**Lecture 7**

GEN_ENG 205-2: Engineering Analysis 2

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§3.3 Three-Dimensional Force Systems[1]

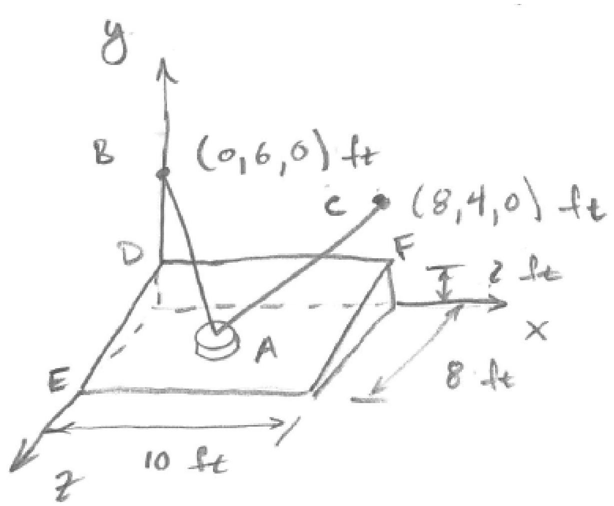
Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Three-dimensional Force Systems (continued)

Example[2]: Problem 3.66 from the textbook

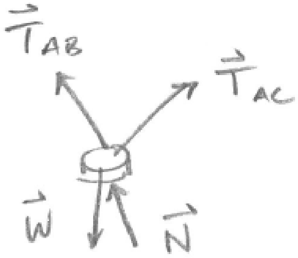
A disc weighing 10 lb is supported by the smooth inclined surface and the strings *AB* and *AC* as shown. The disk is located at coordinates (5, 1, 4) ft. What are the tensions in the strings?




[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

[2] This example, when explained methodically and completely (including the solution of the linear system), consumes a full lecture.

Strategy: Consider the free-body diagram for the disc.




$$\hat{T}_{AB} = T_{AB}\hat{e}_{AB}$$

$$\hat{T}_{AC} = T_{AC}\hat{e}_{AC}$$

$$\vec{N} = N\vec{e}_n$$

$$\vec{W} = \left(-10\text{ lb}\right)\hat{j}$$

$$\hat{e}_{AB} = \frac{\left(0-5\right)\hat{i}+\left(6-1\right)\hat{j}+\left(0-4\right)\hat{k}}{\sqrt{5^2+5^2+4^2}} = -0.6155\hat{i}+0.6155\hat{j}-0.4924\hat{k}$$

$$\hat{e}_{AC} = \frac{\left(8-5\right)\hat{i}+\left(4-1\right)\hat{j}+\left(0-4\right)\hat{k}}{\sqrt{3^2+3^2+4^2}} = 0.5145\hat{i}+0.5145\hat{j}-0.6860\hat{k}$$

Compute $\hat{n}$ the hard way:

$$\hat{e}_n = \frac{\vec{r}_{DE}\times\vec{r}_{DF}}{\left|\vec{r}_{DE}\times\vec{r}_{DF}\right|}$$

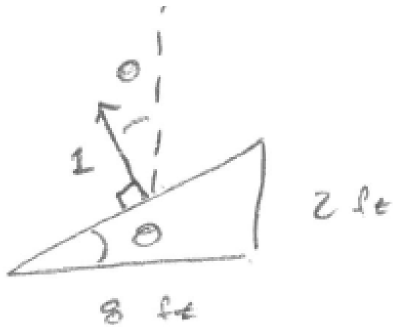
$$\begin{aligned}\vec{r}_{DE} &= \left(0-0\right)\hat{i}+\left(0-2\right)\hat{j}+\left(8-0\right)\hat{k}\text{ (ft)}\\ &= 0\hat{i}-2\hat{j}+8\hat{k}\text{ (ft)}\end{aligned}$$

$$\begin{aligned}\vec{r}_{DF} &= \left(10-0\right)\hat{i}+\left(2-2\right)\hat{j}+\left(0-0\right)\hat{k}\text{ (ft)}\\ &= 10\hat{i}-0\hat{j}+0\hat{k}\text{ (ft)}\end{aligned}$$

$$\vec{r}_{DE} \times \vec{r}_{DF} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 8 \\ 10 & 0 & 0 \end{vmatrix} = (0-0)\hat{i} - (0-80)\hat{j} + \left[0-(-20)\right]\hat{k} = 0\hat{i} + 80\hat{j} + 20\hat{k} \text{ (ft)}$$

$$\hat{e}_n = \frac{\vec{r}_{DE} \times \vec{r}_{DF}}{\left|\vec{r}_{DE} \times \vec{r}_{DF}\right|} = \frac{0\hat{i} + 80\hat{j} + 20\hat{k}}{\sqrt{0^2 + 80^2 + 20^2}} = 0\hat{i} + 0.9701\hat{j} + 0.2425\hat{k}$$

Compute $\hat{e}_n$ the easy way:




$$\theta = \tan^{-1}\left(\frac{2 \text{ ft}}{8 \text{ ft}}\right) = 14.03^\circ$$

$$e_{nx} = 0$$

$$e_{ny} = (1)\cos\theta = 0.9701$$

$$e_{nz} = (1)\sin\theta = 0.2425$$

Equilibrium:

$$\sum \vec{F} = \vec{0}:$$

$$\begin{aligned}\sum \vec{F} &= \vec{T}_{AB} + \vec{T}_{AC} + \vec{N} + \vec{W} \\ &= T_{AB}\left(-0.6155\hat{i} + 0.6155\hat{j} - 0.4924\hat{k}\right) + T_{AC}\left(0.5145\hat{i} + 0.5145\hat{j} - 0.6860\hat{k}\right) \\ &\quad + N\left(0\hat{i} + 0.9701\hat{j} + 0.2425\hat{k}\right) + \left(-10 \text{ lb}\right)\hat{j} \\ &= \vec{0}\end{aligned}$$

Considering that components must sum to zero:

$$\begin{aligned}&-0.6155T_{AB} + 0.5145T_{AC} = 0 \\ &0.6155T_{AB} + 0.5145T_{AC} + 0.9701N - 10 \text{ lb} = 0 \\ &-0.4924T_{AB} - 0.6860T_{AC} + 0.2425N = 0\end{aligned}$$

Eliminate $N$ from the second and third equations:

$$\begin{aligned}&0.6155T_{AB} + 0.5145T_{AC} + 0.9701N - 10 \text{ lb} = 0 \\ &\left(-\frac{0.9701}{0.2425}\right)\left[-0.4924T_{AB} - 0.6860T_{AC} + 0.2425N\right] = 0\end{aligned}$$

Add these to find

$$2.585T_{AB} + 3.259T_{AC} - 10 \text{ lb} = 0 \quad (\#)$$

Use the first equation to write $T_{AB}$ in terms of $T_{AC}$:

$$T_{AB} = \frac{0.5145}{0.6155}T_{AC} = 0.8359T_{AC} \quad (*)$$

Substitute the result of Eq. (*) into Eq. (#) and solve:

$$2.585(0.8359T_{AC}) + 3.259T_{AC} - 10 \text{ lb} = 0$$
$$5.419T_{AC} = 10 \text{ lb}$$
$$T_{AC} = 1.845 \text{ lb}$$

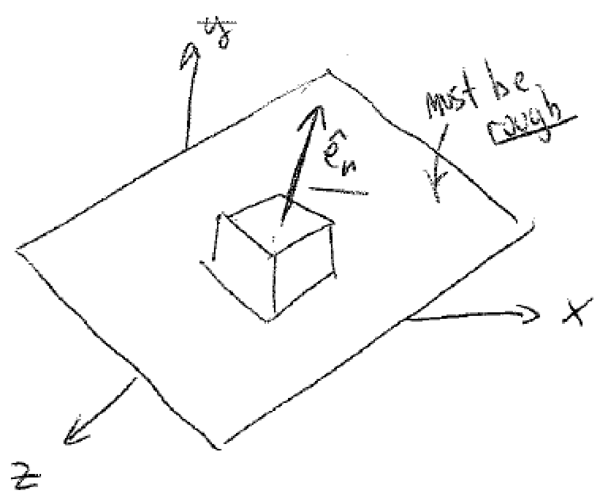
Use Eq. (*) to find

$$T_{AB} = 0.8359(1.845 \text{ lb}) = 1.542 \text{ lb}$$

$$\boxed{T_{AB} = 1.54 \text{ lb}, T_{AC} = 1.85 \text{ lb}}$$

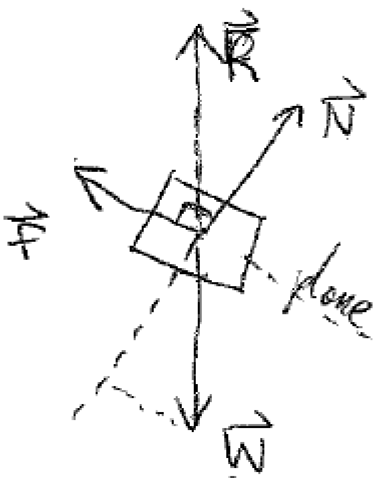
Example:

A box weighing 1000 N rests on an inclined surface with unit normal $\hat{e}_n = 0.3\hat{i} + 0.6\hat{j} + 0.742\hat{k}$. What are the normal and friction forces on the box?



Strategy: Sketch the free-body diagram. Consider equilibrium and the fact that the friction force and normal force are mutually perpendicular.

Free-body diagram (side view):




$$\sum \vec{F} = \vec{N} + \vec{f} + \vec{W} = 0$$

$$\vec{N} = N\hat{e}_n \quad (\#)$$

$$\vec{f} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$$

$$\vec{W} = (-1000 \text{ N})\,\hat{j}$$

Recognizing $\hat{e}_n \cdot \vec{f} = 0$, consider now

$$\hat{e}_n \cdot \left(\vec{N} + \vec{f} + \vec{W}\right) = \hat{e}_n \cdot \vec{N} + \cancel{\hat{e}_n \cdot \vec{f}} + \hat{e}_n \cdot \vec{W} = 0$$

$$\hat{e}_n \cdot \vec{N} = -\hat{e}_n \cdot \vec{W}$$

Substitute for $\vec{N}$ using Eq. (#):

$$\hat{e}_n \cdot \left(N\hat{e}_n\right) = N\left(\hat{e}_n \cdot \hat{e}_n\right) = N = -\hat{e}_n \cdot \vec{W} = -\left(0.3\hat{i} + 0.6\hat{j} + 0.742\hat{k}\right) \cdot \left[\left(-1000 \text{ N}\right)\hat{j}\right] = 600 \text{ N}$$

$$\boxed{\begin{aligned} &\vec{N} = N\hat{e}_n = 180\hat{i} + 360\hat{j} + 445\hat{k} \text{ (N)} \\ &\left|\vec{N}\right| = \left|N\right| = 600 \text{ N} \end{aligned}}$$

Use equilibrium to compute $\vec{f}$:

$$\begin{aligned} \vec{f} &= -\left(\vec{N} + \vec{W}\right) \\ &= -\left[180\hat{i} - 640\hat{j} + 445\hat{k} \text{ (N)}\right] \end{aligned}$$

$$\boxed{\begin{aligned} &\vec{f} = -180\hat{i} + 640\hat{j} - 445\hat{k} \text{ (N)} \\ &\left|\vec{f}\right| = \sqrt{\vec{f} \cdot \vec{f}} = 800 \text{ N} \end{aligned}}$$