Lecture 16

GEN_ENG 205-2: Engineering Analysis 2

Winter Quarter 2018

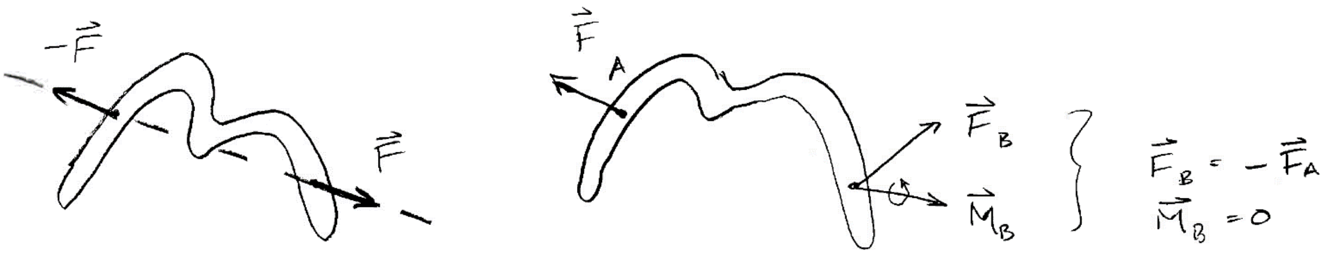
Prof. James P. Hambleton

Chapters 5 and 6: §5.4 Two-Force and Three-Force Members; §6.1 Trusses[1]

Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Two-Force Members[2]



If an object is in equilibrium and subjected only to two forces[3], or multiple forces that can be reduced to two equivalent forces, then the object is referred to as a two-force member, and the two forces

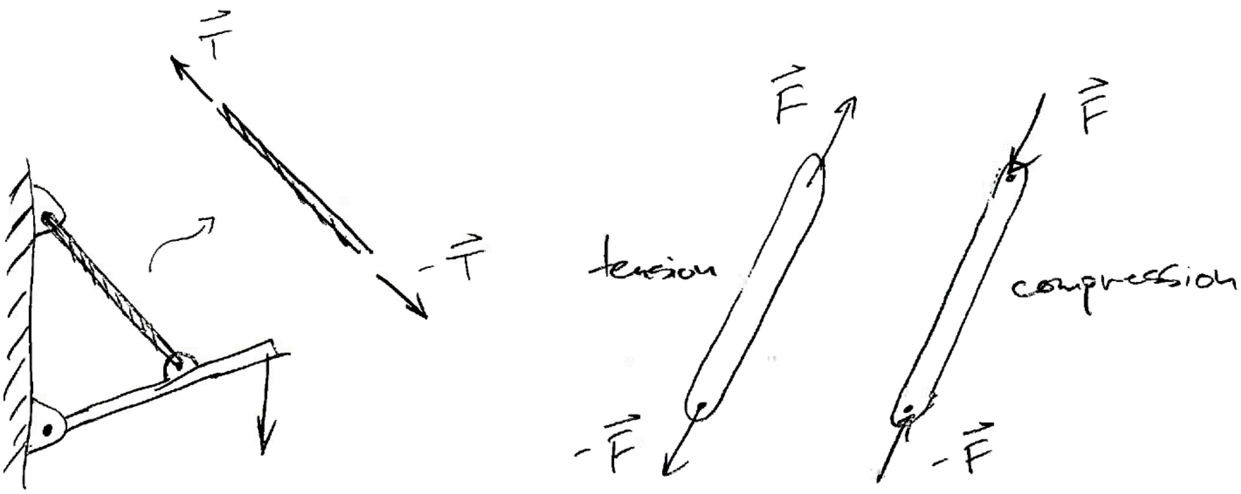
1. are equal in magnitude and opposite in direction (by force equilibrium) and
2. have the same line of action (by moment equilibrium).

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

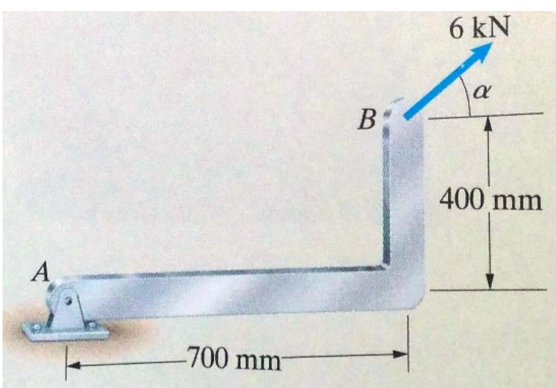
[2] An alternative (and perhaps more compelling) way to introduce two-force members would be to solve a 2D bar with pinned connections. One sees that the forces are undetermined up to a single constant corresponding to the axial force.

[3] And no other forces or couples.

Ropes and cables are now familiar examples. These can only support tension. A bar supported by pinned connections is an example that can support compression. Generally, the geometry can be complex.



Example 5.11. Observe that we can already solve this problem, but go through it on your own to see how recognizing an object as a two-force member can simplify the solution[4].



The L-shaped bar has a pin support a *A* and is loaded by a 6-kN force at *B*. Neglect the weight of the bar.

Determine the angle α and the reactions at *A*.

[4] Note that solving for angles is generally difficult.

In this case, angle α is determined directly by the geometry. The only complexity is in recognizing that there are two possible directions differing by 180 degrees[5] (i.e., “tension” versus “compression”).

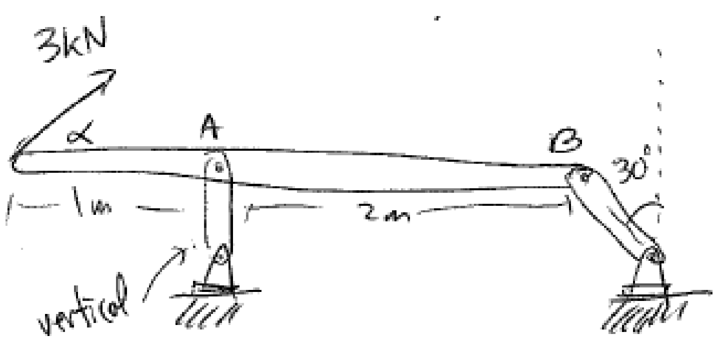
Three-Force Members

If an object is in equilibrium and subjected to three forces[6], or multiple forces that can be reduced to three equivalent forces, then the object is referred to as a three-force member, and the forces are

1. coplanar (by force equilibrium) and
2. either parallel[7] or concurrent[8] (by moment equilibrium).

These assumptions let us analyze complicated structures made of networks of beams.

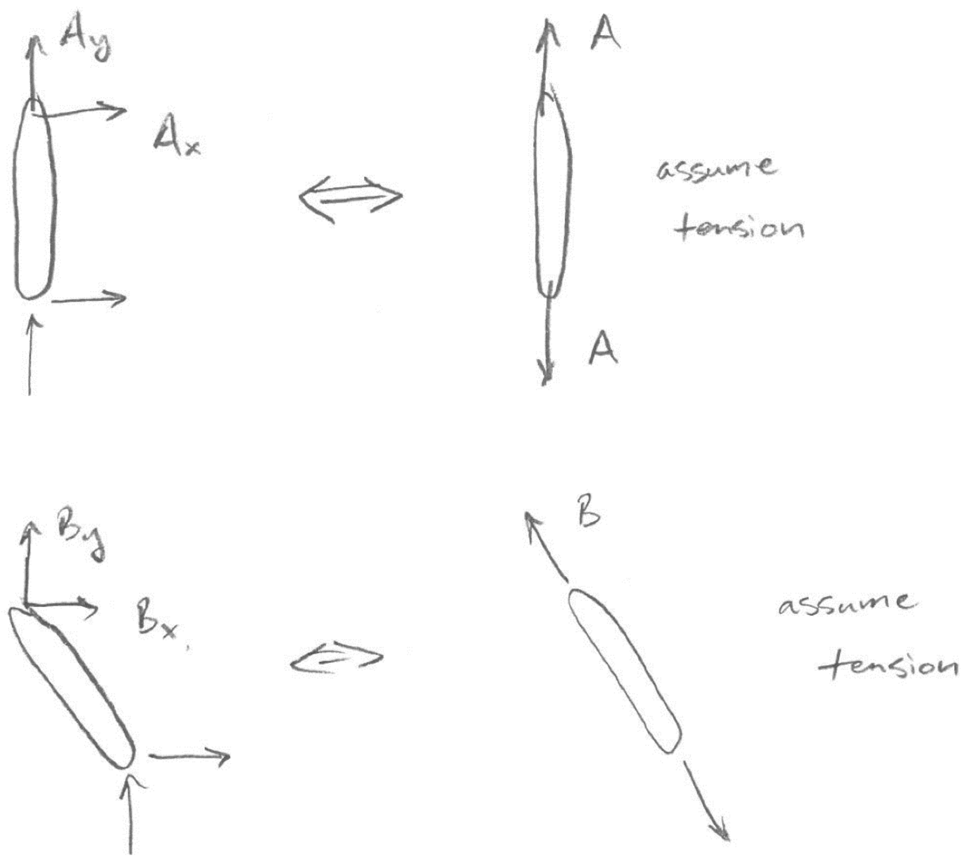
Example[9]:



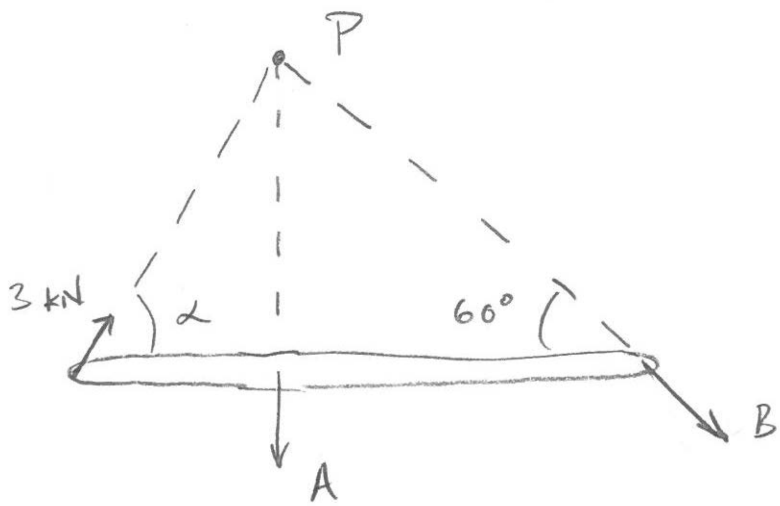
Determine the angle α and the reactions at Points A and B.

[5] Specifically, the two solutions are $\alpha = 29.7°$ and $\alpha = 29.7° + 180° = 209.7°$.
[6] And no other forces or couples.
[7] A “simply supported” beam subjected to a force at midspan is a good example. When any one particular force, the lines of action of the other two are parallel, and the magnitude and direction of the forces are such that moment equilibrium is maintained.
[8] This is entirely analogous to the two-force member. The lines of action must intersect, otherwise there is a net moment.
[9] This example is a good segue to structures.

Strategy: Use the fact that the members attached to Points *A* and *B* are two-force members to assess the lines of action of the reactions. Draw the free-body diagrams and <u>use Newton's third law to determine forces on each object</u>[10]. Recognizing that the horizontal member is a three-force member with <u>concurrent forces</u>, use geometry to solve for angle α.



[10] This example is among the first to apply Newton's third law, and this point should be emphasized. We will use this extensively for frames and machines.



Use triangle on the right-hand side to infer h, and then use h to compute angle α.

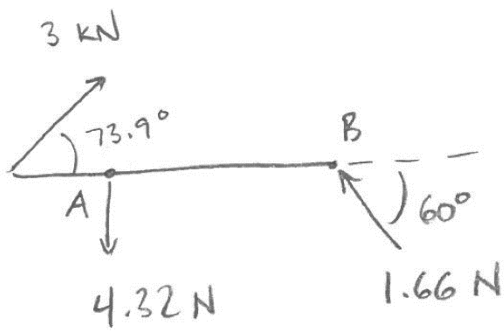
$$\tan 60^\circ = \frac{h}{2\text{ m}} \;\Rightarrow\; h = 3.464\text{ m}$$

$$\tan\alpha = \frac{3.464\text{ m}}{1\text{ m}} \;\Rightarrow\; \alpha = 73.9^\circ$$

Having determined angle α, computing the reactions is now simple:

$$\sum F_x = 0:\; (3\text{kN})\cos\alpha + B\cos 60^\circ = 0 \;\Rightarrow\; B = -166\text{kN}$$
$$\sum F_y = 0:\; (3\text{kN})\sin\alpha - A - B\sin 60^\circ = 0 \;\Rightarrow\; A = +4.32\text{kN}$$

Solved FBD:



Notice that we do not need to consider moment equilibrium, which we have in fact already utilized by recognizing the beam as a three-force member with concurrent forces.

Go through Example 5.10 on your own.

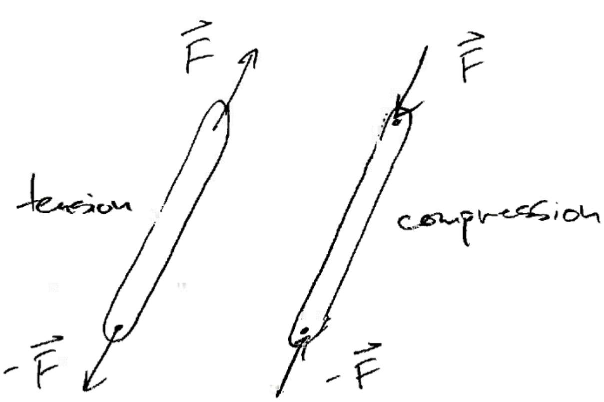
Structures in Equilibrium (Chapter 6)

Trusses

A truss is structure composed purely of two-force members, consisting of members that are assumed to be weightless, pinned at the end, and supported and loaded only at the joints.

When the members are slender, this is often a good approximation for welded and bolted connections.

As two-force members, each member of a truss is subjected to an equal and opposite axial force. The member is in tension when the force tends to cause extension, or lengthening (i.e., the end forces are directed away from each other) and in compression when the force tends to cause shortening.



Observe that the *internal force* is uniform throughout the member and equal to force at the joint.