Lecture 13
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
Prof. James P. Hambleton
Midterm Exam #1 Discussion (25 minutes); Chapter 5: §5.1 Objects in Equilibrium[1]

Acknowledgements

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Objects in Equilibrium (2D)

Conditions for equilibrium:

1. $\sum \vec{F} = \vec{0}$
2. $\sum \vec{M}_P = \vec{0}$

where P is any point, either inside or outside the object.

Scalar equilibrium conditions (2D):

1. $\sum F_x = 0$
2. $\sum F_y = 0$
3. $\sum M_P = 0$

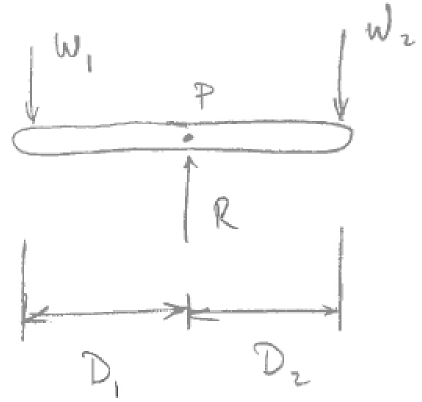
For 2D systems, we will typically consider and write forces in terms of their components.

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

Note that in addition to force equilibrium, we must have moment equilibrium, which effectively requires that the object does not spin.

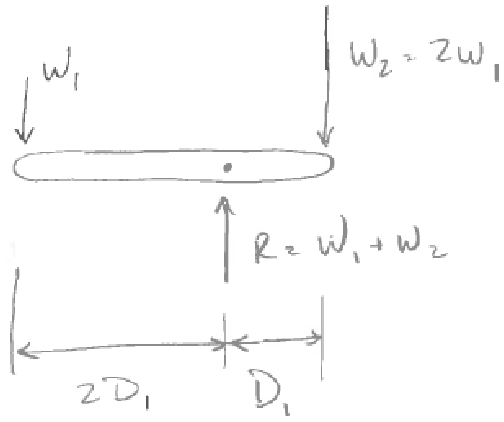
Example:

Returning to our simple example of a balance, we have



$$\sum M_P = D_1W_1 - D_2W_2 = 0 \;\Rightarrow\; W_2 = \frac{D_1}{D_2}W_1$$

If, for example, $D_1 = 2D_2$, then $W_2 = 2W_1$.



The support must be located 2/3 of the way along the bar to be in equilibrium.

Shortly we will generalize this (to 3D systems) by considering the moment vector $\vec{M}_P$ and $\sum \vec{M}_P = 0$.

Basic types of supports

1. Fixed support: Can transmit two components of force and a couple.
2. Pin support: Can transmit two components of force but not a couple, assuming a frictionless pin.
3. Roller support[2]: Can only transmit a force normal to the surface. If the surface is horizontal, then the roller can only transmit a vertical force component.

Show Table 5.1 in textbook and discuss other types of supports.

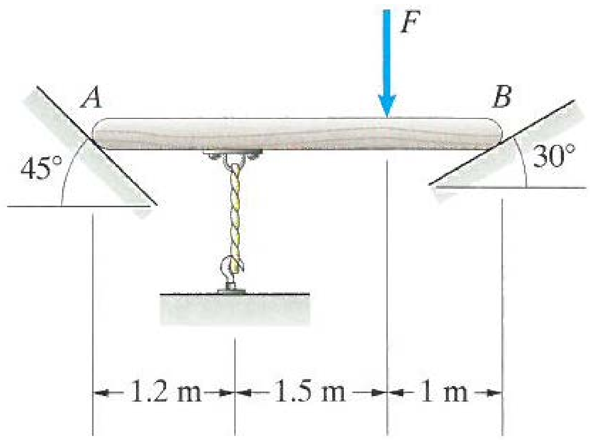
Make sure to read Section 5.1 on your own.

Free-body diagrams

These are (again) essential. The key is to know how to replace supports with the correct system of unknown forces (reactions).

[2] Sketch a beam on two rollers and mention the analogy to a skateboard.

Example: Problem 5.3 in the textbook.

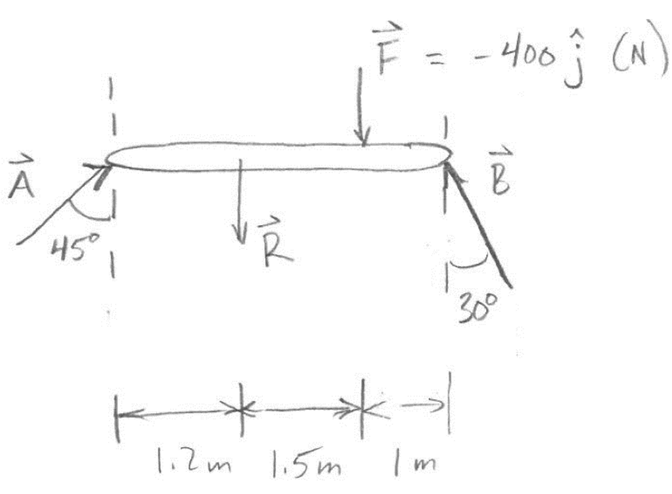


The beam is subjected to a vertical component of force F = 400 N, and it is supported by the rope and the smooth surfaces at points A and B.

a) Draw the free-body diagram of the beam.
b) What are the magnitudes of the reactions at points A and B?

Strategy: To draw the FBD, replace the supports with the correct system of unknown forces, one at a time. Then, use equilibrium to solve Part (b).

Part (a):



Here we have guessed the direction of $\vec{R} = -R\hat{j}$ based on how we think the bar will rotate.

Part (b):

We have 3 unknowns and 3 equations of equilibrium:

$$\sum F_x = 0: \quad A_x - B_x = 0$$

$$A\cos 45^\circ - B\sin 30^\circ = 0$$

$$\sum F_y = 0: \quad A_x - R - 400 + B_x = 0 \quad \text{(N)}$$

$$A\sin 45^\circ - R - 400 + B\cos 30^\circ = 0 \quad \text{(N)}$$

$$\sum M_A = 0: \quad \underset{(\text{CW})}{-(1.2)(R)} \underset{(\text{CW})}{-(2.7)(400)} \underset{(\text{CCW})}{+(3.7)(B\cos 30^\circ)} = 0 \quad (\text{N}\cdot\text{m})$$

Reduce this to a 3×3 system:

$$\begin{aligned} &0.707A - 0.5B = 0 && \text{(N)} \\ &0.707A + 0.866B - R = 400 && \text{(N)} \\ &0.3204B - 1.2R = 1080 && (\text{N}\cdot\text{m}) \end{aligned}$$

Solve this on your own for practice.

$$\begin{aligned} A &= 271 \text{ N} \\ B &= 383 \text{ N} \\ C &= 124 \text{ N} \end{aligned}$$

We are not done until we write these as vectors and draw the “solved” FBD:

$$\vec{A} = 192\hat{i} + 192\hat{j}\ \text{(N)}$$
$$\vec{B} = -192\hat{i} + 332\hat{j}\ \text{(N)}$$
$$\vec{R} = -124\hat{j}\ \text{(N)}$$

Solved FBD: