Lecture 9

GEN_ENG 205-2: Engineering Analysis 2

Winter Quarter 2018

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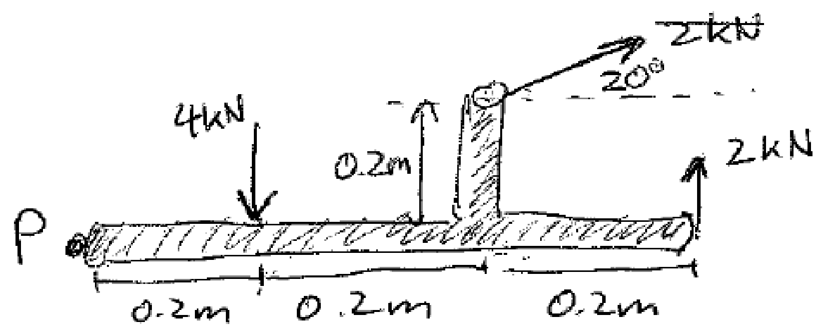
Chapter 4: §4.1 Two-Dimensional Description of the Moment; §4.2 The Moment Vector[1]

Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Moments (continued)

Example[2]: Problem 4.16 in the textbook.

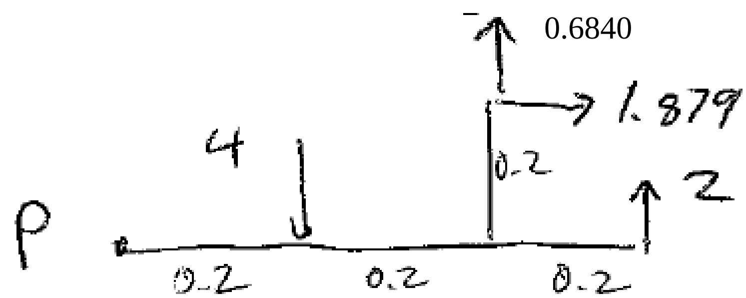


Determine the sum of the moments of the three forces about point *P*.

Strategy: Resolve force into components and use $\sum M_P = \sum (DF)$

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

[2] Experience suggests that this is a key example for students highlighting the importance of direction and the changing moment arm D for each of the various components. It is best to go through it carefully, defining length variables to show where each moment arm comes from ($D_1 = L_1$, $D_2 = L_1 + L_2$, etc.)



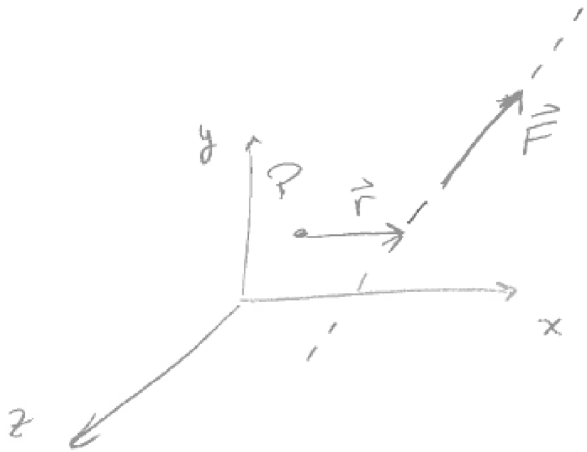
$$\sum M_P = -(4 \text{ kN})(0.2 \text{ m}) + (0.6840 \text{ kN})(0.4 \text{ m}) - (1.879 \text{ kN})(0.2 \text{ m}) + (2 \text{ kN})(0.6 \text{ m})$$

$$\boxed{\sum M_P = +0.298 \text{ kN}\cdot\text{m (CCW)}}$$

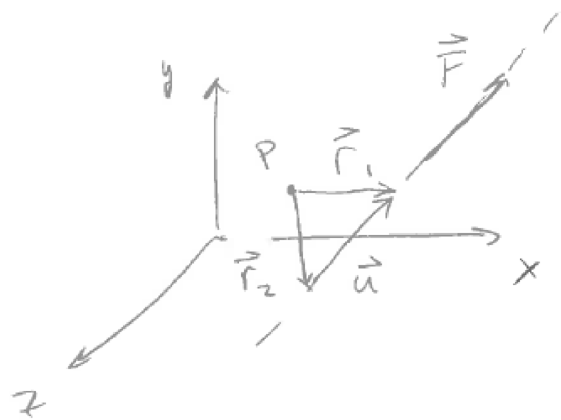
Go through Examples 4.1, 4.2, and 4.3 in the textbook on your own.

Moment Vector (General 3D Definition)

Definition: $\boxed{\vec{M}_P = \vec{r} \times \vec{F}}$



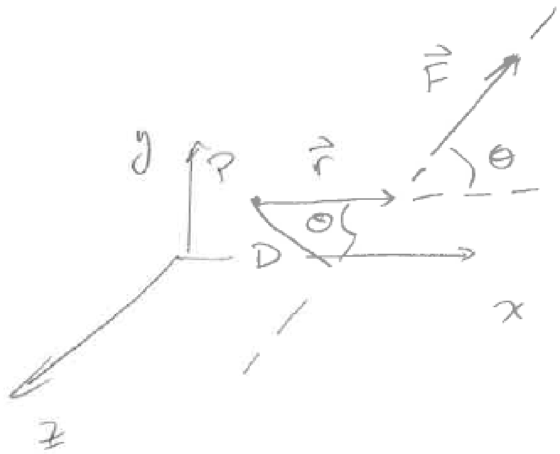
Vector $\vec{r}$ is a position vector from point P to *any* point on the line of action of $\vec{F}$.



$$\vec{r}_1 \times \vec{F} = \left(\vec{r}_2 + \vec{u}\right) \times \vec{F} = \vec{r}_2 \times \vec{F} + \cancel{\vec{u} \times \vec{F}} = \vec{r}_2 \times \vec{F}$$

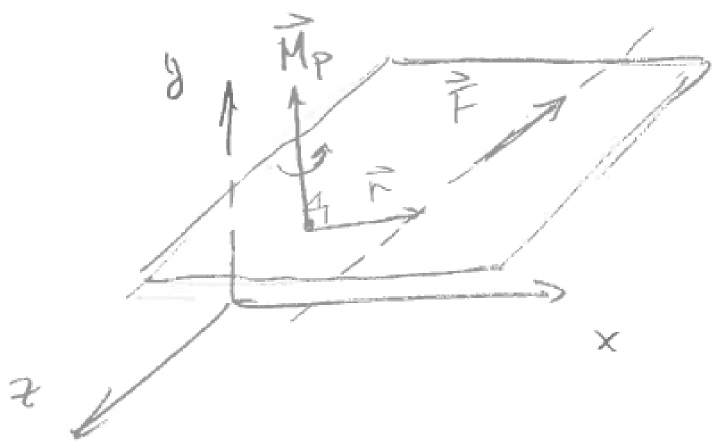
Correspondence to 2D definition

Definition of cross product: $\vec{M}_P = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}|\sin\theta\hat{e} \;\Rightarrow\; |\vec{M}_P| = (|\vec{r}|\sin\theta)|\vec{F}| = D|\vec{F}|$



Direction of the moment

$\vec{M}_P$ is perpendicular to the plane containing P and $\vec{F}$ (again, through the definition of the cross product)



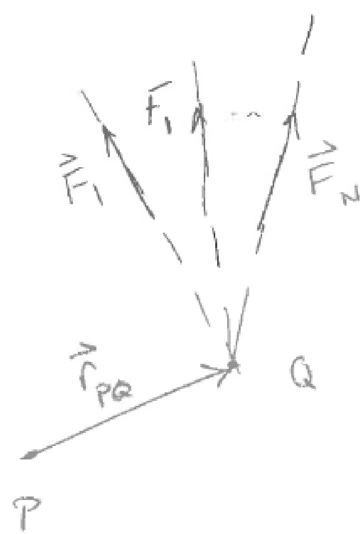
The right-hand rule indicates direction of the moment, and moment vectors are denoted by circular arrow around the vector.

Remember: Order matters!

<u>Varignon's Theorem</u>[3]

Let $\vec{F}_1$, $\vec{F}_1, \ldots,$ $\vec{F}_N$ be a concurrent system of forces for which the lines of action intersect at a point Q. The moment of the system about a point P is

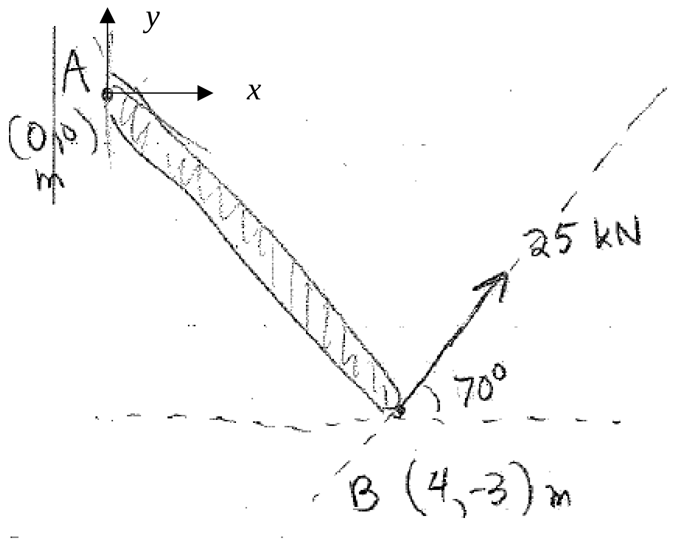
$$\vec{M}_P = \left(\vec{r}_{PQ} \times \vec{F}_1\right) + \left(\vec{r}_{PQ} \times \vec{F}_2\right) + \ldots + \left(\vec{r}_{PQ} \times \vec{F}_N\right) = \vec{r}_{PQ}\left(\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_N\right)$$



This theorem confirms that the moment of a force about a point P is equal to the sum of the moments of its components about P.

[3] This theorem follows directly from the distributive property of the cross product.

Example (relationship to 2D description):



What is the moment of the force about point *A*?

$\vec{r}_{AB} = 4\hat{i} - 3\hat{j} + 0\hat{k}$ (m)

$\vec{F} = 8.551\hat{i} + 23.49\hat{j} + 0\hat{k}$ (kN)

$\vec{M}_A = \vec{r}_{AB} \times \vec{F} = 0\hat{i} + 0\hat{j} + 120\hat{k}$ (kN·m)