Lecture 20
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
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§6.5 Frames and Machines; Ch. 7: Centroids & Center of Mass; §7.1 Centroids of Areas[1]

Acknowledgements

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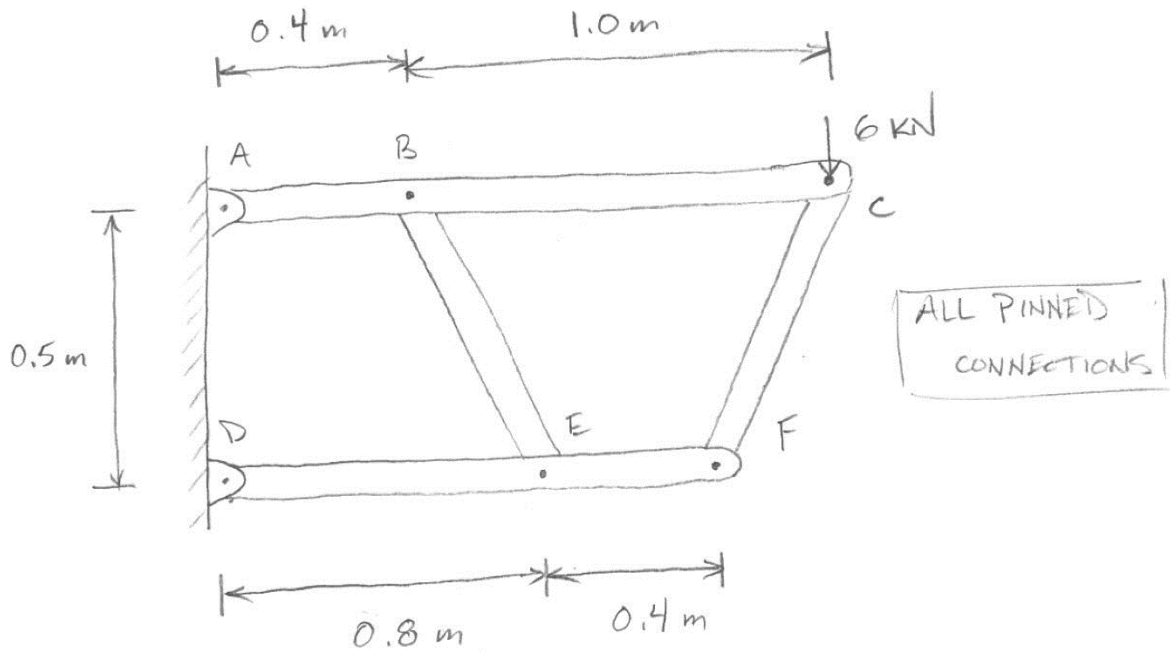
Frames and Machines (continued)

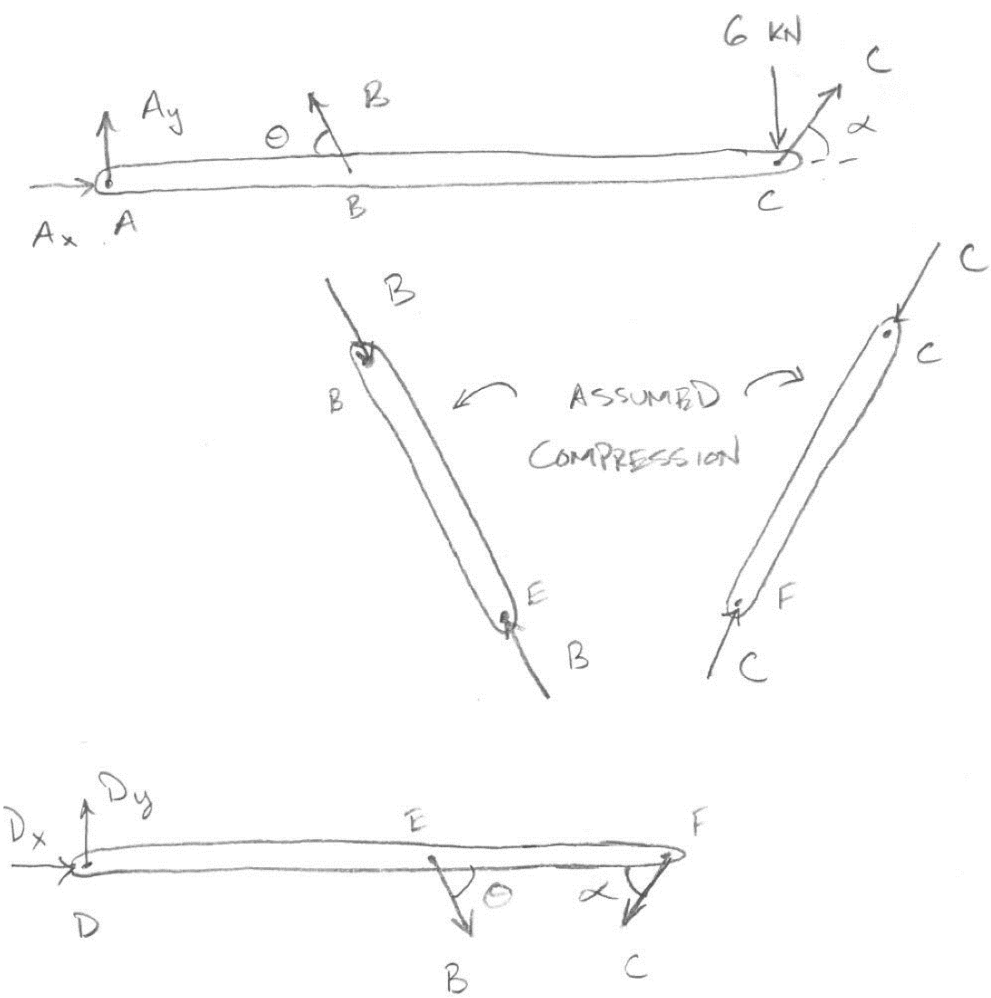
Example: Problem 6.79 in the textbook.



Determine the reactions at the supports and joints.

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

<u>Strategy</u>: There are 4 members and 6 joints for a total of 12 equations and 12 unknowns, but we will see that the solution is not so bad, since *BE* and *CF* are two-force members. <u>Forces on joints can be placed on either member</u>[2] (but not both simultaneously).



The force of 6 kN can go on member *ABC* or member *CF*, but **<u>not</u>** on both.

$$\tan\theta = \frac{0.5 \text{ m}}{0.4 \text{ m}} \;\Rightarrow\; \theta = 50.34^\circ$$

$$\tan\alpha = \frac{0.5 \text{ m}}{0.2 \text{ m}} \;\Rightarrow\; \alpha = 68.20^\circ$$

[2] A detailed discussion is in the textbook.

Member *ABC*:

$$\sum F_x = 0: \; A_x - B\cos\theta + C\cos\alpha = 0$$

$$\sum F_y = 0: A_y + B\sin\theta + C\sin\alpha - 6 = 0 \;\; (\text{kN})$$

$$\sum M_A = 0: B\sin\theta(0.4) + C\sin\alpha(1.4) - (1.4)(6) = 0 \;\; (\text{kN}\cdot\text{m})$$

Member *DEF*:

$$\sum F_x = 0: \; D_x + B\cos\theta - C\cos\alpha = 0$$

$$\sum F_y = 0: D_y - B\sin\theta - C\sin\alpha = 0$$

$$\sum M_D = 0: -B\sin\theta(0.8) - C\sin\alpha(1.2) = 0 \;\; (\text{kN}\cdot\text{m})$$

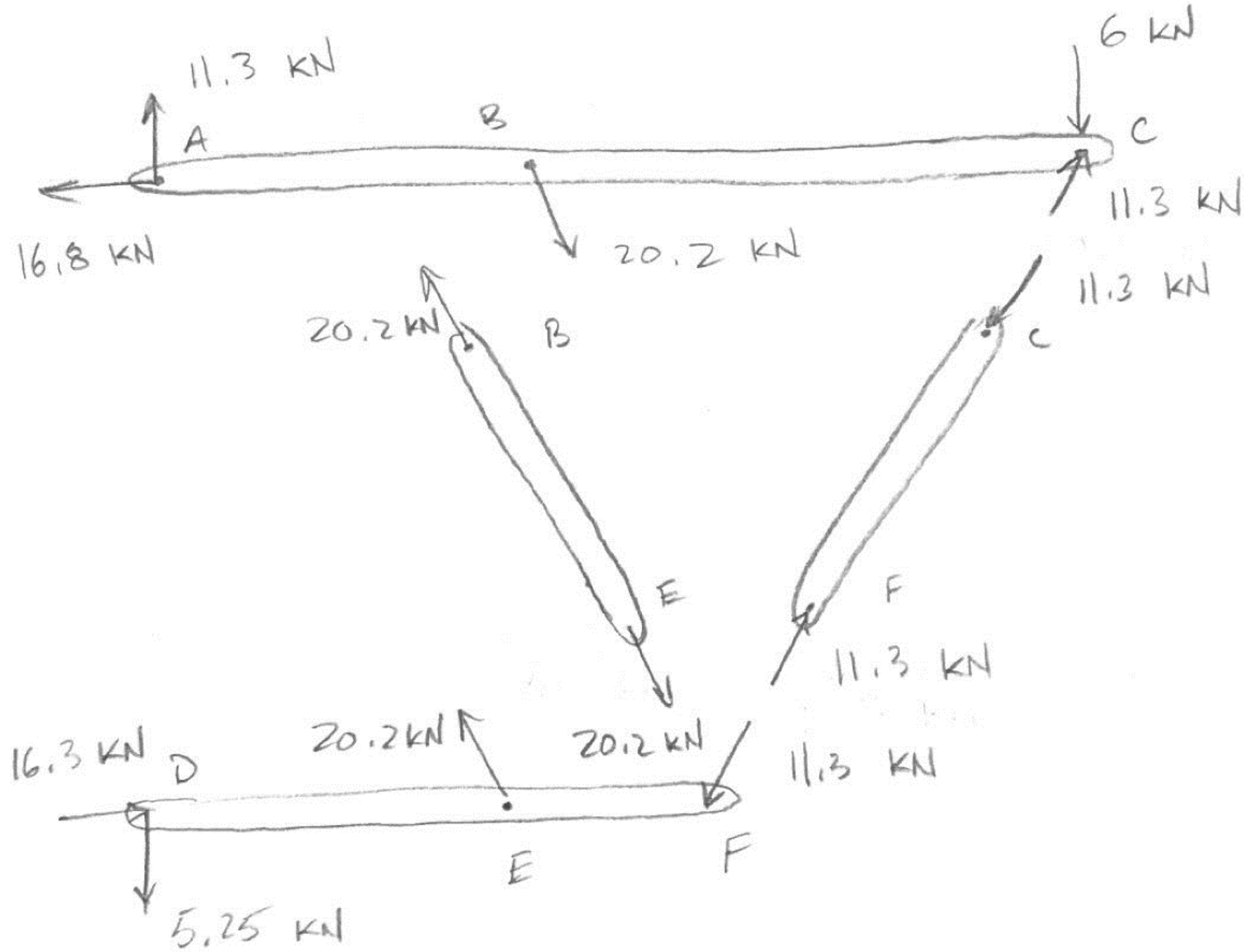
Note that by considering *BE* and *CF* as two-force members, we have already utilized equilibrium for these members.

From Eqs. (3) and (6), we can solve to find $\boxed{B = -20.2\,\text{N}}$ (tension since assumed compression) and $\boxed{C = 11.3\,\text{kN}}$.

Then we can easily solve the remaining equations to find the other forces.

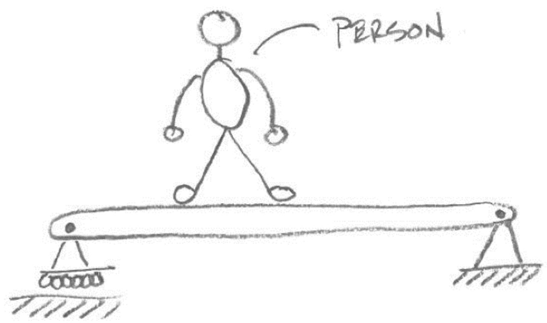
$$\boxed{\begin{aligned} A_x &= -16.8 \text{ kN} \\ A_y &= 11.3 \text{ kN} \\ D_x &= 16.3 \text{ kN} \\ D_y &= -5.25 \text{ kN} \end{aligned}}$$

Solution Summary:

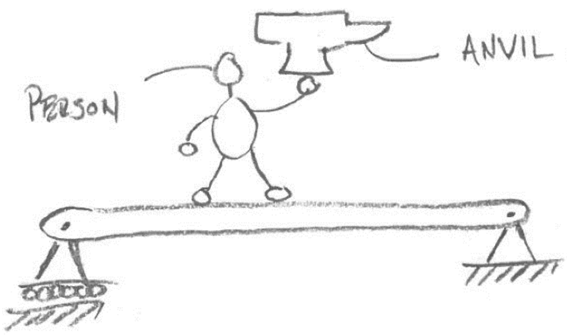


Go through Examples 6.6., 6.7, and 6.8 on your own.

Centroids and Center of Mass (Chapter 7)



How would you determine the reactions at the supports?

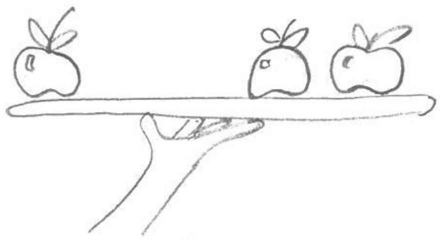


How about now?

The weight of an object, or objects, is distributed over the volume of the object, or objects. However, the weight can be represented by a single equivalent force acting at a point called the center of mass[3].

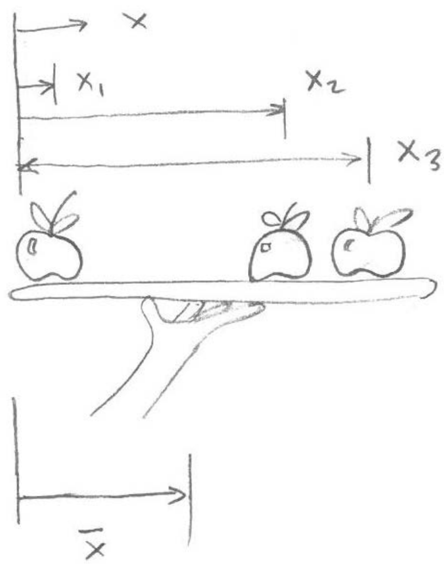
[3] In the book, the center of mass is not introduced until after centroids are introduced.

Another simple (1D) example[4]:



Where should I locate my hand so the tray does not tip?

We can solve the problem using the techniques we have already developed:



Suppose each piece of fruit has the same mass m. From equilibrium in the vertical direction, the force of the hand on the tray is $3mg$ (pointing upward).

[4] The textbook introduces centroids and centers of mass in a rather different way.

Moment equilibrium:

$$\sum M_0 = 0:\ 3\bar{x}mg - x_1mg - x_2mg - x_3mg = 0 \ \Rightarrow \ \boxed{\bar{x} = \frac{x_1 + x_2 + x_3}{3}}$$

What if each piece of fruit has a different mass?

$$\sum M_0 = 0:\ \bar{x}\left(m_1 + m_2 + m_3\right)g - x_1m_1g - x_2m_2g - x_3m_3g = 0 \ \Rightarrow \ \boxed{\bar{x} = \frac{x_1m_1 + x_2m_2 + x_3m_3}{m_1 + m_2 + m_3}}$$

We can generalize this formula to *N* pieces of fruit:

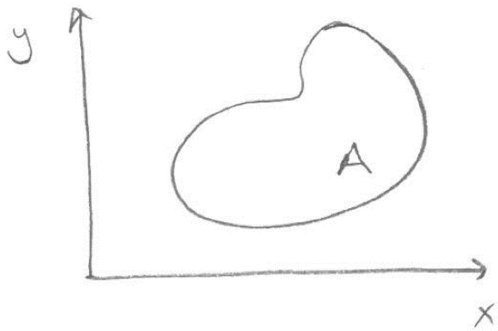
$$\boxed{\bar{x} = \frac{\sum_{i=1}^{N} x_i m_i}{\sum_{i=1}^{N} m_i}}$$

Remarks:

- $\bar{x}$ gives the location of the <u>center of mass</u> of the fruit, which does not necessarily coincide with any pieces of fruit.
- In this example, x was the only relevant coordinate, making it a one-dimensional (1D) problem.

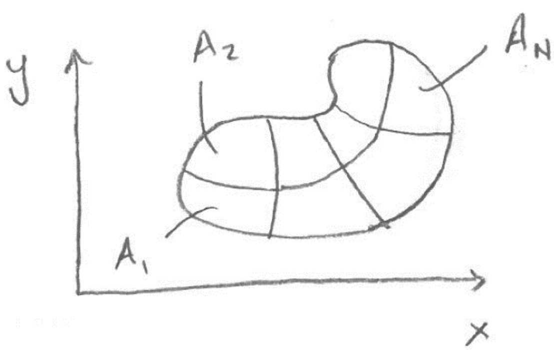
We will now consider centroids and centers of mass of areas (2D). We will <u>not</u> cover volumes (3D).

Centroids of Areas



How would you compute the center of mass?

Consider splitting A into N smaller areas A_1, A_2, …, A_N of mass m_1, m_2, …, m_N, respectively.



By extension from 1D, the center of mass is located at the following coordinates:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i m_i}{\sum_{i=1}^{N} m_i} \quad \text{and} \quad \bar{y} = \frac{\sum_{i=1}^{N} y_i m_i}{\sum_{i=1}^{N} m_i}$$

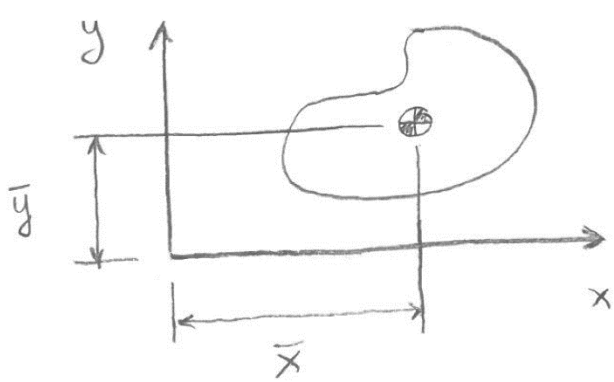
Suppose now that mass is uniformly distributed across this area:

$\boxed{m_i = \rho A_i}$ where ρ is the density with $[\rho]$ = mass/length^2 (e.g., kg/m^2)

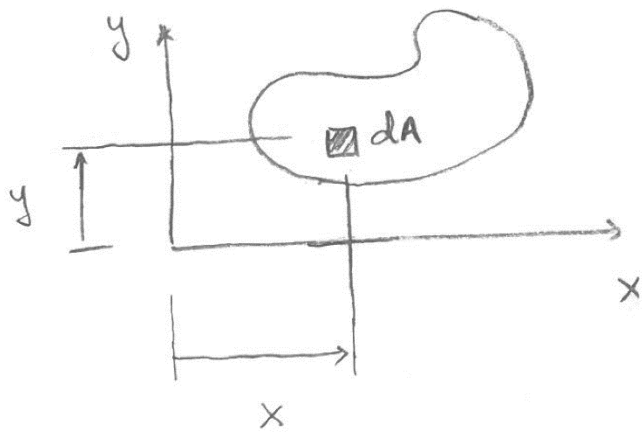
$$\bar{x} = \frac{\sum_{i=1}^{N} x_i m_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} x_i(\rho A_i)}{\sum_{i=1}^{N} (\rho A_i)} = \frac{\rho \sum_{i=1}^{N} x_i A_i}{\rho \sum_{i=1}^{N} A_i} = \frac{\sum_{i=1}^{N} x_i A_i}{\sum_{i=1}^{N} A_i} \text{ and similarly } \bar{y} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i}$$

These give the position of the centroid of the area:

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{N} x_i A_i}{\sum_{i=1}^{N} A_i}} \text{ and } \boxed{\bar{y} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i}}$$



The way that we subdivided was arbitrary. Taking this to the limit to compute the position of the centroid exactly, we find



$$\bar{x} = \frac{\int_A x dA}{\int_A dA}$$ and $$\bar{y} = \frac{\int_A y dA}{\int_A dA}$$

Remarks:

- The centroid gives the average position of an object without respect to mass. In other words, the centroid is a geometric property.
- The centroid and the center of mass coincide only when the mass is uniformly distributed.
- The book does not define the center of mass until Section 7.7, which we will not cover.