Lecture 10
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
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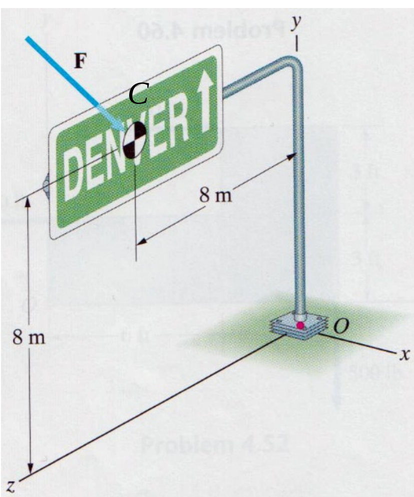
Chapter 4: §4.2 The Moment Vector; §4.3 Moment of a Force about a Line[1]

Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Moment Vector (continued)

Example: Problem 4.63[2] from textbook.



The total force on the sign due to wind is estimated as $\vec{F} = 2.8\hat{i} - 1.8\hat{j}$ (kN). Let $\vec{M}_0$ be the moment due to $\vec{F}$ at the base O of the column. Determine the magnitudes of the y-component of $\vec{M}_O$, or torsion, and the component of $\vec{M}_O$ parallel to the x-z plane, or so-called bending moment.

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.
[2] A good example highlighting the physical interpretation of the components.

Strategy: Determine a position vector from point O to the line of action of $\vec{F}$ and use the cross product to compute $\vec{M}_O$. Assess the component parallel to the x-z simply through vector subtraction.

Position vector:

$$\vec{r}_{OC} = 8\hat{j} + 8\hat{k} \text{ (m)}$$

Compute $\vec{M}_O$:

$$\vec{M}_O = \vec{r}_{OC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 8 & 8 \\ 2.8 & -1.8 & 0 \end{vmatrix} \begin{matrix} \\ \text{(m)} \\ \text{(kN)} \end{matrix}$$

$$= \left[0-(8)(-1.8)\right]\hat{i} - \left[0-(8)(2.8)\right]\hat{j} + \left[0-(8)(2.8)\right] \text{ (kN}\cdot\text{m)}$$

$$= 14.40\hat{i} + 22.40\hat{j} - 22.40\hat{k} \text{ (kN}\cdot\text{m)}$$

Magnitude of y-component:

Let $\vec{M}_O = M_{Ox}\hat{i} + M_{Oy}\hat{j} + M_{Oz}\hat{k}$

$$\boxed{\left|M_{Oy}\right| = 22.4 \text{ kN}\cdot\text{m}}$$

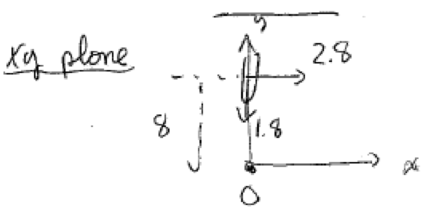
Magnitude of the component of parallel to the x-z plane:

Parallel component is simply $\vec{M}_P = \vec{M}_O - M_{Oy}\hat{j} = M_{Ox}\hat{i} + M_{Oz}\hat{k}$.

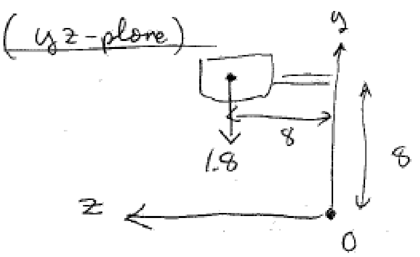
Magnitude $\left|\vec{M}_P\right| = \sqrt{M_{Ox}^2 + M_{Oy}^2} = \sqrt{(14.40)^2 + (-22.40)^2}\ \text{kN}\cdot\text{m}$

$$\boxed{\left|\vec{M}_P\right| = 26.6\ \text{kN}\cdot\text{m}}$$

Observations:

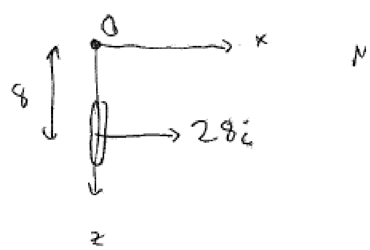


$$M_{Oz} = -(2.8\ \text{kN})(8\ \text{m}) = -22.4\ \text{kN}\cdot\text{m}$$



$$M_{Ox} = (1.8\ \text{kN})(8\ \text{m}) = 14.4\ \text{kN}\cdot\text{m}$$

And also y-axis (xz-plane)



$$M_{Oy} = (2.8\ \text{kN})(8\ \text{m}) = 22.4\ \text{kN}\cdot\text{m}$$

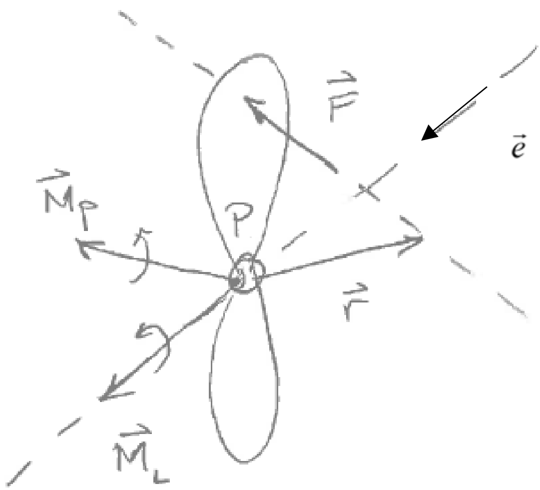
Be careful when constructing the position vector, and be careful with the order!

Go through Example 4.4 on your own.

Moment of Force about a Line

The moment of the force about the line is a measure of the tendency of a force to cause rotation about a line, or axis.

Consider the example of a force applied to a propeller, where it is clear that only certain components of the force tend to cause rotation.



Line L

$$\vec{M}_L = \left(\vec{e} \cdot \vec{M}_P\right)\vec{e}$$

Observing that $\vec{M}_P = \vec{r} \times \vec{F}$, we find

$$\vec{M}_L = \left[\vec{e} \cdot \left(\vec{r} \times \vec{F}\right)\right]\vec{e}$$

$$M_L = \vec{e} \cdot \left(\vec{r} \times \vec{F}\right) = \begin{vmatrix} e_x & e_y & e_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$M_L = \vec{e} \cdot \vec{M}_P = \vec{e} \cdot \left(\vec{r} \times \vec{F}\right)$ gives the magnitude and direction of the moment about the line (can be positive or negative)

By a similar argument as before, we can choose *P* anywhere along the line[3].

Special cases[4]:

- When the line of action of $\vec{F}$ is parallel to line *L*, $\vec{M}_L = 0$.
- When the line of action of $\vec{F}$ intersects line *L*, $\vec{M}_L = 0$.
- When the line of action of $\vec{F}$ is perpendicular to a plane containing line *L*, the magnitude of the moment of is equal to the perpendicular distance *D* multiplied by $\left|\vec{F}\right|$: $\left|\vec{M}_L\right| = D\left|\vec{F}\right|$.

[3] In determining the moment, the position vector can locate anywhere along the line of action of the force. We there for have, *in addition* to this, that point *P* can lie anywhere on the line about which we wish to determine the force.
[4] Consider skipping or skimming through. Students should read Section 4.3 for details.