Lecture 19
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
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§6.5 Frames and Machines[1]

Acknowledgements

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Frames and Machines

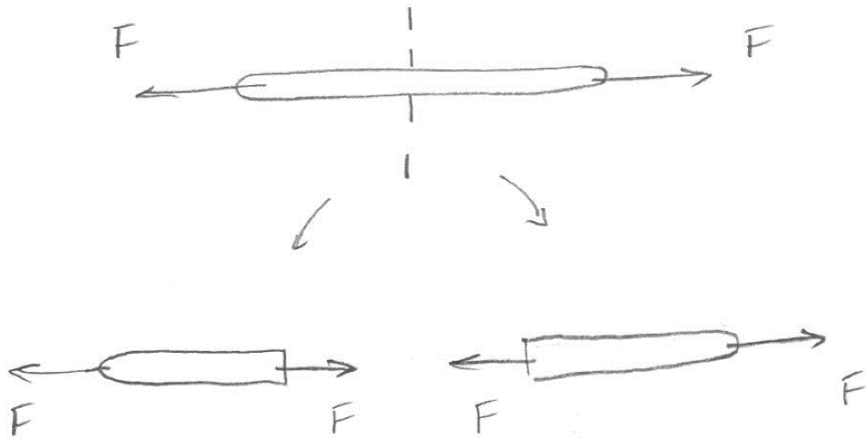
Many structures are made up of members that are not two-force members.

- Frames are structures that support static loads (and contain members other than two-force members).
- Machines are structures that can move.

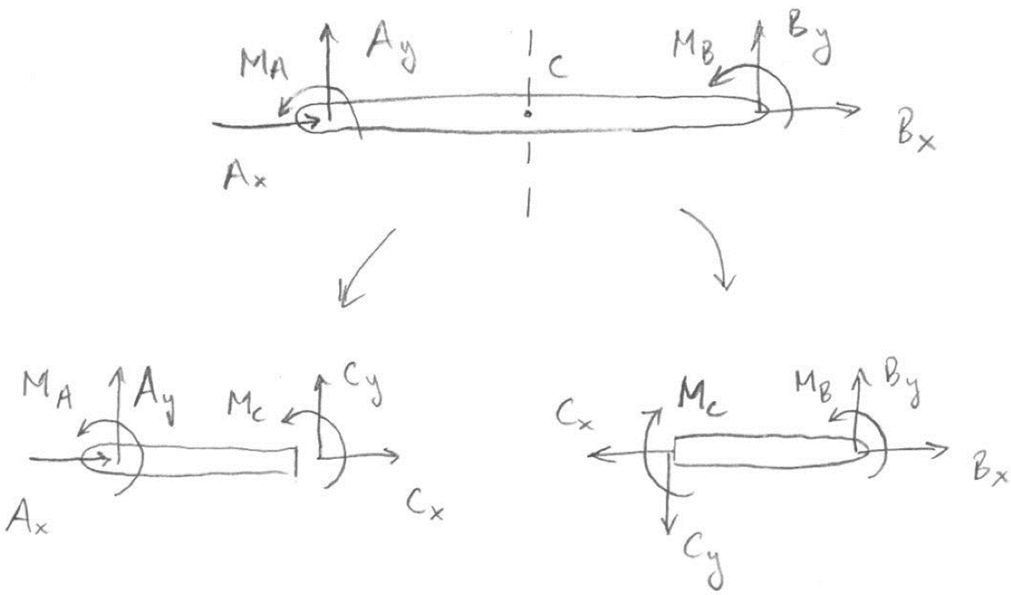
In this course, the members of frames and machines are typically connected by pins so as to make them statically determinate and solvable. Generally, members are connected in other ways (welds, bolts, threading, etc.).

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

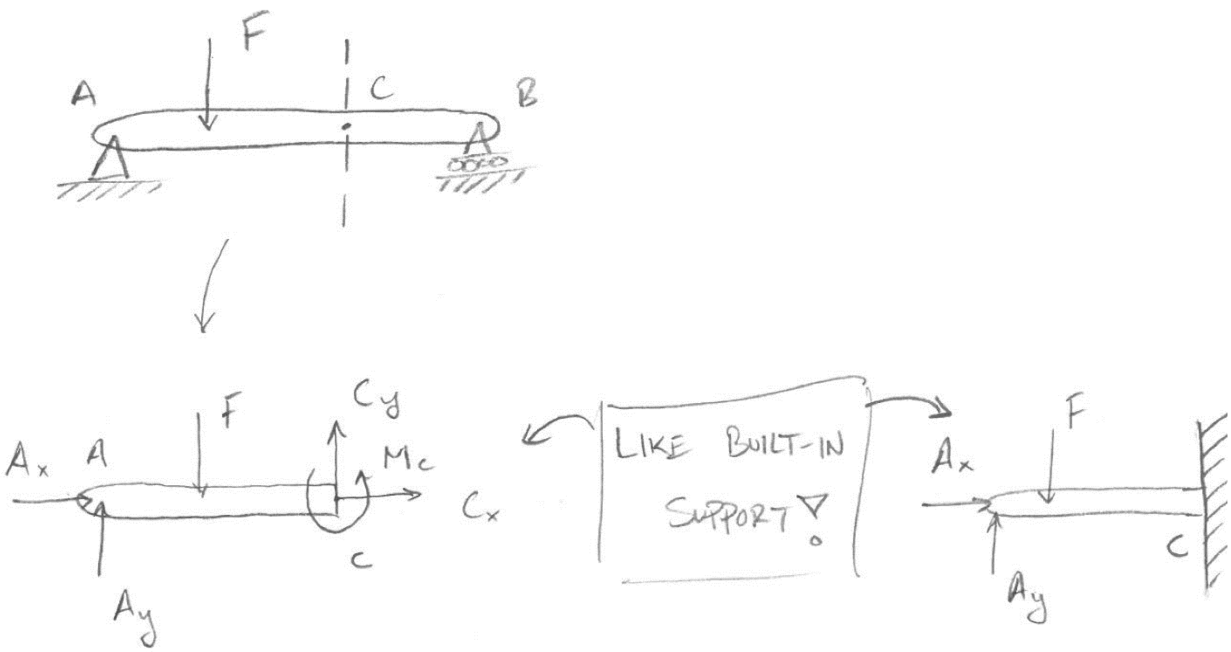
Recall, for a two-force member, we have the following:



Suppose we do not have a two-force member:



Simple example of a member that is not a two force member:



The internal forces change depending on the location of the cut! In this case the moment (M_C) changes continuously. You will do more with this in CIV_ENV 216 (*Mechanics of Materials*).

This affirms that we cannot meaningfully "cut" members that are not two-force members.

So, how do we analyze frames and machines?

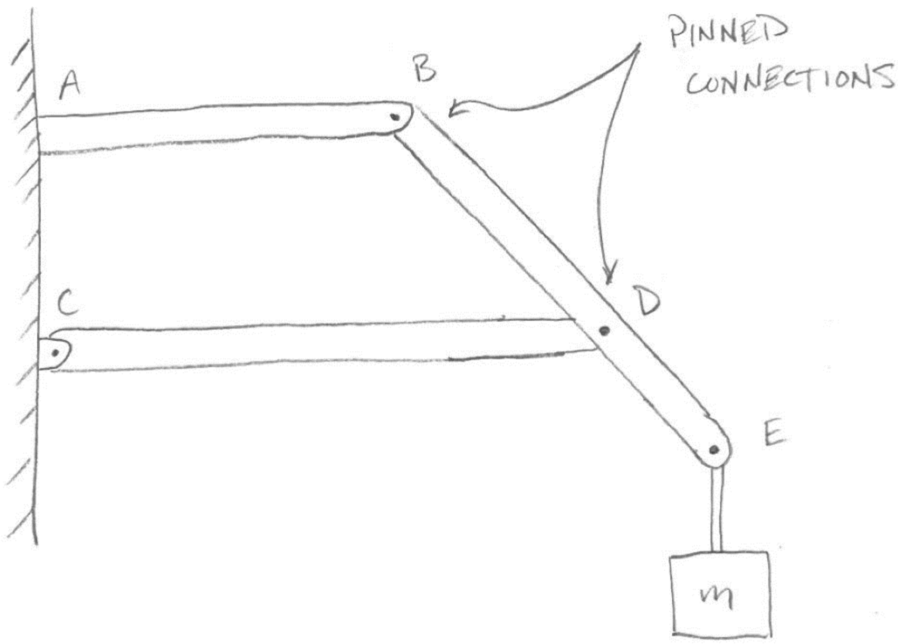
General procedure[2]:

1. Construct the free-body diagram for the whole structure to determine, if possible, unknown reactions.
2. Conceptually take apart the structure at the joints.
3. Draw the free-body diagram for each member, including unknown reactions and external forces.
4. Place equal and opposite forces (reactions) on the member at the location of each joint. These are the unknown internal forces.
5. Place external forces (or reactions determined in Step (1)) that are applied at joints on one and only one member.
6. Look for two-force and three-force members to simplify the calculations.
7. Solve for the unknown forces using $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_P = 0$ for each member.

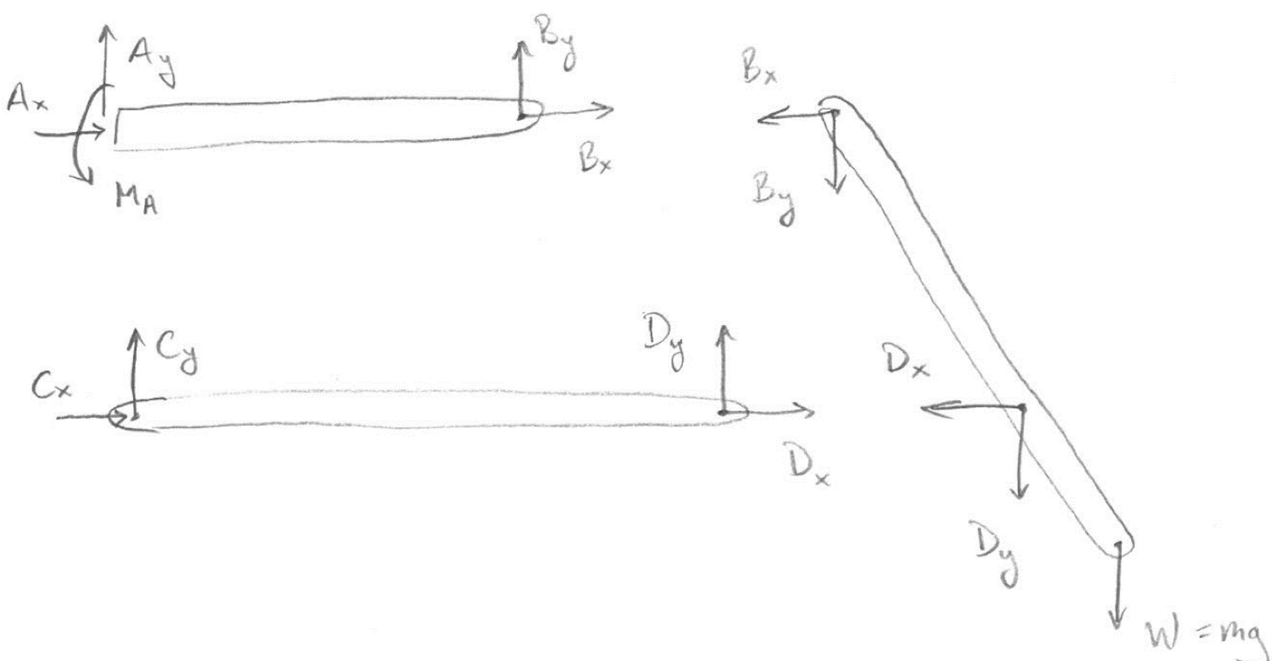
This is in fact the generalization of the method of joints for trusses, except that now forces need not be applied at the joints and we generally consider moment as well as force equilibrium. As we will see, we can also solve for reactions at supports that could not be determined by considering equilibrium of the whole structure alone.

[2] As a first (non-essential) step, we can also consider equilibrium of the entire structure, after drawing the free-body diagrams with appropriate reactions in place of the supports. Even if the structure is statically indeterminate, some reactions often can be determined, thus simplifying the analysis.

<u>Frame "explosion" or "deconstruction"</u> (Illustration of Steps 1-4)



Here we 4 unknown reactions, so we cannot compute anything from global equilibrium.



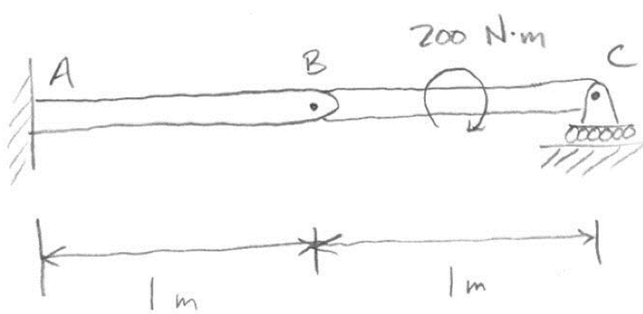
Observations:

- As before, the direction for the unknown reactions can be selected arbitrarily, but they must be equal and opposite, and once decided this cannot be changed. Ultimately the sign will indicate the direction.
- In this example, we have 9 unknowns and, since there are three members, 9 equations. Four of the unknowns are internal forces that cancel out when the structure is assembled.

In this case, member CD is a two-force member, and we see immediately that $C_x = D_x$ and $C_y = D_y = 0$.

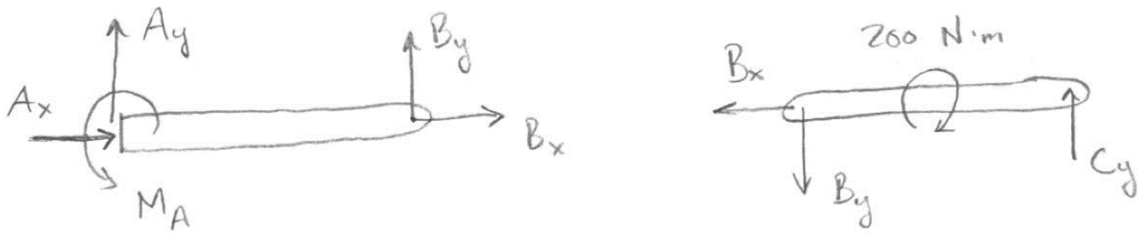
This particular problem would then be solved by first considering equilibrium of member *BDE* to determine B_x, B_y, and C_x, and then considering equilibrium of member *AB* to solve for the reactions from the support at *A*.

Example (simple):



Determine the reactions at *A* and *C* and the internal forces at *B*.

Strategy: First, separate members and draw free-body diagrams with reactions (drawn as positive on member *AB*). Then, reconsider strategy.



Strategy (continued): Looking for two-force members, we see that there are none (member *AC* also carries a moment). First, we will consider equilibrium of member *BC* to solve for these three unknowns, and then we will consider equilibrium of member *AB*.

Equilibrium of member *BC*:

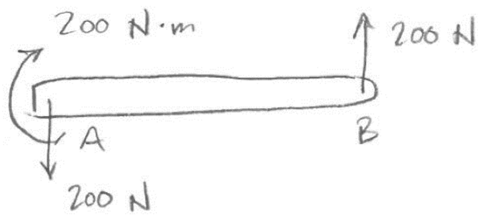
$$\sum F_x = 0: -B_x = 0 \Rightarrow \boxed{B_x = 0}$$
$$\sum F_y = 0: C_y - B_y = 0 \quad (\#)$$
$$\sum M_B = 0: -200(\text{N}\cdot\text{m}) + C_y(1\text{m}) = 0 \Rightarrow \boxed{C_y = 200\,\text{N}} \text{ then from (\#) } \boxed{B_y = 200\,\text{N}}$$

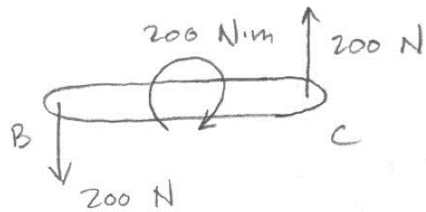
Observe that C_y and B_y are a couple that opposes the applied couple.

Equilibrium of member *AB*:

$$\sum F_x = 0: A_x + B_x = 0 \Rightarrow \boxed{A_x = 0}$$
$$\sum F_y = 0: A_y + B_y = 0 \Rightarrow \boxed{A_y = -200\,\text{N}}$$
$$\sum M_A = 0: M_A + B_y(1\text{m}) = M_A + (200\,\text{N})(1\text{m}) = 0 \Rightarrow \boxed{M_A = -200\,\text{N}\cdot\text{m}}$$

Solution summary:





Same loading on both members[3]!

[3] Recall that couples be moved anywhere without changing the system.