Lecture 6

GEN_ENG 205-2: Engineering Analysis 2
Winter Quarter 2018
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§3.2 Two-Dimensional Force Systems, §3.3 Three-Dimensional Force Systems[1]

Acknowledgements

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Two-dimensional Force Systems

Suppose we have an object in equilibrium and the system of forces is coplanar.

$$\sum \vec{F} = \vec{0}$$

In components, this is

$$\sum \vec{F} = \left(\sum F_x\right)\hat{i} + \left(\sum F_x\right)\hat{j} = \vec{0}$$

Since a vector is zero only if each of its components is zero, we can write this as two scalar equilibrium equations:

$$\boxed{\begin{aligned}\sum F_x = 0\\ \sum F_y = 0\end{aligned}}$$

[1] Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

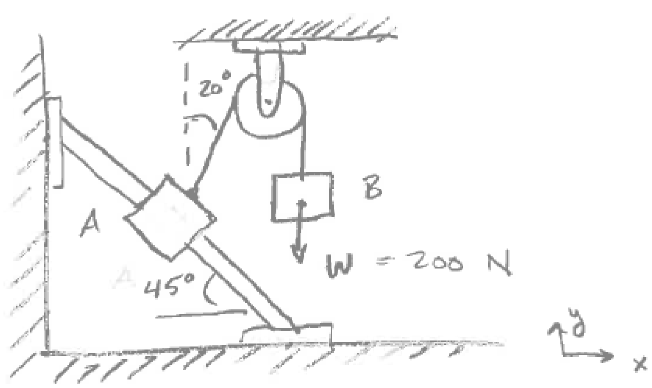
With two equations, we can find a maximum of two (scalar) unknowns.

General procedure for solving problems:

1. Select the object of interest.
2. Draw the free-body diagram such that the unknown forces are external.
3. Express all forces in component form (F_x and F_y).
4. Check that the unknowns are fewer than the number of equations.
5. Solve the equations.
6. Finish up: Does the answer make sense (e.g., magnitude and direction)? Check units and significant figures.

Go through Examples 3.1 (object resting on a surface), 3.2 (system of cables), 3.3 (system of pulleys), and 3.4 in the textbook on your own[2].

Example: Problem 3.32 from the textbook.

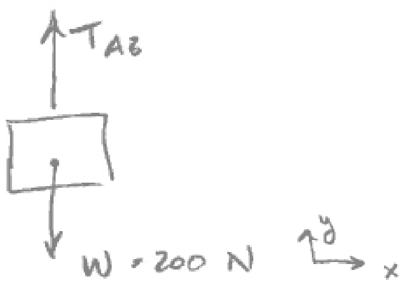


Slider *A* is in equilibrium and the bar is smooth. What is the mass of the slider?

[2] We will go through other examples to provide maximum exposure to different problems.

Strategy: Sketch free-body diagrams of Blocks A and B, consider equilibrium, and solve equations.

Step 1: Sketch free body diagram for Block B.

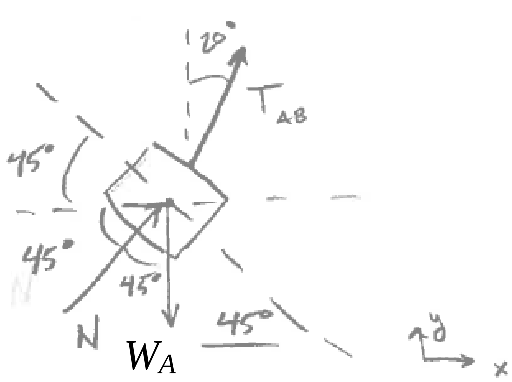


$\sum F_x = 0$: 0 = 0 (no information)

$\sum F_y = 0: \quad T_{AB} - W = 0$

Solve $\sum F_y = 0$ to find $T_{AB} = W = 200$ N

Step 2: Sketch free body diagram for Block B.



$$\sum F_x = 0: \quad N\cos 45^\circ + T_{AB}\sin 20^\circ = 0$$

$$\sum F_y = 0: \quad N\sin 45^\circ + T_{AB}\cos 20^\circ - W_A = 0$$

Solve $\sum F_x = 0$ for *N*:

$$N = -T_{AB}\frac{\sin 20^\circ}{\cos 45^\circ} = -\left(200\text{ N}\right)\frac{\sin 20^\circ}{\cos 45^\circ} = -96.738\text{ N}$$

Note the sign[3] of *N*!

Solve $\sum F_y = 0$ for W_A:

$$\begin{aligned} W &= N\sin 45^\circ + T_{AB}\cos 20^\circ \\ &= \left(-96.738\text{ N}\right)\sin 45^\circ + \left(200\text{ N}\right)\cos 30^\circ \\ &= 119.54\text{ N} \end{aligned}$$

Are we finished? No. We were asked to find the mass.

$$W_A = m_A g \quad \Rightarrow \quad m_A = \frac{W_A}{g} = \frac{\left(119.54\text{ N}\right)}{\left(9.81\,\dfrac{\text{m}}{\text{s}^2}\right)} = \frac{\left(119.54\text{ kg}\cdot\dfrac{\text{m}}{\text{s}^2}\right)}{\left(9.81\,\dfrac{\text{m}}{\text{s}^2}\right)} = 12.2\text{ kg}$$

$$\boxed{m_A = 12.2\text{ kg}}$$

Is this reasonable? Yes.

[3] The normal force and tension induced by the cable are in fact *vectors* for which we know the line of action. The scalars *N* and T_{AB} reflect the magnitude and direction along the line of action (i.e., we allow these quantities to be positive or negative). In other words, we let $\vec{N} = N\vec{n}$ and $\vec{T}_{AB} = T_{AB}\hat{e}_{AB}$, something that we will use often for 3D force systems.

General note: The unknown forces are often referred to as reactions. In this problem, *N* can be regarded as a reaction, viewing *W* as the applied force.

Three-dimensional Force Systems

Suppose we have an object in equilibrium and the system of forces is three-dimensional (i.e., not co-planar). Again, we have

$$\sum \vec{F} = \vec{0}$$

In components, this is

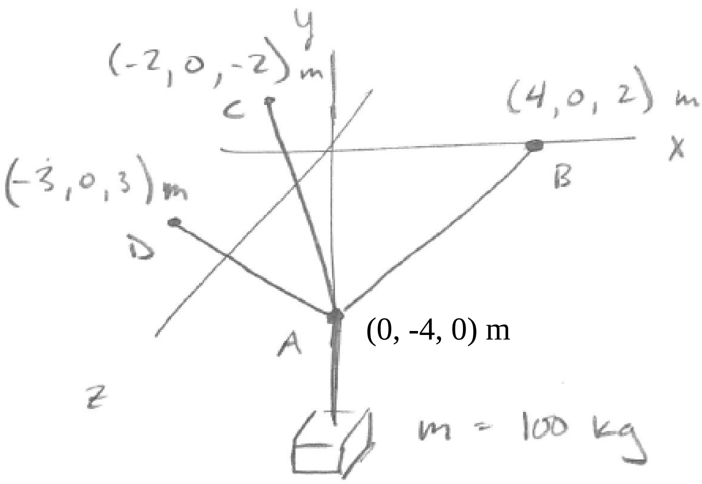
$$\sum \vec{F} = \left(\sum F_x\right)\hat{i} + \left(\sum F_x\right)\hat{j} + \left(\sum F_y\right)\hat{j} = \vec{0}$$

We can write this as three scalar equilibrium equations:

$$\boxed{\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned}}$$

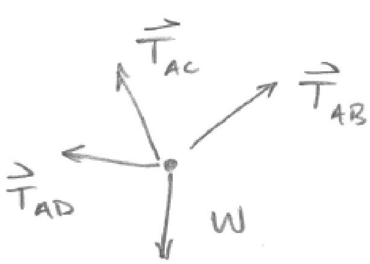
Example: Example 3.5[4] from the textbook.

A mass of 100 kg is suspended by from the ceiling by the system of cables shown. What are the tensions in cables *AB*, *AC*, and *AD*?



Strategy: Sketch the free-body diagram for the connection at point *A* and set up a system of three equilibrium equations to solve for the three unknown tensions, expressing the forces in the cables in terms of their direction (known) and magnitude (unknown).

Free-body diagram for point A:



[4] This problem relates directly to Project #1.

$$W = mg = (100\text{ kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right) = 981\text{ N}$$

$$\vec{W} = -W\hat{j} = -981\hat{j}\text{ (N)}$$

$$\vec{T}_{AB} = T_{AB}\hat{e}_{AB}$$

$$\vec{T}_{AC} = T_{AC}\hat{e}_{AC}$$

$$\vec{T}_{AD} = T_{AD}\hat{e}_{AD}$$

$$\hat{e}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\begin{aligned}\vec{r}_{AB} &= (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k} \\ &= (4-0)\hat{i} + [0-(-4)]\hat{j} + (2-0)\hat{k}\text{ (m)} \\ &= 4\hat{i} + 4\hat{j} + 2\hat{k}\text{ (m)}\end{aligned}$$

$$|\vec{r}_{AB}| = \sqrt{(4\,\text{m})^2 + (4\,\text{m})^2 + (2\,\text{m})^2} = \sqrt{36\,\text{m}^2} = 6\,\text{m}$$

$$\hat{e}_{AB} = \frac{(4\,\text{m})\hat{i} + (4\,\text{m})\hat{j} + (2\,\text{m})\hat{k}}{(6\,\text{m})} = 0.6667\hat{i} + 0.6667\hat{j} + 0.3333\hat{k}$$

Similarly,

$$\begin{aligned}\hat{e}_{AC} &= \frac{(-2-0)\hat{i} - [0-(-4)]\hat{j} + (-2-0)\hat{k}}{\sqrt{2^2+4^2+2^2}} \\ &= -0.4082\hat{i} + 0.8165\hat{j} - 0.4082\hat{k}\end{aligned}$$

$$\begin{aligned}\hat{e}_{AD} &= \frac{(-3-0)\hat{i} - [0-(-4)]\hat{j} + (3-0)\hat{k}}{\sqrt{3^2+4^2+3^2}} \\ &= -0.5144\hat{i} + 0.6856\hat{j} + 0.5144\hat{k}\end{aligned}$$

$$\vec{T}_{AB} = T_{AB}\hat{e}_{AB} = T_{AB}\left(0.6667\hat{i} + 0.6667\hat{j} + 0.3333\hat{k}\right)$$

$$\vec{T}_{AC} = T_{AC}\hat{e}_{AC} = T_{AC}\left(-0.4082\hat{i} + 0.8165\hat{j} - 0.4082\hat{k}\right)$$

$$\vec{T}_{AD} = T_{AD}\hat{e}_{AD} = T_{AD}\left(-0.5144\hat{i} + 0.6856\hat{j} + 0.5144\hat{k}\right)$$

Equilibrium[5]:

$$\sum \vec{F} = \vec{0}:$$

$$\vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} - 981\hat{j}\ \text{(N)} = \vec{0}$$

$$\begin{aligned} &T_{AB}\left(0.6667\hat{i} + 0.6667\hat{j} + 0.3333\hat{k}\right) + T_{AC}\left(-0.4082\hat{i} + 0.8165\hat{j} - 0.4082\hat{k}\right) \\ &\quad + T_{AD}\left(-0.5144\hat{i} + 0.6856\hat{j} + 0.5144\hat{k}\right) - 981\hat{j}\ \text{(N)} = \vec{0} \end{aligned}$$

$$\begin{aligned} &\left(0.6667T_{AB} - 0.4082T_{AC} - 0.5144T_{AD}\right)\hat{i} + \left(0.6667T_{AB} + 0.8165T_{AC} + 0.6856T_{AD} - 981\ \text{N}\right)\hat{j} \\ &\quad + \left(0.3333T_{AB} - 0.4082T_{AC} + 0.5144T_{AD}\right)\hat{k} = \vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k} \end{aligned}$$

Since the components sum to zero, this gives three equations:

$$\begin{aligned} 0.6667T_{AB} - 0.4082T_{AC} - 0.5144T_{AD} &= 0 \\ 0.6667T_{AB} + 0.8165T_{AC} + 0.6856T_{AD} - 981\ \text{N} &= 0 \\ 0.3333T_{AB} - 0.4082T_{AC} + 0.5144T_{AD} &= 0 \end{aligned}$$

[5] We will do this as a vector summation rather than summing components from the free-body diagram, which is generally easier for 3D.

or

$$\begin{bmatrix} 0.6667 & -0.4082 & -0.5144 \\ 0.6667 & 0.8165 & 0.6856 \\ 0.3333 & -0.4082 & 0.5144 \end{bmatrix} \begin{Bmatrix} T_{AB} \\ T_{AC} \\ T_{AD} \end{Bmatrix} = \begin{bmatrix} 0 \\ 981\text{ N} \\ 0 \end{bmatrix}$$

Solving these equations gives

$$\boxed{T_{AB} = 519\text{ N},\ \ T_{AC} = 636\text{ N},\ \ T_{AD} = 168\text{ N}}$$