

Lecture 11

GEN_ENG 205-2: Engineering Analysis 2

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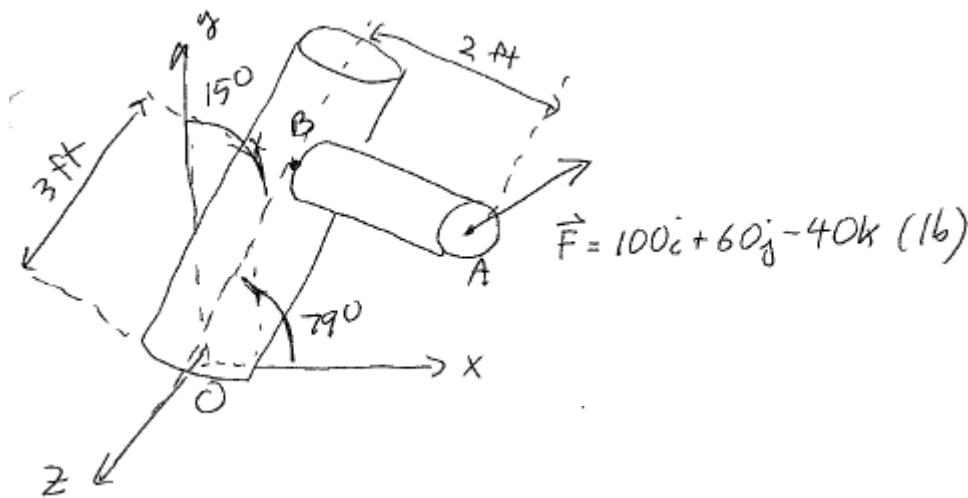
Chapter 4: §4.3 Moment of a Force about a Line; §4.4 Couples¹

Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Moment of a Force About a Line (continued)

Example:



Given $\hat{e}_{BA} = 0.685\hat{i} - 0.250\hat{j} + 0.685\hat{k}$, (a) what is the moment of \vec{F} about point O , and (b) what is the moment of \vec{F} about the main axis of the pipe (line \overline{OB})?

¹ Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

Strategy: For part (a), determine \vec{r}_{OA} using lengths and unit vectors \hat{e}_{OB} and \hat{e}_{BA} , and use the cross product to compute $\vec{M}_O = \vec{r}_{OA} \times \vec{F}$. Then, compute the component of \vec{M}_O parallel to line \overline{OB} .

First, observe the following:

$$\vec{r}_{OA} = \vec{r}_{OB} + \vec{r}_{BA} = 3\hat{e}_{OB} + 2\hat{e}_{BA} \text{ (ft)}$$

Determine \hat{e}_{OB} using direction cosines (§2.3 in textbook):

$$\hat{e}_{OB} = e_x\hat{i} + e_y\hat{j} + e_z\hat{k}$$

$$e_x = \cos 79^\circ = 0.1908$$

$$e_y = \cos 15^\circ = 0.9659$$

$$|\hat{e}_{OB}| = \sqrt{e_x^2 + e_y^2 + e_z^2} = 1 \Rightarrow e_z = \sqrt{1 - e_x^2 - e_y^2} = 0.1749$$

Now, compute \vec{r}_{OA} and \vec{M}_O :

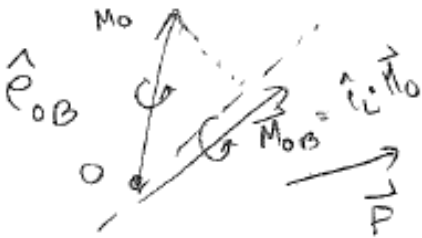
$$\begin{aligned} \vec{r}_{OA} &= 3\left[0.1908\hat{i} + 0.9659\hat{j} + 0.1749\hat{k}\right] + 2\left[0.685\hat{i} - 0.250\hat{j} + 0.685\hat{k}\right] \text{ (ft)} \\ &= 1.943\hat{i} + 2.398\hat{j} + 1.895\hat{k} \text{ (ft)} \end{aligned}$$

$$\begin{aligned} \vec{M}_O = \vec{r}_{OA} \times \vec{F} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.943 & 2.398 & 1.895 \\ 100 & 60 & -40 \end{vmatrix} \text{ (ft)} \\ &= \left[(2.398)(-40) - (1.895)(60)\right]\hat{i} - \left[(1.943)(-40) - (1.895)(100)\right]\hat{j} \text{ (lb}\cdot\text{ft)} \\ &\quad + \left[(1.943)(60) - (1.895)(100)\right]\hat{k} \text{ (lb}\cdot\text{ft)} \end{aligned}$$

$$\vec{M}_O = -210\hat{i} + 267\hat{j} - 123\hat{k} \text{ (lb}\cdot\text{ft)}$$

Component of \vec{M}_O about line \overline{OB} :

First (simple) approach:



$$M_{OB} = \hat{e}_{OB} \cdot \vec{M}_O = (0.1908\hat{i} + 0.9659\hat{j} + 0.1749\hat{k}) \cdot (-209.6\hat{i} + 267.2\hat{j} - 123.2\hat{k})$$

$$M_{OB} = \hat{e}_{OB} \cdot \vec{M}_O = 197 \text{ lb}\cdot\text{ft (CCW)}$$

This is the torsion on the shaft.

Multiply by the unit vector to get the moment vector.

$$\begin{aligned} \vec{M}_{OB} &= (\hat{e}_{OB} \cdot \vec{M}_O) \hat{e}_{OB} \\ &= (197 \text{ lb}\cdot\text{ft}) (0.1908\hat{i} + 0.9659\hat{j} + 0.1749\hat{k}) \text{ (lb}\cdot\text{ft)} \end{aligned}$$

$$\vec{M}_{OB} = 37.5\hat{i} + 190\hat{j} + 34.4\hat{k} \text{ (lb}\cdot\text{ft)}$$

Second (formulaic) approach:

$$\vec{M}_{OB} = \left[\hat{e}_{OB} \cdot (\vec{r} \times \vec{F}) \right] \hat{e}_{OB} \text{ where } \vec{r} \text{ is any vector from line } \overline{OB} \text{ to the line of action of } \vec{F}$$

In this case, we could choose $\vec{r} = \vec{r}_{OA}$ or $\vec{r} = \vec{r}_{BA}$. Choosing $\vec{r} = \vec{r}_{OA}$ gives the mixed triple product

$$\begin{aligned} \hat{e}_{OB} \cdot (\vec{r} \times \vec{F}) &= \begin{vmatrix} 0.1908 & 0.9659 & 0.1749 \\ 1.943 & 2.398 & 1.895 \\ 100 & 60 & -40 \end{vmatrix} \begin{matrix} (\text{ft}) \\ (\text{lb}) \end{matrix} \\ &= 196.5 \text{ (lb} \cdot \text{ft)} \end{aligned}$$

$$\begin{aligned} \vec{M}_{OB} &= 196.5 \hat{e}_{OB} \text{ (lb} \cdot \text{ft)} \\ &= 37.5 \hat{i} + 190 \hat{j} + 34.4 \hat{k} \text{ (lb} \cdot \text{ft)} \end{aligned}$$

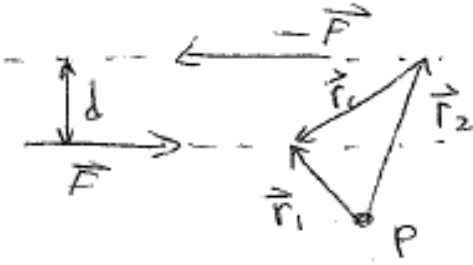
Go through Examples 4.6, 4.7 and 4.8 on your own.

Couples

Can a moment be exerted without a net force? Yes.



Forces are equal in magnitude and opposite in direction, with different lines of action. These are called couples.



Compute $\sum \vec{M}_P$ for an arbitrary point P :

$$\sum \vec{M}_P = \vec{r}_1 \times \vec{F} + \vec{r}_2 \times (-\vec{F}) = (\vec{r}_1 - \vec{r}_2) \times \vec{F} = \vec{r}_c \times \vec{F}$$

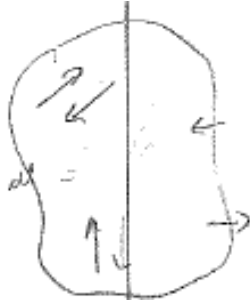
Observe that \vec{r}_c does not depend on point P and can be any vector spanning the lines of action.

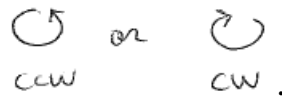
Denote the couple by $\vec{M}_C = \vec{r}_c \times \vec{F}$.


Note that \vec{M}_C is still a vector, one that is normal to the plane containing the forces.

$$|\vec{M}_C| = D|\vec{F}|$$

Why is the concept of a couple important? The moment of a couple, \vec{M}_C , is the same everywhere on a body. Contrast this with the moment of a force, which changes depending on the point.

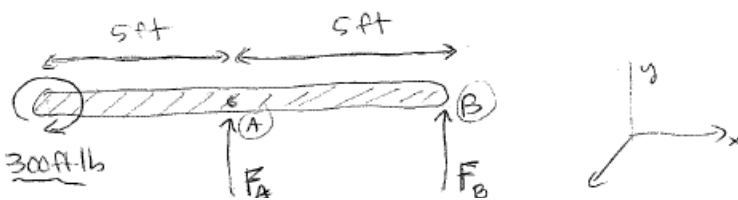


In a 2D problem, we draw a couple as .

In a 3D problem, we draw these as .

These couples can appear anywhere in the system, and we can forget about the forces that generated them.

Example (2D):



The bar shown is in equilibrium. Determine the forces F_A and F_B .

Strategy: Work through the equilibrium equations², $\sum F_y = 0$ and $\sum M_P = 0$, and solve.

² We will write these formally in Chapter 5 (next lecture). As we will see, for 2D structures it is most convenient to use the component form of the equilibrium equations. Again, emphasize that force cannot move vertically only if the net forces in the y-direction are zero. It will not spin if there is no net moment.

$$\sum F_y = F_A + F_B = 0$$

$$\sum M_B = -300 \text{ ft} \cdot \text{lb} - (5 \text{ ft}) F_A + 0 = 0$$

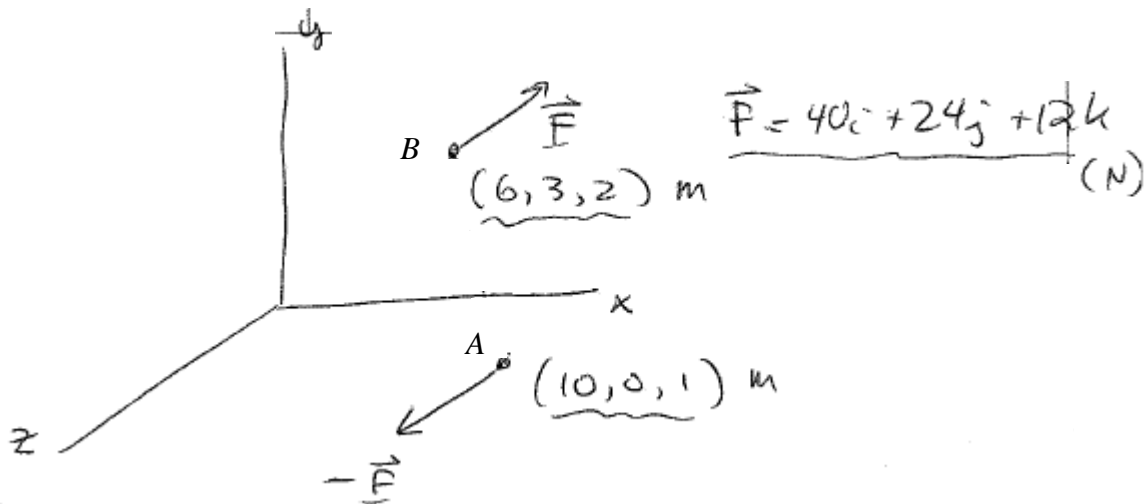
Solve the second equation to find $F_A = -\frac{(300 \text{ ft} \cdot \text{lb})}{(5 \text{ ft})} = -60 \text{ lb}$ and then the first to find

$$F_B = 60 \text{ lb}.$$

$$\boxed{\begin{matrix} F_A = -60 \text{ lb} \\ F_B = 60 \text{ lb} \end{matrix}} \text{ or, formally, } \boxed{\begin{matrix} \vec{F}_A = -(60 \text{ lb}) \hat{j} \\ \vec{F}_B = (60 \text{ lb}) \hat{j} \end{matrix}}$$

Note that these force form a couple! This couple is equal in magnitude but opposite in direction.

Also, we could have summed moments about any point, but this choice led to a direct result.

Example:

- (a) What is \vec{M}_C ?
- (b) What is the perpendicular distance between the forces' lines of action?

Strategy: Select any point, say point A , to compute \vec{M}_C . Then, use $|\vec{M}_C| = D|\vec{F}|$ to compute D .

Part (a):

$$\begin{aligned}\vec{r}_C &= \vec{r}_{AB} = (6-10)\hat{i} + (3-0)\hat{j} + (2-1)\hat{k} \text{ (m)} \\ &= -4\hat{i} + 3\hat{j} + 1\hat{k} \text{ (m)}\end{aligned}$$

$$\begin{aligned}\vec{M}_C &= \vec{r}_C \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 3 & 1 \\ 40 & 24 & 12 \end{vmatrix} \begin{matrix} \text{(m)} \\ \text{(N)} \end{matrix} \\ &= (36-24)\hat{i} - (-48-40)\hat{j} + (-96-120)\hat{k} \text{ (N}\cdot\text{m)}\end{aligned}$$

$$\boxed{\vec{M}_C = 12.0\hat{i} + 88.0\hat{j} - 216\hat{k} \text{ (N}\cdot\text{m)}}$$

Note that we could have selected point B . In this case, we would have arrived at the same result:

$$\vec{r}_{BA} = -\vec{r}_{AB}$$

$$\vec{M}_C = \vec{r}_{BA} \times (-\vec{F}) = (-\vec{r}_{AB}) \times (-\vec{F}) = \vec{r}_{AB} \times \vec{F}$$

Part (b):

$$|\vec{M}_C| = D |\vec{F}|$$

$$|\vec{M}_C| = \sqrt{12^2 + 88^2 + (-216)^2} = 233.5 \text{ N}\cdot\text{m}$$

$$|\vec{F}| = \sqrt{40^2 + 24^2 + (12)^2} = 48.17 \text{ N}$$

$$D = \frac{|\vec{M}_C|}{|\vec{F}|} = \frac{233.5 \text{ N}\cdot\text{m}}{48.17 \text{ N}}$$

$$\boxed{D = 4.85 \text{ m}}$$

Observe that this is the minimum distance between the lines of action ($|\vec{r}_C| = 5.10 \text{ m}$).

Go through Examples 4.9, 4.10, and 4.11 on your own.