Lecture 8
GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
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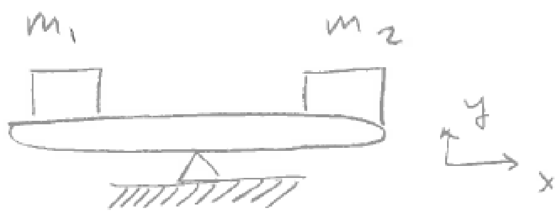
Chapter 4: §4.1 Two-Dimensional Description of the Moment[1]

Acknowledgements

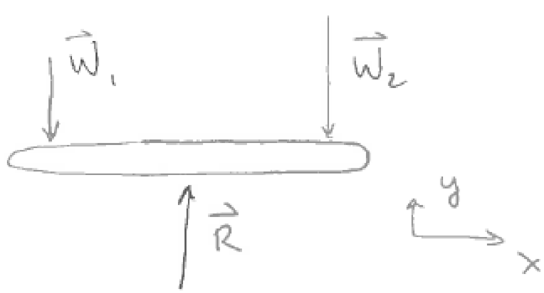
Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Moments[2]

Consider the following system:



Free-body diagram:



Formally

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.
[2] Go through the example before indicating that we are now moving on to moments.

$$\vec{W}_1 = -W_1\hat{j} = \left(m_1 g\right)\hat{j}, \;\; \vec{W}_2 = -W_2\hat{j} = \left(m_2 g\right)\hat{j}, \;\; \vec{R} = R\hat{j}$$

Equilibrium (2D):

$$\sum F_x = 0$$
$$\sum F_y = R - W_1 - W_2 = 0 \;\;\Rightarrow\;\; R = W_1 + W_2$$

However, is the system in equilibrium? It may want to rotate, so equilibrium of forces is not sufficient[3].

Moments

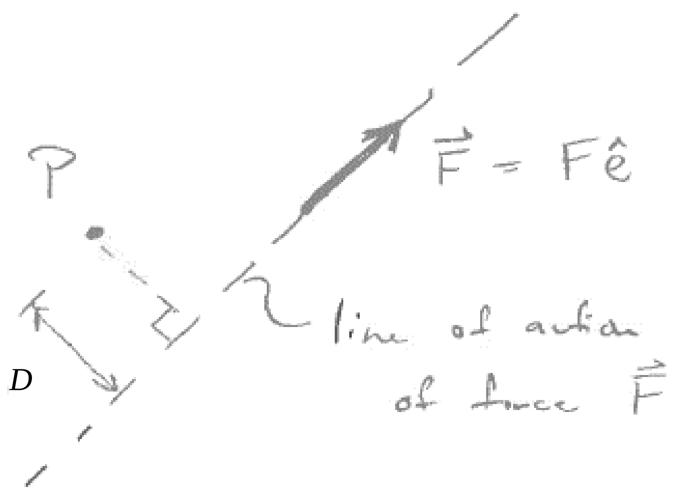
The moment is the tendency for a force to cause rotation about a given point or line.

In 2D, the moment is a scalar given by[4]

$$M_P = DF$$

where D is the perpendicular distance, or moment arm, from point P to the line of action of the force $\vec{F}$.

[3] It is, however, a *necessary* condition.
[4] Be sure to explain notation: the subscript typically denotes the point about which the moment is taken.



The <u>units</u> of a moment are $[M_P]$ = force·length (e.g., N·m or ft·lb).

We define a <u>positive</u> moment as one that causes <u>counterclockwise</u> rotation and a <u>negative</u> moment as one that causes <u>clockwise</u> rotation.

In addition to force equilibrium, we must have moment equilibrium. In two dimensions, this can be expressed as

$$\boxed{\sum M_P = 0}$$

where P can be <u>any point</u>, either inside or outside the object.

We will start using $\sum M_P = 0$ in <u>Chapter 5</u>.

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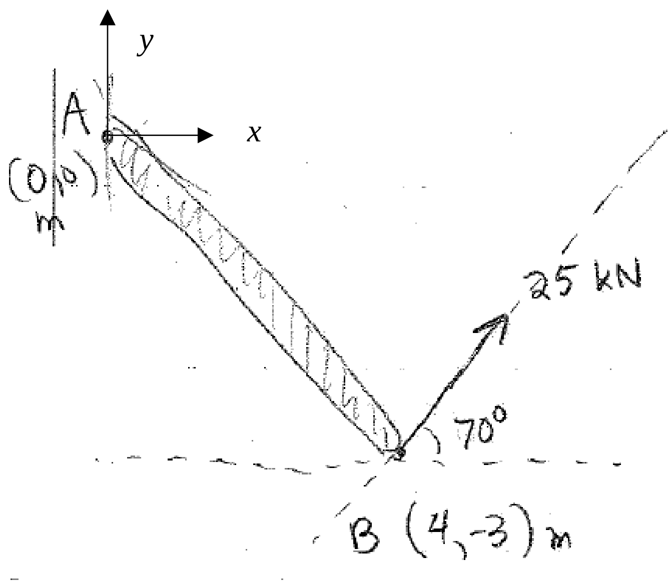
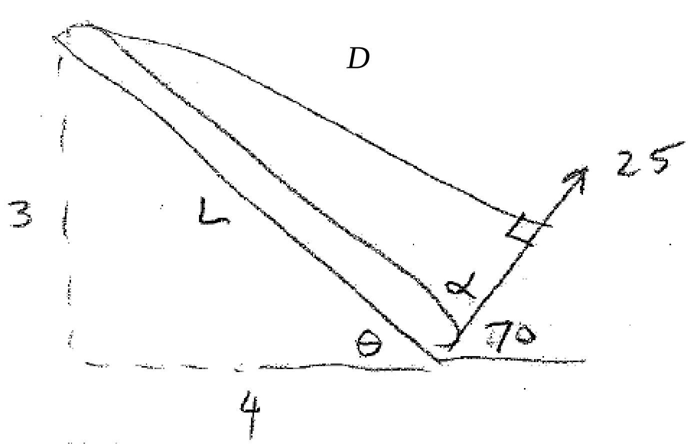
Example:



What is the moment of the force about point *A*?

1st approach: Compute perpendicular distance D



Observe 3-4-5 triangle, so $L = 5$ m.

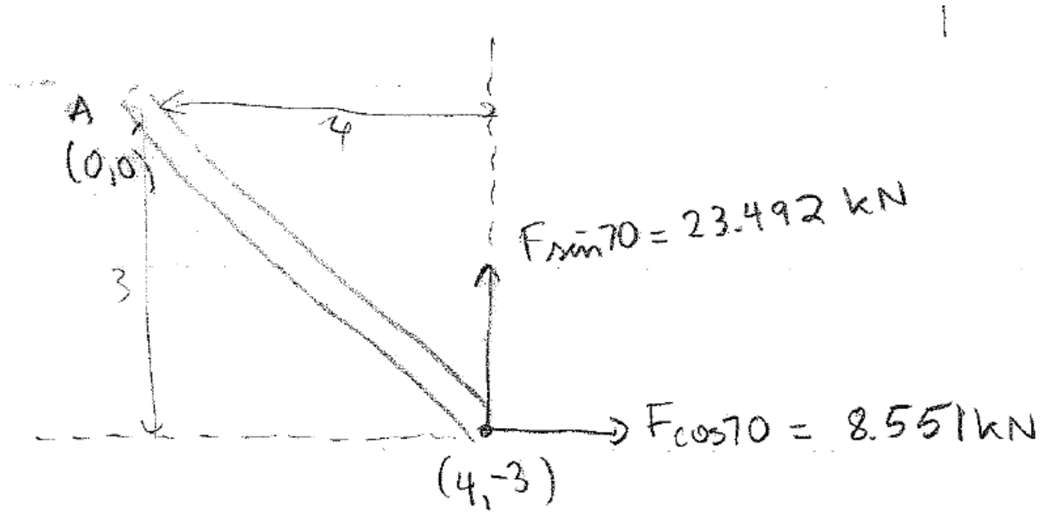
$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$\alpha = 180^\circ - 70^\circ - 36.87^\circ = 73.13^\circ$$

$$D = L\sin\alpha = (5\text{ m})\sin 73.13^\circ = 4.785\text{ m}$$

$$\boxed{M_A = DF = +120\text{ kN}\cdot\text{m (CCW)}}$$

2nd Approach: Use components[5]



$$M_A = D_1F_1 + D_2F_2 = +(4\text{ m})(23.49\text{ kN}) + (3\text{ m})(8.551\text{ kN}) = 120\text{ kN}\cdot\text{m}$$

This is arguably the better way to compute the moment. We will use this approach extensively for trusses and frames.

[5] This is a key example: we will use this approach extensively for trusses and frames.