

Lecture 2

GEN_ENG 205-2: Engineering Analysis 2

Winter Quarter 2018

Prof. James P. Hambleton

Chapter 2: §2.1 Scalars and Vectors¹; Chapter 2: §2.2 Components in 2D; §2.3
Components in 3D

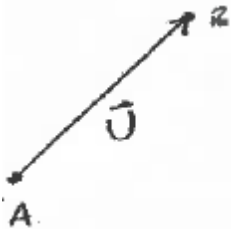
Acknowledgements

Portions of these lecture notes are taken from those of Prof. Jeff Thomas.

Vectors²

A scalar is completely described by a single number (e.g., time and temperature).

A vector is defined by a magnitude and direction (e.g., velocity).



The magnitude of \vec{U} is denoted by $|\vec{U}|$, which gives the length of the vector (distance from A to B).

¹ Bedford, A., & Fowler, W. (2008). *Engineering Mechanics: Statics and Dynamics* (5th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.

² Many quantities cannot be described by a single number but can be described by other quantities (e.g., velocity or position in 2D or 3D space). This is where we need vectors. Higher order quantities (tensors) also exist.

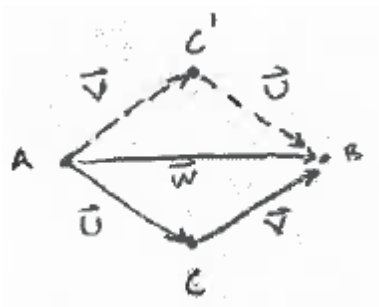
Vector Addition

Vectors are added (graphically) by placing them head to tail.

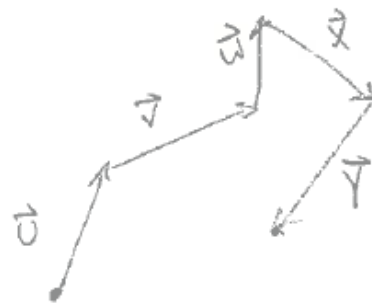


$$\vec{W} = \vec{U} + \vec{V}$$

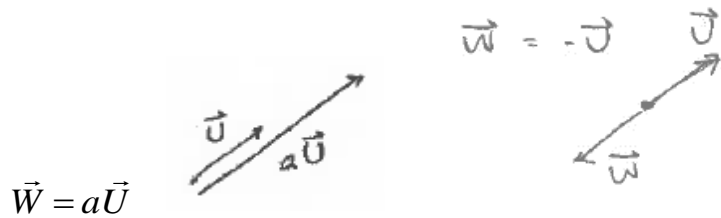
Vector addition is commutative and associative, meaning the order does not matter (parallelogram rule).



$$\vec{W} = \vec{U} + \vec{V} = \vec{V} + \vec{U}$$



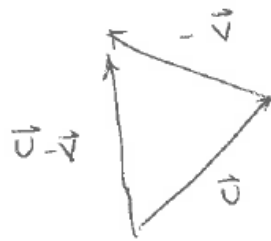
The product of a scalar and a vector is also a vector.



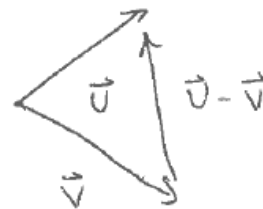
Note that $|\vec{U}| \geq 0$! $-\vec{U}$ points in opposite direction (no such thing as negative magnitude).

Vector Subtraction

$$\vec{W} = \vec{U} - \vec{V} = \vec{U} + (-\vec{V})$$

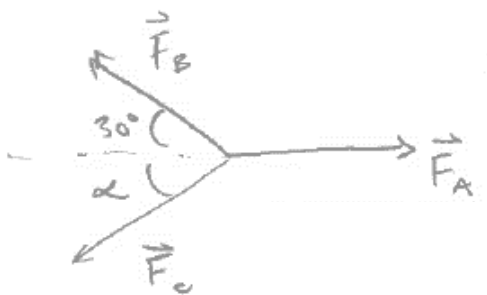


or



Example

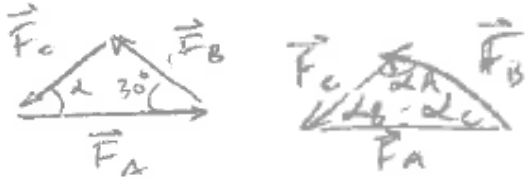
Given:



$$\vec{F}_A + \vec{F}_B + \vec{F}_C = 0, \quad F_A = 100 \text{ N}, \quad F_B = 80 \text{ N}$$

Find: $|\vec{F}_C|, \alpha$

Solution:



Law of cosines (Appendix A.2)

$$\begin{aligned} |\vec{F}_C|^2 &= |\vec{F}_A|^2 + |\vec{F}_B|^2 - 2|\vec{F}_A||\vec{F}_B|\cos\alpha_c \\ &= (100\text{ N})^2 + (80\text{ N})^2 - 2(100\text{ N})(80\text{ N})\cos 30^\circ = 2.544 \times 10^3 \text{ N} \end{aligned}$$

$$|\vec{F}_C| = 50.4 \text{ N}$$

Law of sines (Appendix A.2)

$$\frac{\sin\alpha_B}{|\vec{F}_B|} = \frac{\sin\alpha_c}{|\vec{F}_C|}$$

$$\sin\alpha_B = \sin\alpha_c \frac{|\vec{F}_B|}{|\vec{F}_C|} = \sin 30^\circ \frac{(80\text{ N})}{(50.43\text{ N})} = 0.7932$$

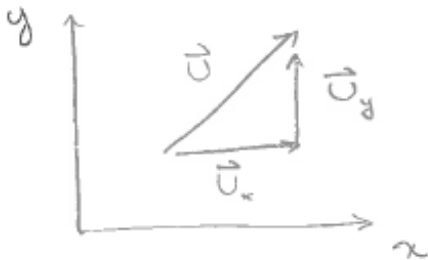
$$\alpha = \alpha_B = \sin^{-1}(0.7932) = 0.916 \text{ rad or } 52.5^\circ$$

Unit Vectors

A unit vector³ is simply a vector whose magnitude is 1.

$$\hat{e} = \frac{\vec{U}}{|\vec{U}|}$$

Observe the one can construct \vec{U} knowing its magnitude $|\vec{U}|$ and its direction \hat{e} .

Cartesian Components in Two Dimensions (2D)

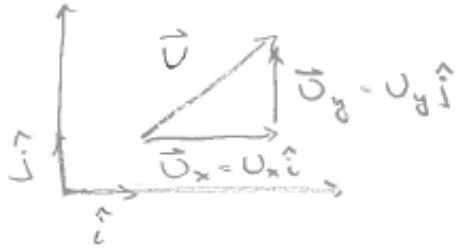
$$\vec{U} = \vec{U}_x + \vec{U}_y$$

Define unit vectors \hat{i} and \hat{j} aligned with x and y axis and observe $\vec{U}_x = |\vec{U}_x| \hat{i}$ and

$$\vec{U}_y = |\vec{U}_y| \hat{j}$$

Define $U_x = |\vec{U}_x|$ and $U_y = |\vec{U}_y|$

³ Dividing any vector by its magnitude yields a unit vector that has the same direction. Unit vectors will be essential for defining vectors in terms of their components.



$$\vec{U} = U_x \hat{i} + U_y \hat{j}$$

Pythagorean theorem:

$$\begin{aligned} |\vec{U}|^2 &= |\vec{U}_x|^2 + |\vec{U}_y|^2 \\ &= U_x^2 + U_y^2 \end{aligned}$$

$$|\vec{U}| = \sqrt{U_x^2 + U_y^2}$$

Observe that we have now defined a single (vector) quantity in terms of two scalars.

Vector Addition

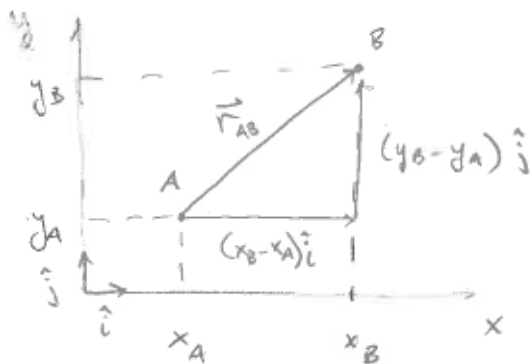
$$\begin{aligned} \vec{U} + \vec{V} &= (U_x \hat{i} + U_y \hat{j}) + (V_x \hat{i} + V_y \hat{j}) \\ &= (U_x + V_x) \hat{i} + (U_y + V_y) \hat{j} \end{aligned}$$

Multiplication by a scalar

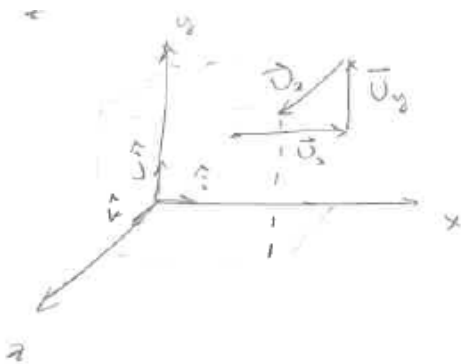
$$\begin{aligned} a\vec{U} &= a(U_x \hat{i} + U_y \hat{j}) \\ &= aU_x \hat{i} + aU_y \hat{j} \end{aligned}$$

Vector subtraction⁴

$$\begin{aligned}\vec{U} - \vec{V} &= (U_x \hat{i} + U_y \hat{j}) - (V_x \hat{i} + V_y \hat{j}) \\ &= (U_x - V_x) \hat{i} + (U_y - V_y) \hat{j}\end{aligned}$$

Position vectors

$$\vec{r}_{AB} = (x_B - x_A) \hat{i} + (y_B - y_A) \hat{j}$$

Cartesian Components in Three Dimensions (3D)

Note: right-handed coordinate system

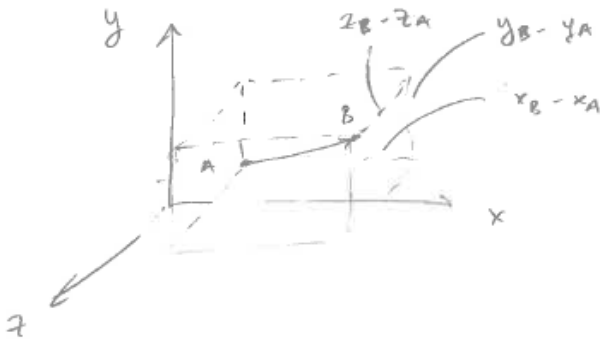
$$\vec{U} = \vec{U}_x + \vec{U}_y + \vec{U}_z$$

⁴ Can skip for time.

$$\vec{U} = U_x \hat{i} + U_y \hat{j} + U_z \hat{k}$$

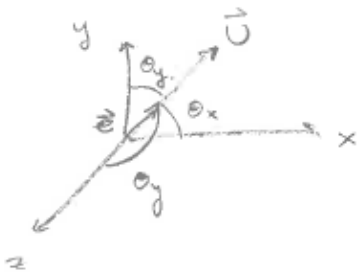
$$|\vec{U}| = \sqrt{U_x^2 + U_y^2 + U_z^2}$$
⁵

Position vectors



$$\vec{r}_{AB} = (x_B - x_A) \hat{i} + (y_B - y_A) \hat{j} + (z_B - z_A) \hat{k}$$

Direction cosines



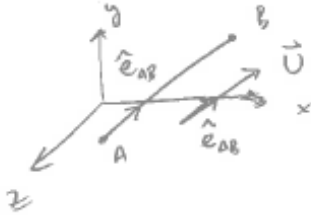
$$\vec{U} = |\vec{U}| \hat{e}, \quad \hat{e} = e_x \hat{i} + e_y \hat{j} + e_z \hat{k}$$

$$e_x = \cos \theta_x$$

$$e_y = \cos \theta_y$$

$$e_z = \cos \theta_z$$

⁵ See derivation in textbook. This involves repeated application of the Pythagorean theorem.

Components of a vector parallel to a line

\vec{U} parallel to line passing through points A and B

$$\vec{U} = |\vec{U}| \hat{e}_{AB}, \text{ where } \hat{e}_{AB} = \frac{\hat{r}_{AB}}{|\hat{r}_{AB}|}$$