Lecture 15

GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
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Chapter 5: §5.3 Three-dimensional applications[1]

Acknowledgements

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Objects in Equilibrium (3D)

The conditions for equilibrium are, as before

1. $\sum \vec{F} = \vec{0}$
2. $\sum \vec{M}_P = \vec{0}$

where P is any point, either inside or outside the object.

In 3D, this generates 6 scalar equilibrium equations[2]:

1. $\sum F_x = 0$
2. $\sum F_y = 0$
3. $\sum F_z = 0$

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.
[2] The notation here varies slightly from the textbook. The book writes simply M_x, for example, whereas here this is written as $M_{x,P}$ to highlight that this is for any point P.

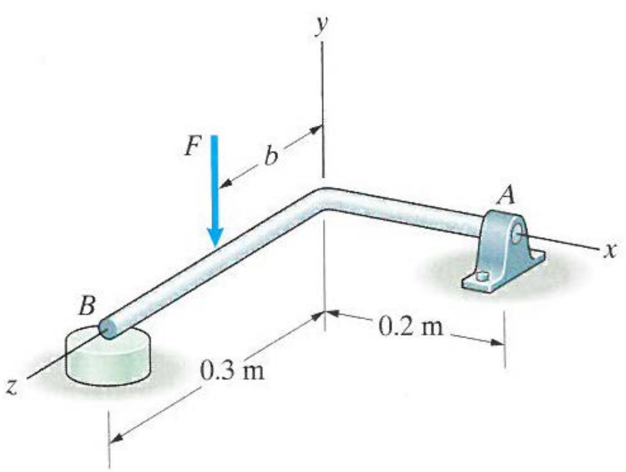
4. $\sum M_{x,P} = 0$
5. $\sum M_{y,P} = 0$
6. $\sum M_{z,P} = 0$

Note that we can have a statically determinate system with as many as 6 unknowns and 6 equations!

Types of supports

Look at Table 5.2 in the textbook for various types of supports.

Example: Problem 5.95[3]

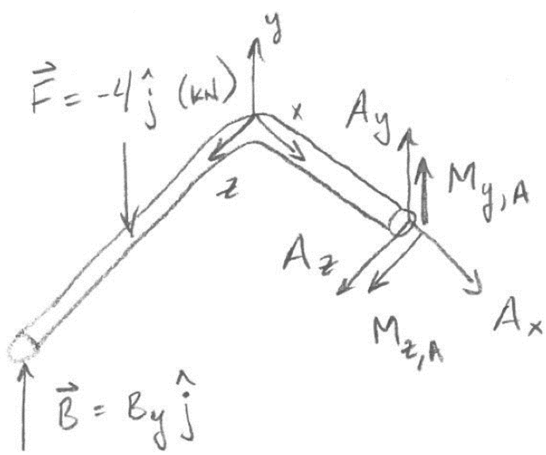


The L-shaped bar is supported by a bearing at A and rests on a smooth horizontal surface at B. The vertical force $F = 4$ kN, and the distance $b = 0.15$ m. Determine the reactions at A and B.

[3] This is a good example to give some exposure to determining reactions in the form of couples.

Strategy: Draw free-body diagram and consider equilibrium, using the vector form of the equilibrium equations for moment equilibrium and considering point A as the point about which to compute moments.

Free-body diagram:



For force equilibrium, it makes sense for such a simple problem to work with components rather than the vector form $\sum \vec{F} = \vec{0}$.

$$\sum F_x = 0: A_x = 0$$
$$\sum F_y = 0: B_y - 4 + A_y = 0 \text{ (kN)}$$
$$\sum F_z = 0: A_z = 0$$

So $\boxed{A_x = 0}$, $\boxed{A_z = 0}$, and $A_y = 4 - B_y$ (kN).

For moment equilibrium, we will work with the vector form rather than work out perpendicular distances. The contributions to the moment about point *A* are from

1. the couple at point *A*, given by $\vec{M}_A = M_{y,A}\hat{j} + M_{z,A}\hat{k}$
2. the reaction at *B*, given by $\vec{B} = B_y\hat{j}$
3. the applied force at *C*, given by $\vec{F}_C = -4\hat{j}$ (kN)

$$\sum \vec{M}_{(A)} = \vec{0}: \quad \vec{M}_A + \vec{r}_{AB}\times\vec{B} + \vec{r}_{AC}\times\vec{F}_C = \vec{0}$$

$$\vec{r}_{AB}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.2 & 0 & 0.3 \\ 0 & B_y & 0 \end{vmatrix} \begin{matrix} \\ \text{(m)} \\ \text{(kN)} \end{matrix} = -0.3B_y\hat{i} - 0.2B_y\hat{k} \text{ (kN}\cdot\text{m)}$$

$$\vec{r}_{AC}\times\left(-4\hat{j}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.2 & 0 & 0.15 \\ 0 & -4 & 0 \end{vmatrix} \begin{matrix} \\ \text{(m)} \\ \text{(kN)} \end{matrix} = 0.6\hat{i} + 0.8\hat{k} \text{ (kN}\cdot\text{m)}$$

$$\sum \vec{M}_{(A)} = \vec{0}: \quad \left(M_{y,A}\hat{j} + M_{z,A}\hat{k}\right) + \left(0.3B_y\hat{i} - 0.2B_y\hat{k}\right) + \left(0.6\hat{i} + 0.8\hat{k}\right) = \vec{0} \quad \text{(units of kN}\cdot\text{m)}$$

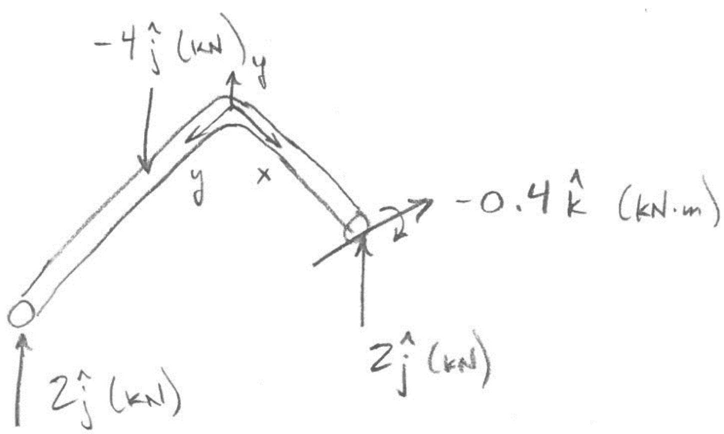
$$\sum M_{x,(A)} = 0: \quad -0.3B_y + 0.6 = 0 \text{ (kN}\cdot\text{m)} \quad \Rightarrow \quad \boxed{B_y = 2 \text{ kN}}$$

$$\sum M_{y,(A)} = 0: \quad \boxed{M_{y,A} = 0}$$

$$\sum M_{z,(A)} = 0: \quad M_{z,A} - 0.2B_y + 0.8 = 0 \left(\text{kN}\cdot\text{m}\right) \quad \Rightarrow \quad \boxed{M_{z,A} = -0.4 \text{ kN}\cdot\text{m}}$$

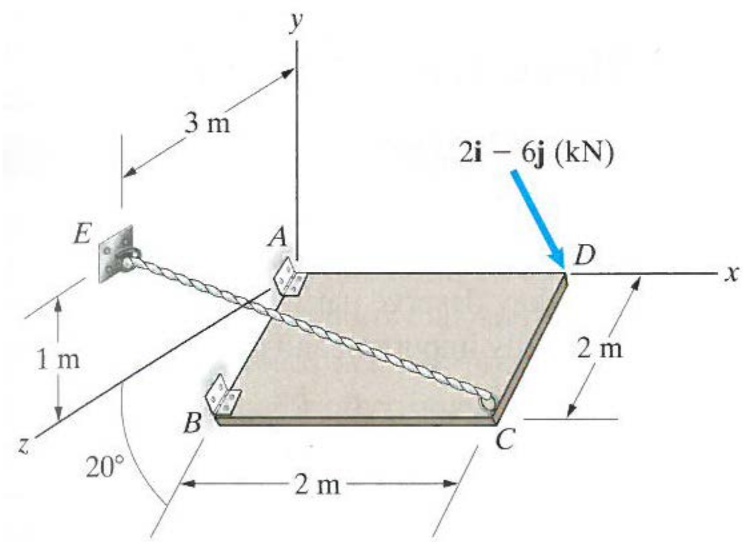
Use previous result from force equilibrium, $A_y = 4 - B_y$ (kN), to compute $\boxed{A_y = 2 \text{ kN}}$.

Solved FBD:



Do these make sense? Yes. In particular, couple is acting the direction we would expect.

Example: Problem 5.113

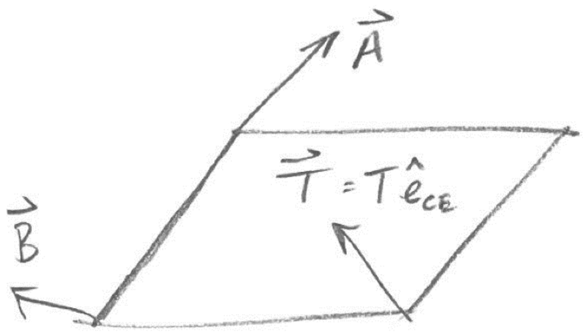


The plate is supported by hinges at *A* and *B* and the cable *CE*, and it is loaded by the force at *D*. The edge of the plate to which the hinges are attached lies in the *y*-*z* plane, and the axes of the hinges are parallel to the line through *A* and *B*[4]. What is the tension in cable *CE*?

[4] The problem statement also indicates that the hinges do not exert couples on the plate. For our purposes, we will discuss this as an assumption.

Strategy: Assume that the hinges exert no couples, because they are properly aligned[5] (i.e., the axes of the individual hinges coincide). For properly aligned hinges, the rotation is resisted mainly by the hinge forces, which are separated by a distance that allows for a development of a significant (resisting) moment. Draw the free-body diagram and consider equilibrium.

Free-body diagram:



Observe that we have 7 unknowns (3 forces at each hinge plus tension) and 6 equations! It would appear we cannot solve the problem, and indeed that is strictly speaking true. The system is statically indeterminate, as the components of the force along the axis of the hinges cannot be determined[6].

We could proceed to solve for all of the forces if we made assumptions. However, none are needed to solve for the tension in the cable.

[5] Section 5.3 discusses this briefly.
[6] In Example 5.9 in the textbook the problem statement indicates that only one of the hinges exerts a force in this direction. The present example highlights why such an assumption is needed.

Express tension in cable as $\vec{T} = T\hat{e}_{CE}$.

Consider (judiciously) the moments about line *AB*. Recall the mixed triple product:

$$M_L = \hat{e}_L \cdot \left(\vec{r} \times \vec{F}\right)$$

Now, moments about line are as follows, recognizing that there is no contribution from the hinges, namely $\hat{e}_{AB} \cdot \left(\vec{r}_{AB} \times \vec{B}\right) = 0$ and $\hat{e}_{AB} \cdot \left(\vec{r}_{BA} \times \vec{A}\right) = 0$:

$$\boxed{\sum M_L = \hat{e}_{AB} \cdot \left(\vec{r}_{AD} \times \vec{F}_D\right) + \hat{e}_{AB} \cdot \left(\vec{r}_{AC} \times T\hat{e}_{CE}\right) = 0}$$

or, alternatively, $\sum M_L = \hat{e}_{AB} \cdot \left(\vec{r}_{BD} \times \vec{F}_D\right) + \hat{e}_{AB} \cdot \left(\vec{r}_{BC} \times T\hat{e}_{CE}\right) = 0$

First, compute coordinates using vector addition or trigonometry to get the quantities that we need:

Coordinates of points *A*, *B*, *C*, and *D*:
A: (0, 0, 0) m
B: (0, −0.684, 1.879) m
C: (2, −0.684, 1.879) m
D: (2, 0, 0) m

Unit vector for line passing through *A* and *B*:

$$\hat{e}_{AB} = \frac{\hat{r}_{AB}}{\left|\hat{r}_{AB}\right|}$$

$$\hat{e}_{AB} = 0\hat{i} - 0.342\hat{j} + 0.9395\hat{k}$$

Position vectors:

$$\hat{r}_{AD} = 2\hat{i} + 0\hat{j} + 0\hat{k} \text{ (m)}$$

$$\hat{r}_{AC} = 2\hat{i} - 0.684\hat{j} + 1.879\hat{k} \text{ (m)}$$

Unit vector $\hat{e}_{CE}$:

$$\hat{e}_{CE} = \frac{\hat{r}_{CE}}{\left|\hat{r}_{CE}\right|} = -0.703\hat{i} + 0.542\hat{j} + 0.394\hat{k}$$

Scalar triple products:

$$\hat{e}_{AB} \cdot \left(\vec{r}_{AD} \times \vec{F}_D\right) = \begin{vmatrix} 0 & -0.342 & 0.9395 \\ 2 & 0 & 0 \\ 2 & -6 & 0 \end{vmatrix} \begin{matrix} \\ \text{(m)} \\ \text{(kN)} \end{matrix}$$

$$= 0.9365\left[(2)(-6) - (0)(2)\right]$$

$$= -11.274 \text{ kN} \cdot \text{m}$$

$$\hat{e}_{AB}\cdot\left(\vec{r}_{AC}\times T\hat{e}_{CE}\right)=\begin{vmatrix} 0 & -0.342 & 0.9395 \\ 2 & -0.684 & 1.879 \\ -0.703T & 0.592T & 0.394T \end{vmatrix}\ \text{(m)}$$

$$=-\left(-0.342\right)\left[\left(2\right)\left(0.394T\right)-\left(1.879\right)\left(-0.703T\right)\right]$$

$$+\left(0.9395\right)\left[\left(2\right)\left(0.592T\right)-\left(-0.684\right)\left(-0.703T\right)\right]$$

$$=1.382T$$

Plug everything in:

$$\sum M_L=\hat{e}_{AB}\cdot\left(\vec{r}_{AD}\times\vec{F}_D\right)+\hat{e}_{AB}\cdot\left(\vec{r}_{AC}\times T\hat{e}_{CE}\right)$$

$$=11.274+1.382T=0\ \text{(kN}\cdot\text{m)}$$

Solve:

$$T=\frac{11.274\ \text{kN}\cdot\text{m}}{1.382\ \text{m}}=8.16\ \text{kN}$$

$$\boxed{T=8.16\ \text{kN}}$$

Go through Examples 5.7, 5.8, and 5.9 on your own.