Area Analogy</u>

Consider the <u>area</u> under the loading curve:



$$\begin{aligned} F &= \int_L w\,dx \\ &= \int_A dA \\ &= A \end{aligned}$$

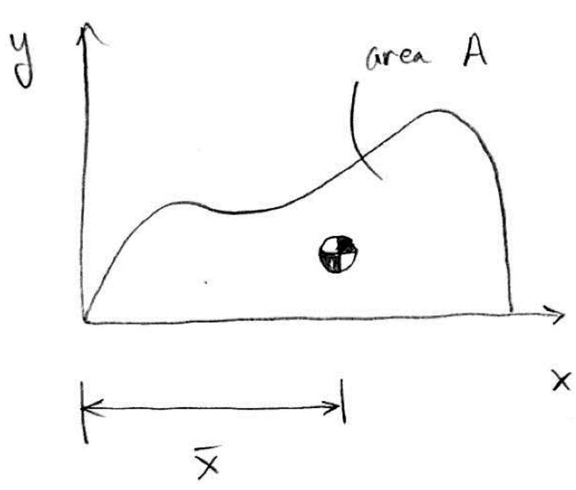
Force F is simply the <u>area</u> under the curve representing the distributed load.

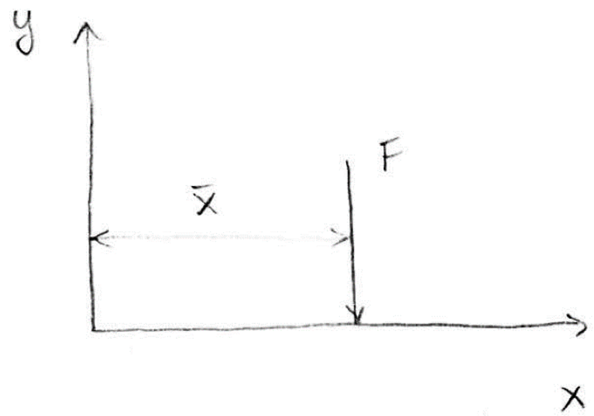
Substitute $wdx = dA$ into our formula for $\bar{x}$:

$$\bar{x} = \frac{\int_L xwdx}{\int_L wdx} = \frac{\int_A xdA}{\int_A dA}$$

This is the formula for the position of the centroid!

$$\boxed{\bar{x} = \frac{\int_A xdA}{\int_A dA}}$$

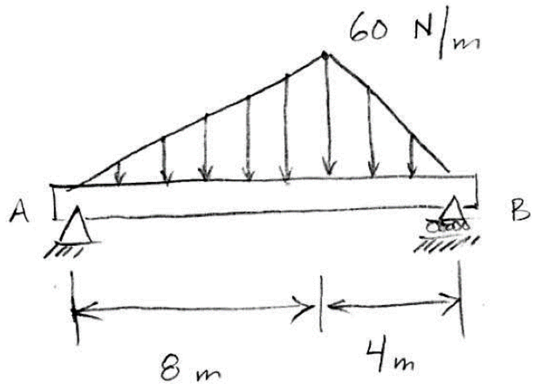




The force F is <u>equivalent</u> to the distributed load if it acts at the <u>centroid</u> of the "area" under the loading curve.

Recall that formulae for centroids of common areas are tabulated in Appendix B.

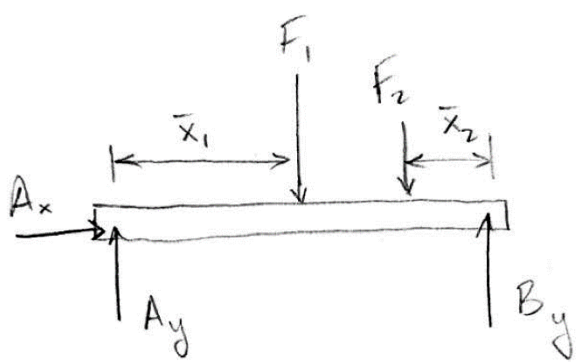
Example: Problem 7.46 in the textbook.



Determine the reactions on the beam at *A* and *B*.

Strategy: Compute two equivalent forces located at the centroids of the two triangular distributed loads, choosing appropriate points from which to determine the distances.

Free-body diagram:



$$F_1 = A_1$$
$$= \frac{1}{2} bh$$
$$= \frac{1}{2}(8 \text{ m})\left(60 \ \frac{\text{N}}{\text{m}}\right)$$
$$= 240 \text{ N}$$

$$F_2 = A_2$$
$$= \frac{1}{2}(4 \text{ m})\left(60 \ \frac{\text{N}}{\text{m}}\right)$$
$$= 120 \text{ N}$$

Position of centroids (take care with the distance and from which point it is measured!)

$$\bar{x}_1 = \frac{2}{3} b \qquad \text{(from Appendix B)}$$
$$= \frac{2}{3}(8 \text{ m})$$
$$= 5.333 \text{ m}$$

$$\bar{x}_2 = \frac{2}{3}(4 \text{ m})$$
$$= 2.667 \text{ m}$$

Equilibrium:

$$\sum F_x = 0 : A_x = 0$$
$$\sum F_x = 0 : A_y + B_y - F_1 - F_2 = 0 \quad \Rightarrow \quad A_y + B_y = 360 \text{ N}$$
$$\sum M_A = 0 : -(5.333)(240) - (12 - 2.666)(120) + (12) B_y = 0 \ (\text{N} \cdot \text{m}) \quad \Rightarrow \quad B_y = 200 \text{ N}$$

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\begin{aligned}
A_y &= 360 - B_y \ \text{(N)} \\
&= 360 - 200 \ \text{(N)} \\
&= 160 \text{ N}
\end{aligned}
$$

$$\boxed{A_x = 0, \quad A_y = 160 \text{ N}, \quad B_y = 200 \text{ N}}$$

Solved FBD (not necessary, but it gives confidence in the answer):