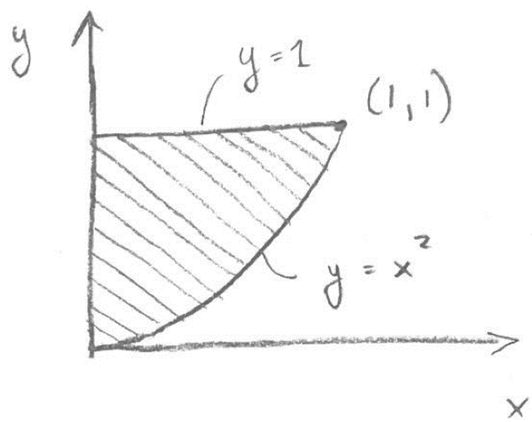
Lecture 21
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
Prof. James P. Hambleton
Ch. 7: §7.1 Centroids of Areas; §7.2 Composite areas[1]

Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Centroids of Areas (continued)

Example: Problem 7.2 in the textbook.



Determine the *x*-coordinate of the centroid.

Strategy: Use the formula based on integration.

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA}$$

What is *dA*?

$$dA = \left(1 - x^2\right)dx$$

$$\int_A x\,dA = \int_0^1 x\left(1 - x^2\right)dx$$

$$= \int_0^1 \left(x - x^3\right)dx$$

$$= \left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Bigg|_0^1$$

$$= \frac{1}{2} - \frac{1}{4}$$

$$= \frac{1}{4}$$

$$\int_A dA = \int_0^1 \left(1 - x^2\right)dx$$

$$= \left(x - \frac{1}{3}x^3\right)\Bigg|_0^1$$

$$= 1 - \frac{1}{3}$$

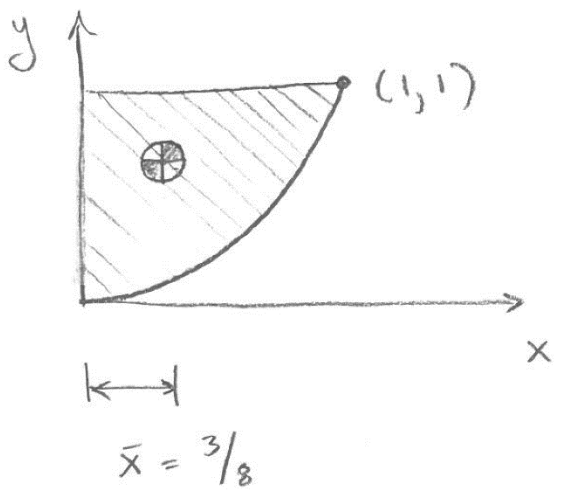
$$= \frac{2}{3}$$

$$\bar{x} = \frac{\int_A x dA}{\int_A dA}$$

$$= \frac{\left(\frac{1}{4}\right)}{\left(\frac{2}{3}\right)}$$

$$= \frac{3}{8}$$

$$\boxed{\bar{x} = \frac{3}{8}}$$

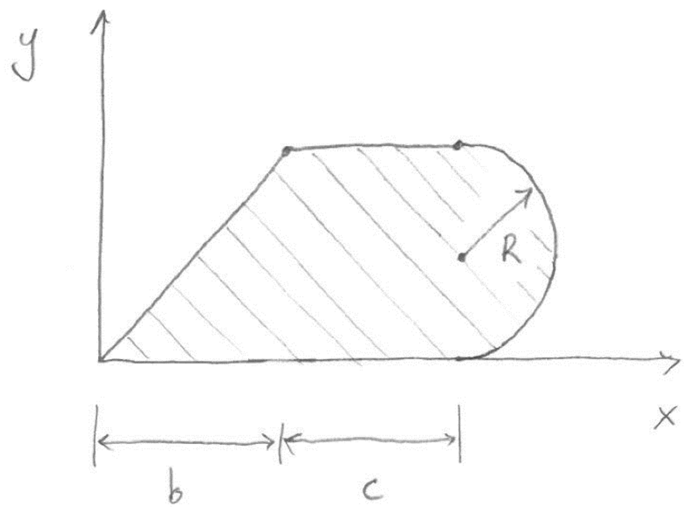


Go through Examples 7.1 and 7.2 in the textbook on your own.

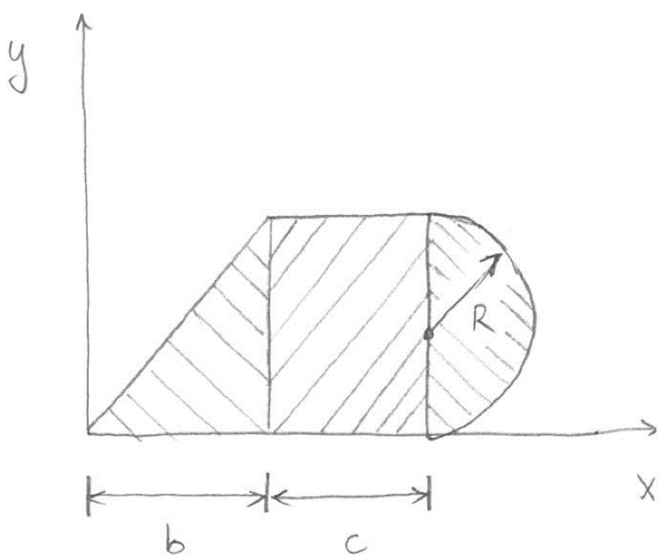
For symmetric areas, the centroid lies on the axis of symmetry. This can simplify calculations greatly. Consider examples.

Composite Areas

Suppose you are asked to determine the centroid of the following area:



Note that it can be subdivided into regions of simpler geometry:



Correspondingly, we can split the integral:

$$\begin{aligned}\bar{x} &= \frac{\int_A x\,dA}{\int_A dA} \\ &= \frac{\int_{A_1} x\,dA + \int_{A_2} x\,dA + \int_{A_3} x\,dA}{\int_{A_1} dA + \int_{A_2} dA + \int_{A_3} dA} \\ &= \frac{\int_{A_1} x\,dA + \int_{A_2} x\,dA + \int_{A_3} x\,dA}{A_1 + A_2 + A_3} \quad (\#)\end{aligned}$$

For the individual areas:

$$\bar{x}_1 = \frac{\int_{A_1} x\,dA}{\int_{A_1} dA} = \frac{1}{A_1}\int_{A_1} x\,dA \quad \Leftrightarrow \quad \int_{A_1} x\,dA = \bar{x}_1 A_1$$

$$\int_{A_2} x\,dA = \bar{x}_2 A_2$$

$$\int_{A_3} x\,dA = \bar{x}_3 A_3$$

Plug these in to Eq. (#):

$$\begin{aligned}\bar{x} &= \frac{\int_{A_1} x\,dA + \int_{A_2} x\,dA + \int_{A_3} x\,dA}{A_1 + A_2 + A_3} \\ &= \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \bar{x}_1 A_3}{A_1 + A_2 + A_3}\end{aligned}$$

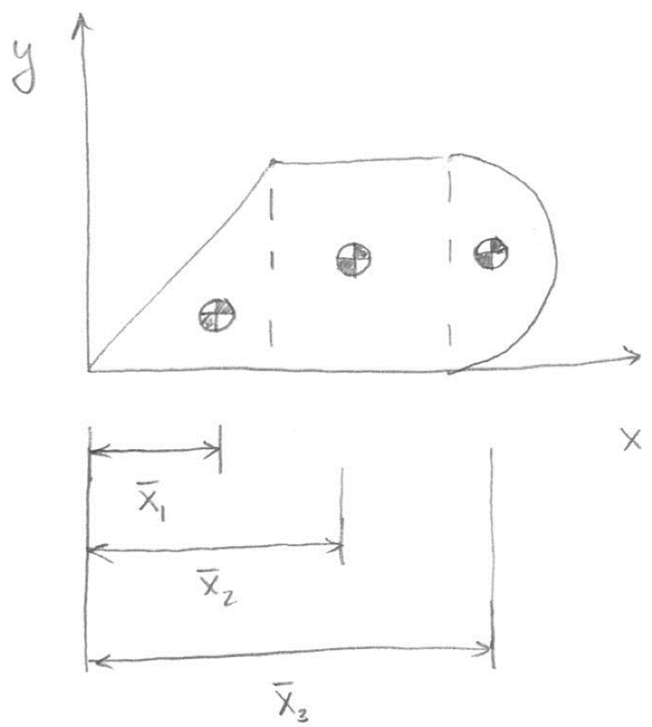
More generally, the *x*-coordinate of the centroid of a composite area is

$$\bar{x} = \frac{\sum_i \bar{x}_i A_i}{\sum_i A_i}$$

By the same argument, the y-coordinate is

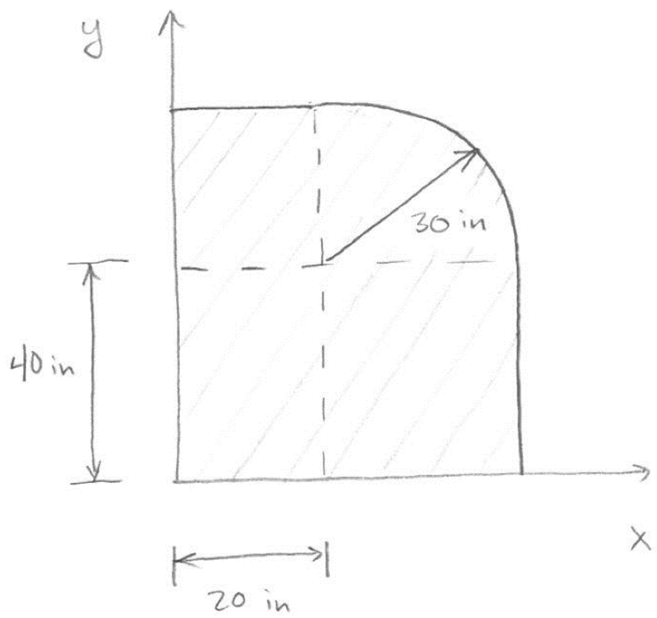
$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$$

We can easily work out the centroid of a composite area from the centroids of its parts:



Example 7.3 goes through this problem in detail. Work through it on your own.

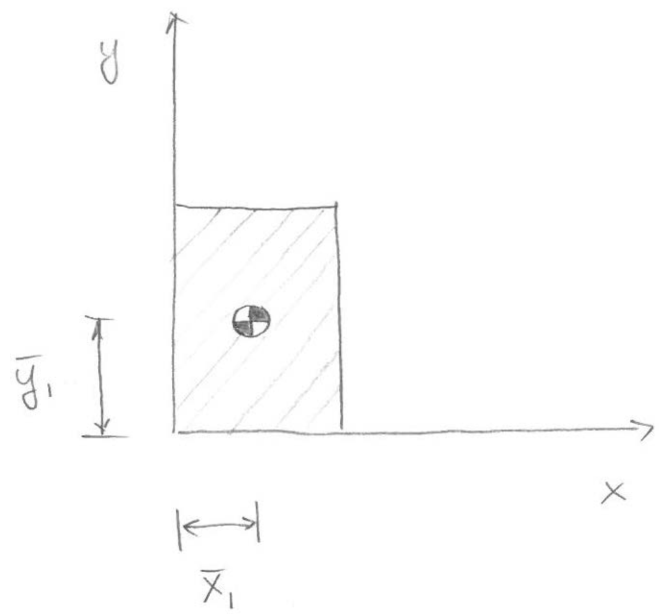
<u>Example</u>: Problem 7.32 from the textbook.



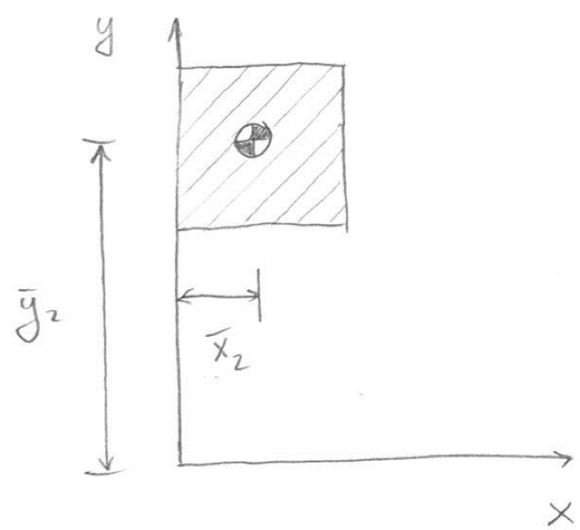
Determine the coordinates of the centroid.

<u>Strategy</u>: Subdivide into 4 regions (3 rectangles + 1 quarter circle) and use the formula for composite areas.

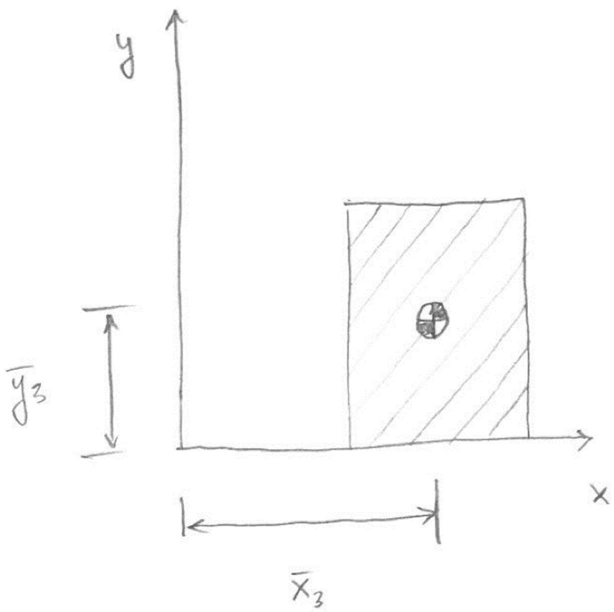
Centroids of simple areas are tabulated in Appendix B of the textbook.



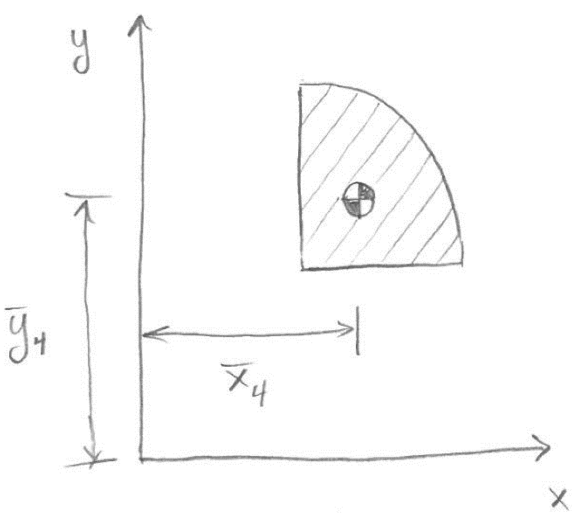
$\bar{x}_1 = 10\text{ in},\quad \bar{y}_1 = 20\text{ in},\quad A_1 = 800\text{ in}^2$



$\bar{x}_2 = 10\text{ in},\quad \bar{y}_2 = 40\text{ in} + \dfrac{30\text{ in}}{2} = 55\text{ in},\quad A_2 = (30\text{ in})(20\text{ in}) = 600\text{ in}^2$



$$\bar{x}_3 = 20 \text{ in} + \frac{30 \text{ in}}{2} = 35 \text{ in}, \quad \bar{y}_3 = 20 \text{ in}, \quad A_3 = (30 \text{ in})(40 \text{ in}) = 1200 \text{ in}^2$$



$$\bar{x}_4 = 20 \text{ in} + \frac{4R}{3\pi} = 20 \text{ in} + \frac{4(30 \text{ in})}{3\pi} = 32.732 \text{ in}$$

$$\bar{y}_4 = 40 \text{ in} + \frac{4R}{3\pi} = 40 \text{ in} + \frac{4(30 \text{ in})}{3\pi} = 52.732 \text{ in}$$

$$A_4 = \frac{\pi R^2}{4} = \frac{\pi\left(30\text{ in}\right)^2}{4} = 706.858\text{ in}$$

$$\bar{x} = \frac{\sum_i \bar{x}_i A_i}{\sum_i A_i}$$
$$= \frac{(10\text{ in})(800\text{ in}^2) + (10\text{ in})(600\text{ in}^2) + (35\text{ in})(1200\text{ in}^2) + (32.73\text{ in})(706.86\text{ in}^2)}{800\text{ in}^2 + 600\text{ in}^2 + 1200\text{ in}^2 + 706.86\text{ in}^2}$$
$$= 23.9\text{ in}$$

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$$
$$= \frac{(20\text{ in})(800\text{ in}^2) + (55\text{ in})(600\text{ in}^2) + (20\text{ in})(1200\text{ in}^2) + (52.73\text{ in})(706.86\text{ in}^2)}{800\text{ in}^2 + 600\text{ in}^2 + 1200\text{ in}^2 + 706.86\text{ in}^2}$$
$$= 33.3\text{ in}$$

$$\boxed{\bar{x} = 23.9\text{ in},\quad \bar{y} = 33.3\text{ in}}$$