Lecture 14
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
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Chapter 5: §5.1 Objects in Equilibrium; §5.2 Statically Indeterminate Objects[1]

Acknowledgements

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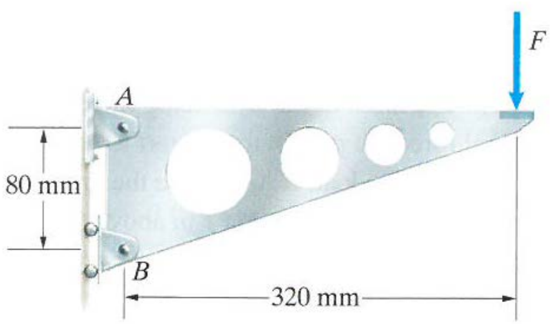
Objects in Equilibrium (2D) (continued)

Example: Problem 5.33 from the textbook.

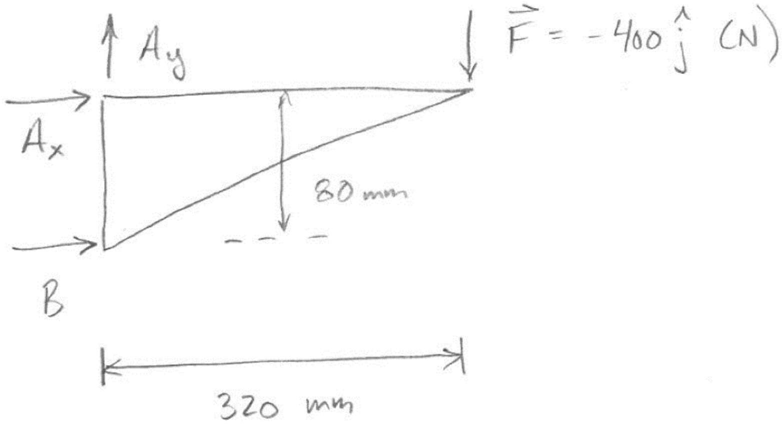


If $\vec{F} = -400\hat{j}$ (N), what are the reactions at points A and B?

Strategy: Draw FBD and use equilibrium equations to solve for unknown reactions.

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

Free-body diagram:



In this case, we drew the components of the unknown reactions as positive, rather than trying to guess the direction ahead of time.

Two approaches for including the unknown forces/reactions:

1. Always draw them positive (up and to the right).
2. Guess at the actual direction as a way to predict and check.

In either case, a negative solution means the direction is opposite of that assumed.

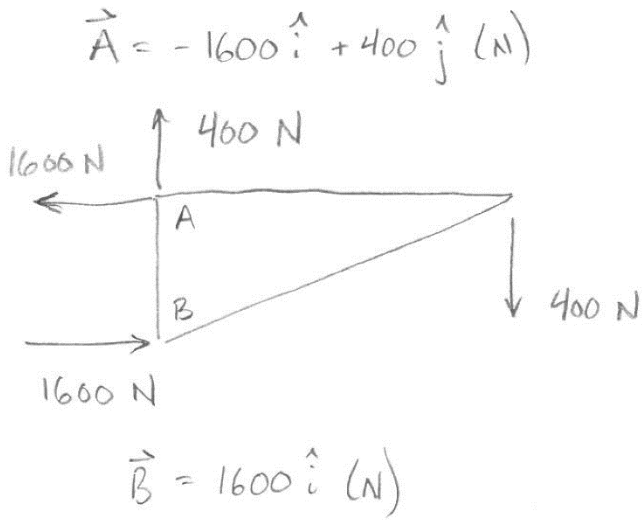
$$\sum F_x = 0: \quad A_x + B = 0 \qquad \text{(N)}$$
$$\sum F_y = 0: \quad A_y - 400 = 0 \qquad \text{(N)}$$
$$\sum M_A = 0: \quad (80)(B) - (320)(400) = 0 \qquad (\text{N} \cdot \text{mm})$$

From the 3rd equation, we find $B = 1600\ \text{N}$.

Then, from the 1st equation, we find $A_x = -1600\ \text{N}$. Note that we assumed the wrong direction for the force; the negative sign indicates that we must reverse direction.

Finally, the 2nd equation gives $A_y = 400\ \text{N}$.

Fully solved FBD (all forces shown drawn in the direction they act):



Go through Examples 5.1-5.4 in the textbook on your own.

<u>Statically Indeterminate Objects</u>

Consider a vertically loaded cantilever with a roller added at the tip[2]:

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[2] Section 5.2 of the book works through this example in detail, considering a force *F*, applied midspan.

Free-body diagram:

Consider a beam on rollers subjected to a force with a horizontal component:

Free-body diagram:

Let

- N = number of unknown forces (reactions)
- M = number of equations from equilibrium

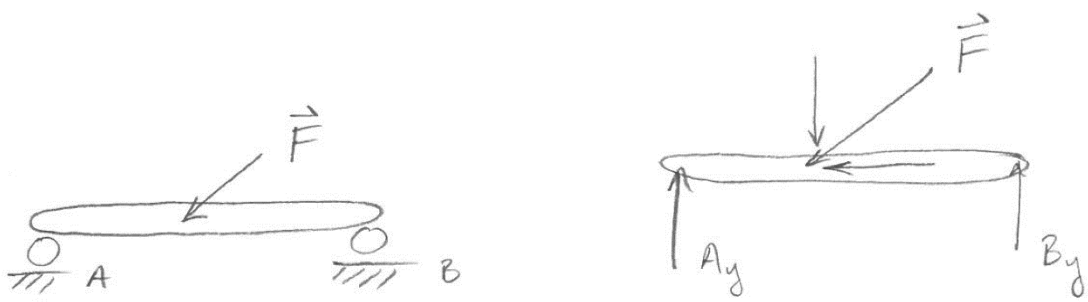
When $N \neq M$, i.e., the number of unknown reactions differs from the number of equations available to solve for these unknowns, the object is said to be statically indeterminate.

Two cases:

- The object has redundant supports, such that $N > M$. The degree of redundancy is $N - M$.
- The object has improper supports, such that $N < M$. This occurs either when (1) the supports can exert only parallel forces or (2) the supports can exert only concurrent forces.

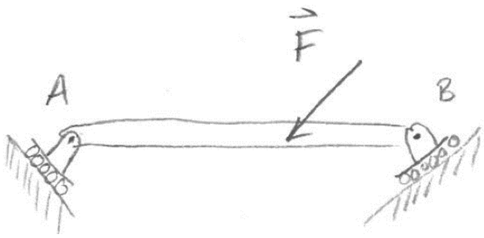
How would you classify the two examples considered previously: (1) the cantilever supported at its tip and (2) the beam on rollers?

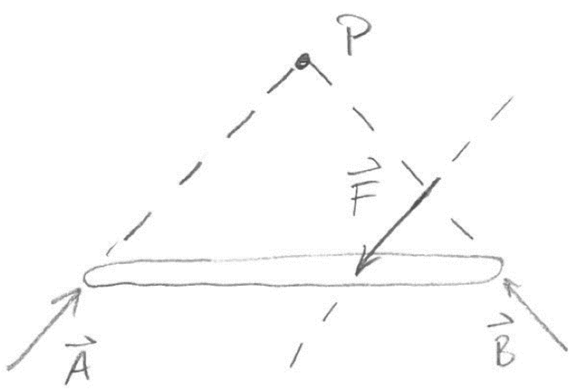
Parallel forces[3]:



[3] Example from textbook.

Concurrent forces[4]:





From the free-body diagram showing the lines of action, we see clearly that the beam may rotate.

The supports can generate both *x*- and *y*- components of force, <u>but they cannot vary independently</u>.

In practical applications, are redundant supports are problem?

No. In fact, they are often introduced deliberately. If one support in a statically determinant fails, the system fails (i.e., loses equilibrium).

[4] Example from textbook. Both supports are inclined at 45 degrees.

Reactions to problems involving redundant supports requires information regarding the material and its deformability. You learn more about this in *Mechanics of Materials* (deformable body mechanics), such as CIV_ENV 216.

Are improper supports a problem?

Generally, yes. The solution to this problem is to add more supports.

In this course, we do not go further than identifying whether a system is indeterminate, and exploring the nature of the indeterminacy.

Go through Examples 5.5[5] and 5.6[6] in the textbook on your own.

[5] This example considers a beam pinned at both ends and loaded vertically. For this problem, there are 4 reactions and 3 equilibrium equations, so the degree of redundancy is $N - M = 4 - 3 = 1$.
[6] This example considers a series of L-shaped bars and asks to determine in each case whether the bar is properly or improperly supported.