

Earthquake Supercycles and Long-Term Fault Memory

Leah Salditch^{a,b,}, Seth Stein^{a,b}, James Neely^a, Bruce D. Spencer^{b,c}, Edward M. Brooks^{a,b},
Amotz Agnon^d, Mian Liu^e*

^aDepartment of Earth and Planetary Sciences, Northwestern University, 2145 Sheridan Rd., Evanston, IL
60208 U.S.A.

^bInstitute for Policy Research, Northwestern University, 2040 Sheridan Rd., Evanston, IL 60208 U.S.A.

^cDepartment of Statistics, Northwestern University, 2006 Sheridan Rd., Evanston, IL 60208 U.S.A.

^dInstitute of Earth Sciences, Hebrew University, Sderot Magnes, Jerusalem 9190401, Israel

^eDepartment of Geological Sciences, University of Missouri, 101 Geological Sciences Bldg., Columbia,
MO 65211, U.S.A

*Corresponding Author, Leah@earth.northwestern.edu

Abstract

Long records often show large earthquakes occurring in supercycles, sequences of temporal clusters of seismicity, cumulative displacement, and cumulative strain release separated by intervals of lower levels of these measures. Supercycles and associated earthquake clusters are challenging to characterize via the traditionally used aperiodicity, which measures the extent that a sequence differs from perfectly periodic. Supercycles are not well described by commonly used models of earthquake recurrence. In the Poisson model, the probability of a large earthquake is constant with time, so the fault has no memory. In a seismic cycle/renewal model, the probability is quasi-periodic, dropping to zero after a large earthquake, then increasing with time, so the probability of a large earthquake depends only on the time since the past one, and the fault has only “short-term memory.” We describe supercycles with a Long-Term Fault Memory (LTFM) model, where the probability of a large earthquake reflects the accumulated strain rather than elapsed time. The probability increases with accumulated strain (and time) until an earthquake happens, after which it decreases, but not necessarily to zero. Hence, the probability of an earthquake can depend on the earthquake history over multiple prior cycles. We use LTFM to simulate paleoseismic records from plate boundaries and intraplate areas. Simulations suggest that over timescales corresponding to the duration of paleoseismic records, the distribution of

32 earthquake recurrence times can appear strongly periodic, weakly periodic, Poissonian, or bursty.
33 Thus, a given paleoseismic window may not capture long-term trends in seismicity. This effect is
34 significant for earthquake hazard assessment because whether an earthquake history is assumed
35 to contain clusters can be more important than the probability density function chosen to describe
36 the recurrence times. In such cases, probability estimates of the next earthquake will depend
37 crucially on whether the cluster is treated as ongoing or over.

38

39

Keywords

40 Earthquake; Supercycle; Cluster; Aperiodicity.

41

42

1. Introduction

43 Since the 1906 San Francisco earthquake, the dominant paradigm in earthquake
44 seismology has been the earthquake cycle, in which strain accumulates between large
45 earthquakes due to interseismic motion between the two sides of a locked fault and is released by
46 coseismic slip on the fault when an earthquake occurs (Reid, 1910). Over time, this process
47 should give rise to approximately periodic earthquakes and a steady accumulation of cumulative
48 displacement (Figure 1a). The fact that earthquake sequences are only approximately periodic
49 prompted a refinement of the model with "time-predictable" recurrence in which a specific strain
50 level must accumulate for an earthquake, but the strain release in the earthquake is variable
51 (Shimazaki and Nakata, 1980).

52 However, long earthquake records often show more complex behavior (Figure 1b).

53 Wallace (1987) found that faults and groups of faults in the Western U.S.'s Great Basin often

54 showed "*grouping, ...a series of displacement events, each being followed by a period of*

55 *quiescence. Slip rates during a group of events along a segment of fault, thus, could be*

56 *considerably greater than the long-term average slip rate. During quiescent periods, the slip*

57 *rate would be lower than the average rate and might even be zero... Additionally, if grouping is*

58 *real, the concept that accumulated elastic strain is released at some regular interval by a single*

59 *displacement event in a seismic cycle should be reexamined. Perhaps strain that has*

60 *accumulated at a more or less constant rate is released in a stuttering, spasmodic manner in a*

61 *group of displacement events."* Subsequent investigation (Friedrich et al., 2003) supported this

62 analysis, finding that "*seismic strain release may be clustered on the 10-kyr timescale... with*

63 *comparatively low, uniform strain accumulation rates on the 100-kyr timescale."* They suggested

64 calling the conventional earthquake cycles "Reid-type" behavior and the longer period variations
65 "Wallace-type" behavior.

66 Wallace (1987) further noted that if this behavior is *"common, as these preliminary*
67 *analyses suggest, care must be exercised in evaluating seismic hazard potentials. It is crucial to*
68 *determine the timing and distribution of individual faulting events because long-term average*
69 *slip rates may give grossly incorrect assessments of the hazard potential."* For example, the
70 estimated probability of a future large earthquake can depend crucially on whether a cluster is
71 treated as ongoing or over.

72 Such variations in earthquake behavior on timescales longer than individual cycles are
73 often termed "supercycles," following Sieh et al.'s (2008) observation from corals near the
74 Sumatra trench that *"because each of the three past episodes of emergence consists of two or*
75 *more discrete events, we refer to the broad periods of strain accumulation and relief as*
76 *supercycles rather than merely cycles"* and Goldfinger et al.'s (2013) analysis showing that large
77 Cascadia subduction zone earthquakes reflect *"strain supercycles that transcend individual*
78 *seismic cycling."*

79 Conceptually, the history of strain accumulation and release is the underlying process that
80 gives rise to patterns in the resulting earthquakes and cumulative displacement. In the schematic
81 example of Figure 1b, supercycles appear as patterns longer than individual earthquake cycles in
82 the earthquake history, cumulative displacement, and cumulative strain records. The fullest
83 picture is given by the strain record. This infers the strain by combining data about strain release
84 via slip in earthquakes over time with the interseismic strain accumulation inferred from the slip
85 between earthquakes taken from present-day geodetic, long-term geological, or other data. The
86 cumulative displacement record shows the dates of earthquakes and the coseismic slip in each,
87 whereas the earthquake history gives only the dates. The displacement record can be viewed as
88 the time derivative of the strain record, and the earthquake history can be viewed as the time
89 derivative of the displacement record, with each differentiation involving a loss of information.
90 Conversely, constructing a displacement record requires supplementing an earthquake history
91 with coseismic slip data, and constructing the strain record then involves also including data or
92 assumptions about the interseismic strain accumulation. Hence the earthquake history has the
93 least uncertainty, and the displacement and strain records have progressively larger uncertainties.

94 Supercycles are difficult to define precisely. One approach is to use major minima in the
95 cumulative strain, which often mark the beginning of intervals during which few large
96 earthquakes and hence little cumulative slip occurs. However, identifying major minima is often
97 challenging and non-unique, especially given the assumptions needed to construct a strain
98 history. Moreover, because data about interseismic strain accumulation and the slip in individual
99 earthquakes are often unavailable, supercycles most often are inferred from an earthquake
100 history that shows temporal clusters of seismicity, separated by intervals of lower seismicity or
101 gaps without large earthquakes.

102 In this paper we use the term "supercycles" broadly, to describe long-term variability
103 shown by aspects of the earthquake record that are difficult to reconcile with commonly used
104 models of earthquake recurrence. The observation of supercycles, especially at plate boundaries
105 and in plate boundary zones, is intriguing because plate boundaries are being loaded by steady
106 plate motion.

107 We first review some proposed examples of supercycles on various faults, and show that
108 these arise in the full range of tectonic environments - at plate boundaries, within plate boundary
109 zones, and in plate interiors. We discuss the fact that supercycles and the sometimes-resulting
110 earthquake clusters are not described by commonly used models of earthquake recurrence. We
111 then introduce a model of Long-Term Fault Memory (LTFM), in which the probability of a large
112 earthquake reflects the accumulated strain, and use it to explore many aspects of supercycles.
113 Finally, we discuss the challenges supercycles pose for earthquake hazard assessment.

114

115

2. Examples of Supercycles

116 Supercycles and/or clustering have been observed in many tectonic environments (Figure
117 2). The best data come from earthquake histories at plate boundaries, because the relatively rapid
118 plate motion (typically > 5 mm/yr) gives shorter and hence easier-to-observe cycles. In some
119 cases, the slip and strain history also show evidence for supercycles.

120 Weldon et al. (2004) used the dates and offset in paleoearthquakes since 500 CE across
121 the San Andreas fault near Wrightwood, California, together with the interseismic slip rate
122 observed from present-day geodesy and long-term geological rates, to reconstruct the history of
123 strain accumulation and release (Figure 2a). They argue that "*it is hard to escape the conclusion*
124 *that strain accumulated over many earthquake cycles was responsible for the flurry of large slip*

125 *events.*" Nearby, at Pallett Creek, Sieh et al. (1989) find that paleoearthquakes occurred in
126 clusters within which they were *"separated by periods of several decades, but the clusters are*
127 *separated by dormant periods of two to three centuries."* To the south, where the San Jacinto
128 fault takes up some of the motion between the Pacific and North America plates, Rockwell et al.
129 (2015) find that *"for much of the past 4,000 years the fault ruptured in a quasi-periodic fashion.*
130 *In the past 1,000 years, in contrast, a flurry or cluster of four earthquakes occurred in a 150-*
131 *year period, and the overall recurrence interval is much shorter."*

132 Sieh et al. (2008) analyzed relative sea level changes recorded by corals from Sumatra,
133 which show interseismic subsidence and coseismic uplift (Figure 2b). They infer that *"this 700-*
134 *kilometer-long section of the Sunda megathrust has generated broadly similar sequences of great*
135 *earthquakes about every two centuries for at least the past 700 years... Because each of the*
136 *three past episodes of emergence consists of two or more discrete events, we refer to the broad*
137 *periods of strain accumulation and relief as supercycles rather than merely cycles."* To the
138 north along the subduction zone, Rubin et al. (2017) studied a 4500 year sequence of at least 11
139 tsunami deposits and find that *"the average time period between tsunamis is about 450 years*
140 *with intervals ranging from a long, dormant period of over 2,000 years, to multiple tsunamis*
141 *within the span of a century... these variable recurrence intervals suggest that long dormant*
142 *periods may follow Sunda megathrust ruptures as large as that of the 2004 Indian Ocean*
143 *tsunami."*

144 The dates and volumes of turbidite deposits, assumed to have been generated by great
145 earthquakes on the Cascadia megathrust (Adams, 1990), show evidence for supercycles. Using
146 these to infer the history of strain energy accumulation and release (Figure 2c), Goldfinger et al.
147 (2013) find that *"the resulting sawtooth pattern reveals what we interpret as a complex pattern*
148 *of long-term energy cycling on the Cascadia megathrust... Overall, what is suggested by this*
149 *pattern is that some events release less energy, whereas others release more energy than*
150 *available from plate convergence (slip deficit) and may have borrowed stored energy from*
151 *previous cycles."* Although an additional event has been identified (Goldfinger et al., 2017), the
152 inferred strain energy history would not be substantially altered. Kelsey et al.'s (2005) analysis of
153 coastal deposits that record local tsunamis and seismic shaking finds that *"over the 4600 yr*
154 *period when Bradley Lake was an optimum tsunami recorder, tsunamis from Cascadia plate-*
155 *boundary earthquakes came in clusters."*

156 A somewhat different style of supercycles has been proposed for the Japan Trench off
157 Tohoku (Figure 2d). Satake (2015) proposed that *"The 2011 Tohoku earthquake source includes*
158 *the Miyagi-oki region, where $M \sim 7.5$ earthquakes repeated with average interval of 37 years.*
159 *The typical slip of such large earthquakes is approximately 2 m, meaning that the cumulative*
160 *coseismic slip is about 6 m per century. Because the subduction rate of the Pacific plate is*
161 *approximately 8 m per century, 2 m slips may remain unreleased. Such a difference was*
162 *previously interpreted as aseismic slip, but can be accumulated at the plate interface and cause a*
163 *large coseismic slip of approximately 15 m with a recurrence interval of approximately 700*
164 *years... Such [a] supercycle model can explain the unusually large slip of the 2011 Tohoku*
165 *earthquake. The term 'supercycle' was first used for a seismic cycle consisting of a series of*
166 *large events, but often used for long-term cycle imposed on shorter cycles ('superimposed*
167 *cycle'). "*

168 The Sumatra and Tohoku records have interesting similarities and differences. In both,
169 supercycles reflect infrequent events that have slip much greater than typical events. However,
170 the Sumatra earthquake history has long gaps separating clusters, whereas for Tohoku smaller
171 earthquakes occur frequently between the largest events, so the supercycles in the strain record
172 do not appear in the earthquake history as gaps and clusters. It is worth noting that a Sumatra-
173 type record could result if the detection limit in a paleoseismic record is too high to record the
174 smaller events, or a Tohoku-type record could result if the recent rate of smaller events could not
175 be extrapolated into the past.

176 In other areas, supercycles have been inferred from the earthquake history, even though
177 the strain history requires data on the slip in individual events. Agnon's (2014) analysis of a long
178 record of seismites, sediment records of earthquake shaking (Marco et al., 1996), along the Dead
179 Sea Transform in Israel (Figure 2e) finds *"a pattern of long quiescence periods between quasi-*
180 *periodic clusters. During each cluster of seismicity the recurrence interval is quite uniform,*
181 *varying among clusters between 200 and 1,400 years. Quiescence periods may linger 3–10,000*
182 *years."* Further north on this transform, Wechsler et al. (2014) find that *"the interevent time of*
183 *surface-rupturing earthquakes varies by a factor of two to four during the past 2 ka at our site,*
184 *and the fault's behavior is not time predictable."*

185 Supercycles and/or clustering have also been identified in plate boundary zones, where
186 diffuse deformation is spread over multiple faults with long-term slip rates typically slower than

187 on the primary plate boundary faults, and in continental interiors, which typically deform at < 1
188 mm/yr. As noted earlier, paleoseismic data from faults and groups of faults in the Western U.S.'s
189 Great Basin (Wallace, 1987), part of the broad boundary zone that accommodates motion
190 between the Pacific and North American plates, often show "*clustered strain release and*
191 *uniform, low strain accumulation*" (Friedrich et al., 2003), shown schematically in Figure 2f.

192 Topographic data within the Australian plate, where erosion is very slow, provide some
193 of the best evidence available of how continental intraplate faults slip over time, shown
194 schematically in Figure 2g. Clark et al. (2012) found that "*a common characteristic of*
195 *morphogenic earthquake occurrence in Australia appears to be temporal clustering. Periods of*
196 *earthquake activity comprising a finite number of large events are separated by much longer*
197 *periods of seismic quiescence, at the scale of a single fault and of proximal faults. In several*
198 *instances there is evidence for deformation at scales of several hundred kilometers switching on*
199 *and off over the last several million years.*" As result, "*assigning an 'active/inactive' label to a*
200 *fault in a slowly deforming area based upon the occurrence (or non-occurrence) of an event in*
201 *the last few thousands to tens of thousands of years is not a useful indicator of future seismic*
202 *potential*" (Clark et al., 2011) and "*it is debatable whether a 'recurrence interval' on individual*
203 *faults applies*" (Clark, 2003).

204 These examples illustrate that long-term variability in earthquake behavior is a common
205 effect, although the specifics vary between different areas. Hence in this paper, we take the view
206 that observations of clustering likely reflect supercycles.

207

208 **3. Earthquake recurrence models**

209 The most easily studied aspect of supercycles is that they often - but need not always -
210 cause variability in earthquake recurrence interval times, notably temporal clusters (Figure 2),
211 which have important consequences for hazard estimation. As a result, many studies focus on
212 possible clusters in a fault's earthquake history and their implications for the recurrence of future
213 large earthquakes.

214 Neither of the commonly used classes of models for the recurrence of large earthquakes
215 (Stein and Wyssession, 2009) includes the possible effect of supercycles. The models are posed in
216 terms of the conditional probability of an earthquake in a time period, based on a conceptual
217 model of earthquake recurrence. The parameters for an area are inferred from its history of large

218 earthquakes and the rate of smaller earthquakes. The models do not predict actual event timing,
219 due to their stochastic nature.

220 One model treats earthquake occurrence as a Poisson process, in which the probability of
221 a large earthquake is constant with time (Figure 3a). This probability depends on the mean
222 recurrence interval μ , such that the probability of at least one event in a time interval t that is
223 short compared to μ is t/μ . In this model the occurrence of a large earthquake does not reduce the
224 probability of another. Hence the fault has no “memory,” the dates of previous earthquakes have
225 no effect on when the next will occur, and any clusters resulting from short intervals between
226 events arise purely by chance. As the earthquake record's length increases, the standard
227 deviation of the recurrence intervals approaches the mean. Equality of the mean and standard
228 deviation of inter-event times is a property of a Poisson process, but - as shown later in the paper
229 - other stochastic processes can also have this property. Because the Poisson model is the
230 simplest recurrence model, it is traditionally used in earthquake hazard modeling and provides a
231 null hypothesis against which other models can be tested (Rundle and Jackson, 1977; Smalley et
232 al., 1987; Kagan and Jackson, 1991; Michael, 1997; Biasi et al., 2002).

233 An alternative class of probability models is based on the concept of an earthquake cycle
234 (Figure 1), in which strain accumulates between large earthquakes and is completely released
235 when one occurs (Reid, 1910; Savage and Burford, 1973; Sykes and Nishenko, 1984; Matthews
236 et al., 2002; Field et al., 2015). In these models, the probability of a large earthquake increases
237 with time until one occurs, at which point the probability drops to zero and the cycle begins
238 again (Figure 3b). This assumption corresponds to the fault releasing all the strain accumulated
239 on it in each cycle, so strain would not accumulate on timescales longer than individual cycles.

240 The length of time between earthquakes is described by one of a number of probability
241 distributions (Gaussian, lognormal, Weibull, Brownian passage, etc.) for the recurrence times.
242 The fault “remembers” only the last event, when the probability was renewed - reset to zero - so
243 recurrence times in successive cycles are independent. Because the probability of a large
244 earthquake depends only on the time since the past one, the fault has only “short-term memory.”
245 Renewal models are increasingly used in earthquake hazard analysis (WGCEP, 2003). The
246 probability distributions describing the recurrence intervals are peaked around the average
247 expected interval, so much longer or shorter intervals are rare, and earthquakes should occur

248 quasi-periodically rather than in clusters. Thus as an earthquake record length increases, the
249 standard deviation of the observed recurrence times should become small relative to their mean.

250 Hence clusters in an earthquake record could have various causes, each of which is likely
251 to apply in some cases. First, they could be apparent clusters, artifacts of the limits of the
252 paleoseismic record such as missing events or errors in earthquake dating (Weldon et al., 2005;
253 Akciz et al., 2010). Second, if recurrence is described by Poisson or earthquake cycle models,
254 clusters could result by chance when short recurrence intervals arise. Third, clusters could result
255 from interactions between nearby faults or fault segments (Ward, 1992; Goes, 1996; Rundle et
256 al., 2006; Dolan et al., 2016).

257 However, the fact that strain accumulation and/or clusters are observed on many fault
258 systems has led to proposals that they are, at least in part, a real effect due to intrinsic properties
259 of the faulting process (Ben-Zion et al., 1999). Hence in this paper, we take the view that
260 observations of clustering likely reflect supercycles. We thus explore the possibility that faults
261 have "long-term memory," such that the occurrence of large earthquakes depends on earthquake
262 history over multiple previous earthquake cycles (Figure 3c).

263 Faults having long-term memory would have important consequences. Weldon et al.
264 (2004) point out that *"resetting of the clock during each earthquake not only is conceptually*
265 *important but also forms the practical basis for all earthquake forecasting because earthquake*
266 *recurrence is statistically modeled as a renewal process (Cornell and Winterstein, 1988). In a*
267 *renewal process, intervals between earthquakes must be unrelated so their variability can be*
268 *expressed by (and conditional probabilities calculated from) independent random variables.*
269 *Thus, if the next earthquake depends upon the strain history prior to that earthquake cycle, both*
270 *our understanding of Earth and our forecasts of earthquake hazard must be modified... there can*
271 *be little doubt that the simple renewal model of an elastic rebound driven seismic cycle will need*
272 *to be expanded to accommodate variations that span multiple seismic cycles."*

273

274

4. Characterizing Earthquake Sequences

275

4.1 Aperiodicity

276

277

278

In discussing long-term fault memory, it is useful to consider how earthquake sequences are characterized. A common characterization uses the aperiodicity, which measures the extent that a sequence differs from perfectly periodic. Aperiodicity, also termed the coefficient of

279 variation (CV), is defined by $\alpha = \sigma/\mu$ where μ is the mean of the recurrence intervals (interevent
 280 times) and σ is their standard deviation (Kagan and Jackson, 1991; Goes, 1996; Vere-Jones
 281 1970). An aperiodicity of zero corresponds to a perfectly periodic sequence, because $\sigma = 0$. An
 282 aperiodicity of one could correspond to a sequence produced by an ideal Poisson process with σ
 283 $= \mu$ but could also arise from other stochastic processes. $\alpha > 1$ corresponds to a "bursty"
 284 sequence that is so strongly clustered that $\sigma > \mu$. Because the entire range between perfectly
 285 periodic and perfectly Poissonian is termed "quasiperiodic," we divide it into the portion with α
 286 < 0.5 as "strongly periodic" or "weakly aperiodic" - closer to purely periodic than purely
 287 Poissonian - and that with $1 > \alpha > 0.5$ as "weakly periodic" or "strongly aperiodic" - closer to
 288 purely Poissonian than purely periodic (Figure 4). Although sequences with $\alpha > 1$ are often
 289 termed "clustered," we use the term "bursty" because sequences with $\alpha < 1$ can be quite
 290 clustered, as discussed shortly.

291 A related characterization uses the burstiness parameter

$$292 \quad B = (\alpha - 1) / (\alpha + 1) = (\sigma - \mu) / (\sigma + \mu)$$

293 (Goh and Barabasi, 2008). An ideal periodic sequence has $B = -1$, a perfectly Poisson sequence
 294 has $B = 0$, and bursty sequences have $0 < B < 1$. Goh and Barabasi (2008) also characterize
 295 sequences by a memory parameter

$$296 \quad M = \sum_{i=1}^{N-1} (\tau_i - \mu_1)(\tau_{i+1} - \mu_2) / \sigma_1 \sigma_2$$

297 where N is the number of recurrence intervals τ_i , μ_1 and σ_1 are the mean and standard deviations
 298 of τ_i ($i = 1, 2, \dots, N-1$), and μ_2 and σ_2 are the mean and standard deviations of τ_{i+1} ($i = 1, 2, \dots, N-$
 299 1). M ranges from -1 to 1 , with $M > 0$ when short interevent times are generally followed by
 300 short ones, and long interevent times are generally followed by long ones. $M < 0$ when short
 301 interevent times are generally followed by long ones, and vice versa. These arise because M is a
 302 normalized form of the autocorrelation of lag one, i.e. the crosscorrelation between the series of
 303 interevent times and that series shifted by one.

304 Figure 4 shows the aperiodicities for the earthquake sequences in Figure 2. The
 305 Wrightwood and Cascadia (Figures 2a and 2c) sequences have $\alpha = 0.47$ and 0.51 , so the time
 306 series alone do not indicate the supercycle behavior shown by the strain records. In contrast, the
 307 Sumatra and Dead Sea transform (Figures 2b and 2e) sequences have $\alpha = 1.05$ and 1.6 ,
 308 indicating the supercycle behavior. The Great Basin and Australia sequences (Figures 2f and 2g)

309 were described schematically without specific dates, so the aperiodicity illustrated is also
310 schematic. Also shown is the global result from Goes (1996), who compiled 52 earthquake
311 sequences from the San Andreas fault and the Middle America, Alaska, Chile, and Japan
312 trenches. She found aperiodicities varying from 0.0 to 1.7, with "a large average aperiodicity" of
313 0.72 ± 0.36 that she interpreted as showing that earthquake recurrence is more irregular "than
314 often assumed in hazard analysis."

315 These examples illustrate some of the issues in using aperiodicity to characterize
316 sequences:

317 i) *Sequences with the same aperiodicity can be quite different.* Because the aperiodicity
318 depends only on the mean and standard deviation of the interevent times, it does not depend on
319 the order of events. Thus quite different sequences can have the same aperiodicity (Cowie et al.,
320 2012). Figure 5a shows a sequence of paleoearthquakes composed of clusters of events several
321 decades apart, separated by gaps of two to three centuries. The sequence has $\alpha = 0.79$, showing
322 strong aperiodicity. Grouping the short-interval events together (Figure 5b) does not change α ,
323 but we would probably view the sequence as showing a change from longer recurrence times in
324 the past to more recent short recurrence times. The memory parameter illustrates the difference,
325 in that the more clustered sequence has a negative value, $M = -0.28$, whereas the grouped
326 sequence has $M = 0.70$. This difference between the two sequences can also be seen in the
327 interevent time plots shown to the right of each sequence. In these, major gaps appear as
328 interevent times longer than the mean, which is shown by a horizontal line. In the first sequence,
329 short and long intervals generally alternate, giving clusters and negative values of M . In the
330 second sequence, short and long intervals are grouped, giving a positive memory.

331 ii) *Sequences with "quasiperiodic" aperiodicity can be quite clustered.* Earthquake
332 sequences that we would consider clustered can fall below the nominal burstiness criterion of α
333 > 1 . Figure 6b shows that lengthening the three major gaps in Figure 6a by 100 years increases
334 the aperiodicity from 0.79 to 0.92, making the clustering stronger and the weak periodicity even
335 weaker. Lengthening the gaps by 300 years (Figure 6c) increases the aperiodicity to 1.08. In all
336 three panels we assume that observations begin at the earliest observed event (at the right side of
337 the time axis showing years before present), so no gap is observed prior to the earliest event.
338 This example illustrates that a sequence must be very strongly clustered to be bursty.

339 iii) *Sequences with aperiodicity close to 1 need not result from a Poisson process.*
340 Earthquake records with aperiodicity close to 1 could resemble those that would be generated by
341 a Poisson process. However, other stochastic processes, including the Long-Term Fault Memory
342 process discussed later in this paper, can also generate earthquake records with interevent times
343 whose mean and standard deviation are similar. Hence given the evidence in some areas of an
344 underlying process involving strain supercycles, we think it useful to consider such sequences as
345 clustered in many senses. In particular, considering clustering in such cases means that estimates
346 of the probability that the next earthquake will occur within a given time window will depend
347 crucially on whether the cluster is treated as ongoing or over.

348 iv) *Aperiodicity can vary within an earthquake record.* In particular, it is likely to be
349 underestimated by short records. Because a short record is likely to contain events with
350 recurrence times shorter than the mean of a longer record, shorter sequences underestimate
351 aperiodicity (Ellsworth et al., 1999; Mucciarelli, 2007). This effect is seen in both synthetic
352 catalogs (Ward, 1992) and earthquake records (Goes, 1996). Parsons (2008a) used Monte Carlo
353 simulations to estimate the parameters of a parent distribution of recurrence times most likely to
354 yield an observed time series. For example, an observed 1800-year-long earthquake record on
355 the South Hayward fault with mean recurrence of 180 years and aperiodicity 0.48 is most likely
356 to have arisen from a parent distribution with mean recurrence of 210 years and aperiodicity 0.6
357 (Parsons, 2008b).

358

359

4.2 Cluster Analysis

360 Another way to characterize earthquake sequences is through clustering. The statistical
361 literature provides several criteria for defining a cluster and how many exist in a sequence.
362 Categorizing clusters could facilitate definition of a supercycle, for example one cluster plus one
363 gap. Hence we briefly review different clustering methods which either assign events to a cluster
364 or choose the number of clusters. Clustering algorithms are broadly classified as either
365 partitioning or hierarchical. To illustrate, we use Sieh et al.'s (1989) record from Pallett Creek,
366 California (Figure 7a).

367 Partitioning methods such as the popular k -means algorithm are used to divide a sequence
368 of observations, forming a given number of clusters, k , each observation assigned to one cluster.

369 Other methods, discussed later, are used to determine the number of clusters for a given
370 sequence. In our application, the observations in a sequence are the dates, in years, of n
371 earthquakes in an earthquake record and the clusters are defined as time intervals encompassing
372 the range of dates. In the Pallett Creek record, $n = 10$. A k -means algorithm starts by guessing k
373 cluster centers, which are averages of dates. The process then alternates two steps: 1) The closest
374 cluster center is identified for each observation, measured by time in years between earthquakes
375 and cluster centers, and the observation is assigned to that cluster. 2) Each cluster center is
376 recalculated as the average date of its members (Hastie et al., 2009). This process repeats until it
377 minimizes the sum or total within-cluster sum of squares (TWSS) of distances from cluster
378 centers, i.e., it minimizes the sum of within-cluster variances of clusters $i=1, \dots, k$ multiplied by
379 the number of observations in that cluster, n_i (Hartigan, 2006). Commonly, this analysis is
380 performed for a range of k and different aspects of the resulting cluster assignments are assessed
381 to determine the number of clusters. The choice of k will strike a balance between too many
382 clusters and not enough. The methods for choosing k do not always agree, as discussed next.

383 Some methods, such as the Elbow method, examine the graph of TWSS versus k (Figure
384 7b) and choose the value of k corresponding to a kink in the plot resembling a bent elbow
385 (Tibshirani et al., 2001). Increasing k beyond this value conveys a lesser reduction in TWSS. By
386 this method, Pallett Creek has 4 clusters. The Silhouette method compares the tightness (length
387 of clusters) and separation (distance between clusters) to determine whether the cluster lengths
388 are small compared to the distances between-clusters (Rousseeuw, 1987). Each observation
389 receives a silhouette value, ranging from -1 to $+1$, indicating the extent to which the observation
390 is well matched to its assigned cluster and poorly matched to the others. The number of clusters k
391 is chosen to maximize the average values for all observations; again $k = 4$ for Pallett Creek
392 (Figure 7c). The Gap method plots two curves that are functions of k , the logarithm of TWSS
393 and its expected value under a uniform distribution of earthquake dates within the record
394 (Tibshirani et al., 2001). The Gap statistic is the distance (gap) between the curves. The chosen
395 value for k has the maximum Gap statistic, which again is $k = 4$ (Figure 7d).

396 Hierarchical methods do not rely on advance specification of the number of clusters, k ,
397 but rather create clusters for all $k = 1, \dots, n$ possibilities. This process is illustrated by
398 dendrogram plots (tree diagrams), showing the order in which different clusters are merged

399 through connecting branches (Figure 7e). The vertical axis shows the cumulative difference in
400 dates (in years) between cluster centers being merged. Hierarchical methods are of two kinds: 1)
401 Divisive, in which all observations start in a single cluster, $k = 1$, and are iteratively separated
402 until $k = n$. 2) Agglomerative nesting (termed AGNES), in which all observations start in their
403 own cluster ($k = n$), with the closest clusters (defined here by years between cluster centers)
404 iteratively joined until $k = 1$ (Kaufman and Rousseeuw, 1990). AGNES may be better at
405 identifying small clusters, while divisive methods may be better at identifying large clusters,
406 although this choice makes no difference for our example. A popular AGNES algorithm, Ward's
407 (1963) method, minimizes the within-cluster sum of squares using an update formula which
408 assigns a new cluster's height on the vertical axis as the cumulative distance between the cluster
409 centers being merged at that step and each step below it (Murtagh and Legendre, 2014). We
410 show Ward's method because it is intended for interval-scaled data such as the dates of
411 earthquakes (Kaufman and Rousseeuw, 1990). Figure 7e shows the tree resulting from applying
412 Ward's method to Pallett Creek. Clusters that merge at high levels on the vertical axis (indicating
413 large distances between cluster centers being merged at that step) relative to the level of the
414 clusters within them can be interpreted as a 'natural' number of clusters (Hastie et al., 2009).
415 This determination is subjective, so in this example one could reasonably choose 2 or 4 clusters
416 (Figure 7e). The four clusters $\{1,2,3\}$, $\{4,5,6\}$, $\{7,8\}$, $\{9,10\}$ are the same as obtained from k -
417 means with $k = 4$.

418 Goldfinger's (2012) hierarchical clustering analysis on the 10,000-year-long Cascadia
419 earthquake record found either four or five clusters, using AGNES with complete linkage
420 (furthest neighbor) method. Furthest neighbor defines the distance between two clusters as the
421 distance between two observations, one in each cluster, that are farthest away from one another
422 (Tibrishani et al., 2001b). The two clusters with the shortest distance between them are merged at
423 each step. Applying the complete linkage method to the Pallett Creek record yields the same tree
424 as shown in Figure 7e using Ward's method. Goldfinger performed several tests of the statistical
425 significance of the clusters with most resulting in a rejection of an underlying Poisson
426 distribution. He cautions, however, "*there is no requirement that physical systems pass statistical*
427 *tests*" (Goldfinger et al., 2012).

428 Hierarchical methods are complementary to partitioning methods such as k -means. For
 429 example, one can use the cluster centers from Ward's method as the initial cluster centers in k -
 430 means. As discussed above, results from k -means for chosen k can be compared to the results of
 431 AGNES. Our results are moderately robust to slight changes in dates, as illustrated by comparing
 432 the slightly differing dates of Pallett Creek from Sieh et al. (1989), Biasi et al. (2002), and
 433 Scharer et al. (2011). The tree diagrams are the same, because they largely reflect only the
 434 events' order. Differences in the k -means evaluations are shown in Table 1. The Gap statistic for
 435 Scharer et al.'s dates yields 1 cluster, and the next best number is 4, with the difference between
 436 their statistics being quite small compared to the differences between other numbers of clusters.
 437 A similar situation occurs in the silhouette for the Sieh et al. dates (Figure 7c) where one could
 438 argue for 2, 3, or 4 clusters because of the similar values. The Elbow method is the most stable
 439 between these different records and the Gap statistic is the least.

440 Table 1: Differences in the number of clusters indicated by three methods for records of
 441 earthquakes at Pallett Creek with slightly differing dates.

Record	Gap Statistic	Elbow	Silhouette
Sieh et al., 1989	4	4	4
Biasi et al., 2002	2	4	2
Scharer et al., 2011	1	4	4

442

443

444

5. Long-Term Fault Memory Model

445 To explore how earthquake supercycles and clusters arise, we use a simple Long-Term
 446 Fault Memory (LTFM) model, which is a modified version of the standard earthquake cycle
 447 model. In it, the probability of an earthquake reflects the accumulated strain. This increases
 448 steadily with time until an earthquake happens, after which it decreases, but not necessarily to
 449 zero (Figure 8). Hence, the probability of an earthquake depends on the earthquake history over
 450 multiple prior cycles. Clusters happen because after a gap, a period of quiescence, the probability
 451 can remain higher than the long-term average for several cycles. The model simulates large
 452 earthquakes releasing only part of the strain accumulated on the fault, in contrast to the standard
 453 model in which all of the accumulated strain is released.

454 LTFM is a simple model with only a few parameters. The annual probability $P(t)$ grows
455 with time at rate $dP/dt = A = 2/\tau^2$, simulating steady strain accumulation. τ is an initial mean
456 recurrence interval, such that if no earthquake occurs during the initial time interval $t = \tau$, the
457 average annual probability is $1/\tau$. If the probability is above a threshold value δ , which we
458 typically set as zero, an earthquake can occur. When an earthquake occurs, the probability drops
459 by $\Delta P = -R$, simulating a partial strain release. Hence on average R/A years of accumulated
460 strain is released in an earthquake.

461 The accumulation parameter A controls the long-term seismicity rate, and the release
462 parameter R controls the clustering. Small R yields long-term memory and more clusters,
463 whereas in the limit large R gives the standard earthquake cycle model with only short-term
464 memory because it forces the probability to zero after each earthquake. The probability is not
465 allowed to go below 0 or to exceed 1.

466 We generate earthquake histories by using the Mersenne Twister pseudo-random number
467 generator (Matsumoto and Nishimura, 1998), sampling from a uniform distribution between 0
468 and 1. If the value exceeds the probability for that year, no earthquake occurs and the probability
469 increases by A for the next year. If the value is less than that year's probability, an earthquake
470 occurs and the probability drops by R for the following year. Linearly increasing probabilities
471 have been used by other authors, e.g., Pinedo and Shpilberg (1981).

472 The saw-tooth behavior of LTFM simulates the proposed long-term variations in stored
473 elastic strain or strain energy (Figure 2). Supercycles and clusters arise because longer intervals
474 between earthquakes generally begin at times of low probability, consistent with the pattern
475 noted in terms of cumulative strain by Weldon et al. (2004). A lower probability corresponds to
476 the fault having less memory of previous earthquakes. Thus, as the probability (i.e. cumulative
477 strain) approaches zero, the corresponding supercycle can be viewed as approaching a renewal
478 process.

479 Because LTFM is a stochastic model, the resulting earthquake sequences depend on both
480 the model parameters and chance. As a result (Figure 9) sequences can appear strongly periodic,
481 weakly periodic, Poissonian, or bursty. The four sequences in this example have the same
482 probability (i.e. strain) accumulation rate ($A = 2/125^2$) but different release parameters ($R =$
483 $200A, 175A, 80A, 50A$). As shown, the aperiodicity increases as R decreases. The strongly
484 periodic sequence arises in a way similar to a standard earthquake cycle model because R is so

485 large that the probability drops to zero after each earthquake, so the fault has no memory. The
486 effects of fault memory increase for successively smaller values of R , making the sequences less
487 periodic. However, A and R control only the overall sequence properties via the probability of
488 earthquake occurrence, because when earthquakes occur is random. As a result, the aperiodicity
489 varies between different portions of the sequence.

490 In some cases, we use two thresholds, $\delta_2 > \delta_1$ and corresponding probability drops, $R_2 >$
491 R_1 , to describe the earthquakes with larger and smaller strain changes implied by some records
492 (Figure 2). Hence if $P(t) > \delta_i$, the probability drops by R_i . Using two probability thresholds and
493 probability drops to describe both rare larger and more frequent smaller strain changes allows
494 LTFM to simulate the range of observed supercycle behavior (Figure 10). The higher threshold
495 and probability drop simulate infrequent events that have slip and strain release much greater
496 than typical events, and so end a supercycle. Using two similar thresholds simulates a Sumatra-
497 style earthquake history with long gaps separating clusters, because earthquakes can occur only
498 late in a supercycle. This case would correspond to a very strong fault. Conversely, a low
499 threshold for smaller earthquakes and a much higher one for larger earthquakes simulates a
500 Tohoku-style record where smaller earthquakes occur frequently between the largest events, so
501 the supercycles in the strain record do not appear in the earthquake history as gaps and clusters.
502 The threshold and drop parameters can be chosen to simulate the very long gaps associated with
503 intraplate and plate boundary zone earthquakes. In such situations, because strain accumulates
504 slowly relative to plate boundaries, the lower threshold is quite low. Hence this threshold can be
505 used in most applications with higher strain rates (e.g., Figure 10b), since it would have
506 essentially the same effect as a zero threshold (e.g., Figure 9).

507

508

5.1 Example

509 To explore choosing LTFM parameters to match key aspects of an earthquake history, we
510 simulated the record from Pallett Creek, California. Although recent studies have reestimated
511 the dates (Biasi et al., 2002; Scharer et al., 2011), we used Sieh et al.'s (1989) dates because the
512 resulting clusters provide a better test case. We ran the model 100 times for pairs of input
513 parameters, R and τ , and averaged the mean and standard deviation of recurrence intervals for
514 each pair. Contouring these averages identified regions of the model space, and hence ranges of
515 the input parameters, that produce simulations with comparable mean and standard deviation to

516 those observed in the paleoseismic record. We then searched these regions for parameters
517 giving a memory parameter close to that observed.

518 A simulation with accumulation rate $A = 2/289^2$ and release parameter $R = 130A$ that
519 gives clustering behavior similar to that observed is shown in Figure 11. The data have $\mu = 132$
520 yr, $\sigma = 105$ yr, $\alpha = 0.79$ and $M = -0.28$, and the simulation has $\mu = 136$ yr, $\sigma = 102$ yr, $\alpha = 0.75$
521 and $M = -0.33$, indicating weak periodicity. The event timing differs between the simulation
522 and the observed record due to the model's stochastic nature. The longer intervals between
523 earthquakes begin at times of low probability, consistent with the pattern noted in terms of
524 cumulative strain by Weldon et al. (2004).

525 We used the LTFM model to explore the long-term variability of fault behavior by
526 creating simulations much longer than paleoseismic records, and then sampling them for
527 intervals corresponding to paleoseismic records. Figure 12 shows results for a 50,000 year long
528 simulation using parameters appropriate for Pallet Creek. The mean and standard deviation of
529 recurrence times averaged over a moving 1345-year window, corresponding to a paleoseismic
530 record, are relatively stable over long time periods. This stability would be consistent with the
531 idea of steady loading and unloading by plate motion and large earthquakes. However the mean
532 and standard deviation of recurrence times vary somewhat. The aperiodicity shows that the
533 simulated paleoseismic record sometimes appears strongly periodic (standard deviation small
534 relative to the mean) implying a seismic cycle model, while at other times it looks weakly
535 periodic, Poissonian (standard deviation similar to the mean), or bursty. This variability is
536 illustrated by the earthquake history between model years 19,000 and 22,000. Hence the
537 recurrence variability due to long-term fault memory can give rise to paleoseismic records that at
538 different times appear to have different underlying statistical distributions. Thus a given
539 paleoseismic or instrumental window may give a biased view of the long-term seismicity.

540

541 **5.2 LTFM and intraplate earthquakes**

542 Long-term fault memory may also be an important contributor to the space-time
543 variability of continental intraplate earthquakes. Considerable recent attention (reviewed by Liu
544 and Stein, 2016, Calais et al., 2016, and Stein et al., 2017a) has been directed to how and why
545 earthquakes within continents behave differently in space and time from those on plate
546 boundaries. Faults at plate boundaries are loaded at constant rates by relatively rapid and steady

547 relative plate motion. Consequently, earthquakes concentrate along the plate boundary faults and
548 show quasi-periodic (relative to intraplate earthquakes) occurrences, although the actual
549 temporal patterns are often complicated. The spatial gaps that appear are filled in over time.

550 However, in mid-continent, the slower tectonic loading is shared by a complex system
551 of interacting faults spread over a large region, such that a large earthquake on one fault could
552 increase the loading rates on other faults in the system. Because the low tectonic loading rate is
553 shared by many faults, individual faults may remain dormant for a long time before they
554 accumulate enough strain for a short period of activity. The resulting earthquakes are therefore
555 episodic, clustered, and spatially migrating (Li et al., 2009; Stein et al., 2009). These effects can
556 be seen in many areas, including North China (Liu et al., 2011), Europe (Camelbeeck et al.,
557 2007; 2014), and the central United States (Crone and Luza, 1990; Newman et al., 1999;
558 Holbrook et al., 2006; Tuttle et al., 2006; Gold et al., 2018).

559 Topographic data from Australia, where erosion is very slow, provide some of the best
560 evidence available of how intraplate faults slip over time. Figure 2f illustrates this pattern of
561 clusters of activity separated by much longer and irregular intervals of quiescence. Liu and Stein
562 (2016) note that the pattern of displacement accumulated over time is similar to the Devil's
563 Staircase function, a fractal property of chaotic dynamic systems (Devaney et al., 1989; Turcotte,
564 1997). The apparent long-distance roaming of large mid-continental earthquakes also suggests
565 dynamic system behavior. In such a system, change of any part of the system (such as rupture of
566 a fault) could impact nonlinearly the behavior of the whole system.

567 Although this view of intraplate seismicity fits what is known in general terms, the
568 specifics are still unclear. In particular, how effectively stress can be transferred to distant faults
569 is unknown. We thus used the LTFM model to explore the possibility that long-term fault
570 memory may also contribute to the space-time variability.

571 A noticeable difference between the clustering in Australia and that on plate boundaries
572 is that in Australia the gap durations are more than ten times as long as the clusters, whereas on
573 plate boundaries the gaps are only 2-3 times as long as the clusters. As shown in Figure 9,
574 LTFM can describe this effect via assuming the level of strain accumulation required for an
575 earthquake. A proposed alternative is that clusters of large intraplate earthquakes reflect the fault
576 weakening after the first major event, so as to permit repeated failure (Li et al., 2009). Models
577 have been proposed for how weakening and subsequent healing might occur (Sibson, 1992;

578 Lyakhovsky et al., 2001). In Lyakhovsky et al.'s model, as the rate ratio between loading and
579 healing increases, behavior changes from regular to clustered. This is because healing tends to
580 purge long-term memory. These models were developed with a view toward describing the
581 evolution of fault properties over multiple earthquake cycles, i.e. a different type of long-term
582 fault memory.

583

584

5.3 LTFM model discussion

585 Our results illustrate that a modified version of the standard earthquake cycle model can
586 be used to simulate and explore key features of supercycles that are observed at many plate
587 boundaries and in plate interiors. This is gratifying, given the model's simplicity. LTFM can be
588 thought of as an idealized model like those used in many disciplines, including physics,
589 astronomy, meteorology, biology, and economics, that allow investigations to focus on some key
590 characteristics of a complex phenomenon and explore whether they can be explained by simple
591 assumptions. Reutlinger et al. (2018) explain that *"we call such models 'toy models'—a term
592 that is not meant to have belittling or derogatory connotations... First, models of this type are
593 strongly idealized... Second, such models are extremely simple in that they represent a small
594 number of causal factors (or, more generally, of explanatory factors) responsible for the target
595 phenomenon. Third, these models refer to a target phenomenon."* A good example would be the
596 simple analytical model of subduction zones that extracts key aspects of sophisticated numerical
597 models and thus can be used how the temperature structure and resulting plate driving force
598 depend on the age of the subducting plate and convergence rate (Stein and Wysession, 2009)

599 In this spirit, we have used a simple model that simulates general properties of
600 supercycles. We plan to explore its possible applicability to paleoseismic records in other areas
601 and in different tectonic regimes. For example, clusters have been observed in paleoseismic data
602 in plate boundary zones, where diffuse deformation is spread over multiple faults and long-term
603 slip rates are slower than on primary plate boundary faults (which typically move at > 10 mm/yr)
604 but higher than in continental interiors (which typically deform at < 1 mm/yr) (Wallace, 1987;
605 Rockwell et al., 2000; Friedrich et al., 2003; Oskin et al., 2008; Dolan et al., 2016; Gold et al.,
606 2017). Some clusters seem to arise on individual faults, whereas others involve groups of faults.
607 The Wasatch fault and adjacent faults show a strain release and slip pattern similar to that in
608 Australia (Figure 2e) (Wallace, 1987; Friedrich et al., 2003). In the Eastern California shear

609 zone, regional strain release appears to occur via "*distinct periods or bursts of seismic activity*
610 *punctuated by periods of relative quiescence. Individual faults, however, appear to behave in a*
611 *quasiperiodic fashion, with the clustering produced by the in-phase earthquake generation of the*
612 *system of faults*" (Rockwell et al., 2000). Hence LTFM may be involved in plate boundary zone
613 faults, but fault interactions and changes in loading across the zone may also contribute.

614 Additional features could be added to the model without overcomplicating it. Its current
615 form allows for two classes of earthquakes causing different probability decreases, or strain
616 releases. In some cases, only one may be needed, as motivated by observations that slip in large
617 events on individual fault segments appears similar (Schwartz and Coppersmith, 1984) and
618 Weldon et al.'s (2004) observation that on the area of the San Andreas they studied "*there*
619 *appears to be no relationship between strain level and the size of earthquakes.*" However,
620 Goldfinger et al. (2013) note a "weak tendency" for clusters to terminate with an "outsized"
621 event, as found for the Tohoku and Sumatra records (Figure 2). Moreover, some of the strain
622 release may occur via slow slip events (Rogers and Dragert, 2003; Jiang et al., 2017) that may
623 not appear in the paleoseismic record.

624 Fault interactions could be introduced into the model by having multiple faults that affect
625 the probability of large earthquakes on each other. In some situations these may increase
626 clustering, and in others they may reduce it. This effect is likely to contribute to the variability in
627 earthquake size often observed at subduction zones (Thatcher, 1990; Stein and Okal, 2007). One
628 example is the trench segment that produced the $M_w \sim 9.6$ 1960 Chilean earthquake. Its rupture
629 mode must be variable because the seismic-slip rate inferred assuming that the 1960 earthquake
630 is this segment's characteristic earthquake exceeds the convergence rate. Hence Stein et al.
631 (1986) proposed that either the characteristic earthquake is smaller than the 1960 event, the
632 average recurrence interval is greater than observed in the past 400 years, or both. Recent
633 paleoseismic studies support this analysis (Cisternas *et al.*, 2005). Paleoseismic studies also find
634 evidence for variable size of thrust events, presumably due to the differences between
635 multisegment and single-segment rupture, in areas including the Nankai Trough (Ando, 1975)
636 and the Kuril trench (Nanayama et al., 2003).

637 Viewing supercycles as a result of long-term fault memory fits into a general framework
638 in the literature of complex dynamic systems. Clustered events, described as "bursts," are
639 observed in many disparate systems, from the firing system of a single neuron to the outgoing

640 mobile phone sequence of an individual (Karsai et al., 2012). Such systems display “...a bursty,
 641 intermittent nature, characterized by short timeframes of intense activity followed by long times
 642 of no or reduced activity,” (Goh and Barabasi, 2008). As a result, the system’s state depends on
 643 its history, so it has long-term memory (Beran et al., 2013).

644 An additional point worth noting is that we generally limit our discussion to cases where
 645 the supercycle is shorter than the climatic forcing cycles such as global glaciation periods.

646

647 **5.4 Mathematics of the LTFM model**

648 The LTFM model is a stochastic process, specifically a Markov chain with a finite
 649 number of states at discrete times $0, 1, 2, \dots$. The states correspond to possible values of
 650 accumulated strain, reflected in the probability $P(t)$, which are finite in number.¹ The probability
 651 that an earthquake occurs at time t , conditional on the full history of strain accumulation and
 652 release at all times prior to t , depends only on the most recent level of strain, i.e., at time $t-1$.²
 653 Given $P(t)$, the probability does not otherwise depend on time. Thus, the history prior to t is
 654 fully captured by $P(t-1)$. The process starts over each time accumulated strain is equal to the
 655 strain at time $t=0$ (or, for practical purposes, is close to that amount). The length of time until
 656 the process starts over can be interpreted as the length of a supercycle. The theory of Markov
 657 chains (Çinlar, 1975) allows us to directly specify the full probability distribution for the length
 658 of a supercycle, and hence its mean and standard deviation. The theory also allows us to specify
 659 the conditional probability of an earthquake at a time $t > s$ given the accumulated strain at
 660 current time s . The theory implies that the probability at a far future time t does not depend on
 661 the accumulated strain at time s and provides a formula for that probability. From this
 662 probability, the expected number of earthquakes in a distant time span of length T can be
 663 calculated, along with the approximate standard deviation.

¹ Possible values of $P(t)$ have the form $\min\{(\alpha A - \beta R_1 - \gamma(\lambda - 1)R_1)^+, 1\}$ with $\lambda = R_2 / R_1$,
 $(x)^+ = \max\{x, 0\}$, and α, β, γ taking non-negative integer values.

² If the probability at time s is $P(s) = C_s$ then the conditional probability of an earthquake at
 time t is equal to $\min\{(A + C_{t-1}) \times \chi_{pos}(A + C_{t-1} - \delta_1), 1\}$, with $\chi_{pos}(x)$ equal to 1 if $x > 0$ and
 equal to 0 otherwise.

664 LTFM can also be posed in terms of the classic probability model of drawing balls from
665 an urn. (Stein and Stein, 2013). If e balls are labeled "E" for earthquake, and n balls are labeled
666 "N" for no earthquake, the probability of an earthquake is that of drawing an E-ball, which is the
667 ratio of the number of E-balls to the total number of balls. If after drawing a ball, we replace it,
668 the probability of an event is constant or time-independent in successive draws, because one
669 happening does not change the probability of another happening. Thus an event is never
670 "overdue" because one has not happened recently, and the fact that one happened recently does
671 not make another less likely. LTFM corresponds an alternative, sampling such that the fraction
672 of E-balls and the probability of another event change with time. We add A E-balls after a draw
673 when an earthquake does not occur, and remove R E-balls when an earthquake occurs. This
674 makes the probability of an event increase with time until one happens, after which it decreases
675 and then grows again. Events are not independent, because one happening changes the
676 probability of another.

677

678

6. Implications for hazard assessment

679 Advances in understanding supercycles would be important for seismic hazard
680 assessment. Such assessments depend heavily on assumptions about the magnitude and
681 recurrence rate of future large earthquakes (Stein et al., 2012), both of which are often more
682 variable than assumed. A larger assumed aperiodicity will cause cumulative or conditional
683 probabilities to decrease, all else fixed (Ward, 1992).

684 Current earthquake probability estimates depend on assuming a probability density
685 function for the recurrence intervals with input parameters inferred from the available earthquake
686 history. Figure 13 illustrates the resulting uncertainties for Cascadia. Figure 13a shows the
687 effects of additional paleoseismic data. Goldfinger et al.'s (2012) chronology yielded a mean
688 recurrence interval of 530 yrs and a standard deviation of 271 yrs for the entire 10,000 year
689 record, and a mean recurrence interval of 326 yrs and a standard deviation of 88 yrs for the most
690 recent cluster. Including a newly-identified event in a revised chronology (Goldfinger et al.,
691 2017) has a small effect on the 10,000-year record's parameters, changing the mean recurrence
692 interval to 502 yrs and a standard deviation of 239 yrs. However, adding this event makes all
693 events in the past 5,000 years part of the same cluster, with a recurrence interval of 452 yrs and a
694 standard deviation of 142 yrs.

695 Whether to assume that a recent cluster is continuing or has ended can lead to quite
696 different estimates of earthquake probabilities (Stein et al., 2017b). Figure 13b shows the
697 different distribution of recurrence intervals corresponding to the two different chronologies and
698 various probability density functions with parameters corresponding to the two chronologies. By
699 far the largest difference arises from assuming either that the recent cluster continues, or that the
700 cluster is over so the appropriate parameters are those for the entire record. Assuming that we are
701 still in the cluster predicts higher probability than using the entire record. This effect is more
702 important than the specific probability density function assumed. The corresponding effect
703 appears from considering the conditional probability of a large earthquake in the next 50 years,
704 which results from integrating the probability density functions (Figure 13c).

705 More generally, if faults have long-term memory, then individual earthquake cycles, and
706 hence the recurrence times between successive large earthquakes, are not independent. Hence the
707 renewal approach of modeling their probability as a function of time since the previous large
708 earthquake can give misleading results. The problem is not that a renewal model is inappropriate,
709 but rather that the renewal depends on release of nearly all accumulated strain, and that may
710 occur at different times than large earthquakes. As shown in Figure 12, the recurrence variability
711 due to long-term fault memory can cause short earthquake records to give a biased view of the
712 long-term seismicity. As a result, further investigation of long-term earthquake recurrence
713 variability is important both for understanding the nature and causes of supercycles and for
714 improving hazard assessment.

715 Acknowledgements

716 This research was supported by Northwestern University's Institute for Policy Research and
717 Buffet Center for International Studies. Liu acknowledges support of NSF grant 1519980.

718

719

720 References

721

722 Adams, J., 1990. Paleoseismicity of the Cascadia subduction zone: Evidence from turbidites off
723 the Oregon-Washington margin, *Tectonics* 9(4), 569-583.

724

- 725 Agnon, A., 2014. Pre-instrumental earthquakes along the Dead Sea Rift, In: Z. Garfunkel et al.
726 (eds.), *Dead Sea Transform Fault System: Reviews, Modern Approaches in Solid Earth Sciences*
727 6, doi: 10.1007/978-94-017-8872-4_8
728
- 729 Akciz, S. O., Ludwig, L. G., Arrowsmith, J. R., Zielke, O., 2010. Century-long average time
730 intervals between earthquake ruptures of the San Andreas fault in the Carrizo Plain, California,
731 *Geology* 38 (9), 787-790, doi: 10.1130/G30995.1.
732
- 733 Ando, M., 1975. Source mechanisms and tectonic significance of historical earthquakes along
734 the Nankai Trough, Japan, *Tectonophysics* 27, 119–140.
735
- 736 Ben-Zion, Y., K. Dahmen, V. Lyakhovsky, D. Ertas, Agnon, A., 1999. Self-driven mode
737 switching of earthquake activity on a fault system, *Earth Planet. Sci. Lett.* 172(1–2), 11–21.
738
- 739 Beran, J., Feng, Y., Ghosh, S., Kulik, R., 2013. *Long-Memory Processes*, Springer-Verlag Berlin
740 Heidelberg, doi: 10.1007/978-3-642-35512-7_1.
741
- 742 Biasi, G. P., Weldon, R. J., Fumal, T. E., Seitz, G. G., 2002. Paleoseismic event dating and the
743 conditional probability of large earthquakes on the southern San Andreas Fault, California, *Bull.*
744 *Seismol. Soc. Am.* 92(7), 2761-2781.
745
- 746 Calais, E., Camelbeeck, T., Stein, S., Liu, M., Craig, T.J., 2016. A new paradigm for large
747 earthquakes in stable continental plate interiors, *Geophys. Res. Lett.* 4320, 10-621.
748
- 749 Camelbeeck, T., Vanneste, K., Alexandre, P., Verbeeck, K., Petermans, T., Rosset, P., Everaerts,
750 M., Warnant, R., Van Camp, M., 2007. Relevance of active faulting and seismicity studies to
751 assess long term earthquake activity in northwest Europe, In: Stein, S., Mazzotti, S. (Eds.),
752 *Continental Intraplate Earthquakes: Science, Hazard, and Policy Issues*, *Geol. Soc. Amer.*
753 *Special Paper* 425, 193–224.
754

- 755 Camelbeeck, T., Alexandre, P., Sabbe, A., Knuts, E., Garcia Moreno, D., Lecocq, T., 2014. The
756 impact of the earthquake activity in western Europe from the historical and architectural heritage,
757 In: Talwani, P. (Ed.), *Intraplate Earthquakes -Solid Earth Geophysics*. Cambridge University
758 Press.
- 759
- 760 Çinlar, E., 1975. *Introduction to Stochastic Processes*, Prentice Hall, Englewood Cliffs, N.J.
761
- 762 Cisternas, M., Atwater, B.F., Torrejón, F., Sawai, Y., Machuca, G., Lagos, M., Eipert, A.,
763 Youlton, C., Salgado, I., Kamataki, T., Shishikura, M., 2005. Predecessors of the giant 1960
764 Chile earthquake, *Nature* 437(7057), 404.
- 765
- 766 Clark, D., McCue, K., 2003. Australian paleoseismology: towards a better basis for seismic
767 hazard estimation, *Ann. Geophys.* 46(5), 1087–1106.
- 768
- 769 Clark, D., McPherson, A., Collins, C., 2011. Australia's seismogenic neotectonic record,
770 *Geosci. Aust. Rec.* 11.
- 771
- 772 Clark, D., McPherson, A., Van Dissen, R., 2012. Long-term behavior of Australian stable
773 continental region (SCR) faults, *Tectonophysics* 566, 1–30.
- 774
- 775 Cornell, C.A., Winterstein, S.R., 1988. Temporal and magnitude dependence in earthquake
776 recurrence models, *Bull. Seismol. Soc. Am.* 78(4), 1522-1537.
- 777
- 778 Cowie, P.A., Roberts, G.P., Bull, J.M., Visini, F., 2012. Relationships between fault geometry,
779 slip rate variability and earthquake recurrence in extensional settings, *Geophysical Journal*
780 *International* 189(1), 143-160.
- 781
- 782 Crone, A.J., Luza, K.V., 1990. Style and timing of Holocene surface faulting on the Meers
783 fault, southwestern Oklahoma, *Geol. Soc. Amer. Bull.* 102, 1–17.
- 784

- 785 Devaney, R.L., Devaney, L., Devaney, L., 1989. An Introduction to Chaotic Dynamical Systems,
786 Addison-Wesley, Reading.
787
- 788 Dolan, J.F., McAuliffe, L.J., Rhodes, E.J., McGill, S.F., Zinke, R., 2016. Extreme multi-
789 millennial slip rate variations on the Garlock fault, California: Strain super-cycles, potentially
790 time-variable fault strength implications for system-level earthquake occurrence, *Earth Planet.*
791 *Sci. Lett.* 446, 123-136.
792
- 793 Ellsworth, W.L., Matthews, M.V., Nadeau, R.M., Nishenko, S.P., Reasenber, P.A., Simpson,
794 R.W., 1999. A physically-based earthquake recurrence model for estimation of long-term
795 earthquake probabilities, *US Geol. Surv. Open-File Rept.* 99, 522.
796
- 797 Field, E. H., et al., 2015. Long-term time-dependent probabilities for the Third Uniform
798 California Earthquake Rupture Forecast (UCERF3), *Bull. Seismol. Soc. Am.* 105(2A), doi:
799 10.1785/0120140093.
800
- 801 Friedrich, A.M., Wernicke, B.P., Niemi, N.A., Bennett, R.A., Davis, J.L., 2003. Comparison of
802 geodetic and geologic data from the Wasatch region, Utah, and implications for the spectral
803 character of Earth deformation at periods of 10 to 10 million years, *J. Geophys. Res.: Solid Earth*
804 108(B4).
805
- 806 Goes, S. D.B., 1996. Irregular recurrence of large earthquakes: An analysis of historic and
807 paleoseismic catalogs, *J. Geophys. Res.* 101(B3), 5739-5749.
808
- 809 Goh, K.I., Barabasi, A.L., 2008. Burstiness and memory in complex systems, *Europhys. Lett.*
810 81(4), 48002, doi: 10.1209/0295-5075/81/48002.
811
- 812 Gold, R.D., Cowgill, E., Arrowsmith, J.R., Friedrich, A.M., 2017. Pulsed strain release on the
813 Altyn Tagh fault, northwest China, *Earth Planet. Sci. Lett.* 459, 291-300.
814

- 815 Gold, R. D., Duross, C. B., Delano, J., Jibson, R. W., Briggs, R. W., Mahan, S. A., Williams,
816 R., 2018. Four major Holocene earthquakes on the Reelfoot Fault, New Madrid Seismic Zone,
817 GSA Annual Meeting (abstract).
- 818
- 819 Goldfinger, C., Nelson, C. H., Morey, A., Johnson, J. E., Gutierrez- Pastor, J., Eriksson, A. T.,
820 Karabanov, E., Patton, J., Gracia, E., Enkin, R., Dallimore, A., Dunhill, G., Vallier, T., 2012.
821 Turbidite event history: Methods and implications for Holocene Paleoseismicity of the Cascadia
822 Subduction Zone, U.S. Geol. Survey Professional Paper 1661-F, 184.
- 823
- 824 Goldfinger, C., Ikeda, Y., Yeats, R.S., Ren, J., 2013. Superquakes and Supercycles, *Seismol.*
825 *Res. Lett.* 84(1), 24-32.
- 826
- 827 Goldfinger, C., Galer, S., Beeson, J., Hamilton, T., Black, B., Romsos, C., Patton, J., Nelson,
828 C.H., Hausmann, R., Morey, A., 2017. The importance of site selection, sediment supply, and
829 hydrodynamics: A case study of submarine paleoseismology on the Northern Cascadia margin,
830 Washington USA, *Marine Geology* 384, 4-46.
- 831
- 832 Hartigan, J.A., 2006. Classification – 1, *Encyclopedia of Statistical Sciences*, 1-9, John Wiley &
833 Sons, Inc., doi: 10.1002/0471667196.ess0356.pub2.
- 834
- 835 Hastie, T., Tibshirani, R., and Friedman, J., 2009. *The Elements of Statistical Learning: Data*
836 *Mining, Inference, and Prediction*, 2nd ed, Springer, New York, doi: 10.1007/b94608.
- 837
- 838 Holbrook, J., Autin, W.J., Rittenour, T.M., Marshak, S., Goble, R.J., 2006. Stratigraphic
839 evidence for millennial-scale temporal clustering of earthquakes on a continental-interior fault:
840 Holocene Mississippi River floodplain deposits, New Madrid seismic zone, USA,
841 *Tectonophysics* 420(3–4), 431–454.
- 842
- 843 Jiang, Y., Liu, Z., Davis, E.E., Schwartz, S.Y., Dixon, T.H., Voss, N., Malservisi, R. Protti, M.,
844 2017. Strain release at the trench during shallow slow slip: The example of Nicoya Peninsula,
845 Costa Rica, *Geophys. Res. Lett.* 44(10), 4846-4854.

846

847 Kagan, Y. Y., Jackson, D. D., 1991. Long-Term earthquake clustering, *Geophys. J. International*.
848 104, 117-133.

849

850 Karsai, M., Kimmo, K., Barabasi, A., Kertesz, J. 2012. Universal features of correlated bursty
851 behavior, *Scientific Reports* 2, 397, doi: 10.1038/srep00397.

852

853 Kaufman, L. and Rousseeuw, P.J., 1990. *Finding Groups in Data: An Introduction to Cluster*
854 *Analysis*, Wiley-Interscience, New Jersey.

855

856 Kelsey, H.M., Nelson, A.R., Hemphill-Haley, E., Witter, R.C., 2005. Tsunami history of an
857 Oregon coastal lake reveals a 4600 yr record of great earthquakes on the Cascadia subduction
858 zone, *Geological Society of America Bulletin* 117(7-8), 1009-1032.

859

860 Li, Q., Liu, M., Stein, S., 2009. Spatial-temporal complexity of continental intraplate seismicity:
861 Insights from geodynamic modeling and implications for seismic hazard estimation, *Bull.*
862 *Seismol. Soc. Am.* 99, doi: <http://dx.doi.org/10.1785/0120080005>.

863

864 Liu, M., Stein, S., 2016. Mid-continental earthquakes: Spatiotemporal occurrences, causes, and
865 hazards, *Earth-Science Reviews* 162, 364-386.

866

867 Liu, M., Stein, S., Wang, H., 2011. 2000 years of migrating earthquakes in north China: How
868 earthquakes in midcontinents differ from those at plate boundaries, *Lithosphere* 3(2), 128–132.

869

870 Lyakhovsky, V., Ben-Zion, Y., Agnon, A., 2001. Earthquake cycle, fault zones, and seismicity
871 patterns in a rheologically layered lithosphere, *J. Geophys. Res.: Solid Earth* 106(B3), 4103-
872 4120.

873

874 Marco, S., Stein, M., Agnon, A., Ron, H., 1996. Long-term earthquake clustering: A 50,000-year
875 paleoseismic record in the Dead Sea Graben, *J. Geophys. Res.: Solid Earth* 101(B3), 6179-6191.

876

- 877 Matsumoto, M., Nishimura, T., 1998. Mersenne twister: a 623-dimensionally equidistributed
878 uniform pseudo-random number generator, *ACM Transactions on Modeling and Computer*
879 *Simulation* 8(1), 3-30, doi:10.1145/272991.272995.
- 880
- 881 Matthews, M. V., Ellsworth, W. L., Reasenber, P. A., 2002. A Brownian model for recurrent
882 earthquakes, *Bull. Seismol. Soc. Am.* 92(6), 2233-2250.
- 883
- 884 Michael, A. J., 1997. Testing prediction methods: Earthquake clustering versus the Poisson
885 model, *Geophys. Res. Lett.* 24(15), 1891-1894.
- 886
- 887 Mucciarelli, M., 2007. A test for checking earthquake aperiodicity estimates from small
888 samples, *Natural Hazards and Earth System Science* 7(3), 399-404.
- 889
- 890 Murtagh, F. and Legendre, P., 2014. Ward's hierarchical agglomerative clustering method:
891 which algorithms implement Ward's criterion?, *Journal of Classification* 31, 274-295, doi:
892 10.1007/s00357-014-9161-z.
- 893
- 894 Nanayama, F., Satake, K., Furukawa, R., Shimokawa, K., Atwater, B.F., Shigeno, K., Yamaki,
895 S., 2003. Unusually large earthquakes inferred from tsunami deposits along the Kuril
896 trench, *Nature* 424(6949), 660.
- 897
- 898 Newman, A., Stein, S., Weber, J., Engeln, J., Mao, A., Dixon, T., 1999. Slow deformation and
899 lower seismic hazard at the New Madrid seismic zone, *Science* 284(5414), 619–621.
- 900
- 901 Oskin, M., Perg, L., Shelef, E., Strane, M., Gurney, E., Singer, B., Zhang, X., 2008. Elevated
902 shear zone loading rate during an earthquake cluster in eastern California, *Geology* 36(6), 507-
903 510.
- 904
- 905 Parsons, T., 2008a. Monte Carlo method for determining earthquake recurrence parameters from
906 short paleoseismic catalogs: Example calculations for California, *Journal of Geophysical*
907 *Research: Solid Earth* 113(B3), doi:10.1029/2007JB004998.

908

909 Parsons, T., 2008b. Earthquake recurrence on the south Hayward fault is most consistent with a
910 time dependent, renewal process, *Geophysical Research Letters* 35(21).

911

912 Pinedo, M. and Shpilberg, D., 1981. Stochastic models with memory for seismic risk, *The*
913 *Journal of Risk and Insurance* 48(1), 46-58.

914

915 Reid, H.F. 1910. The mechanics of the earthquake, the California earthquake of April 18, 1906,
916 Report of the State Investigation Commission 2, Carnegie Institution of Washington,
917 Washington, D.C.

918

919 Reutlinger, A., Hangleiter, D., Hartmann, S. 2018. Understanding (with) toy models, *British J.*
920 *for Philosophy of Science* 69, 1069–1099.

921

922 Rockwell, T.K., Lindvall, S., Herzberg, M., Murbach, D., Dawson, T., Berger, G., 2000.
923 Paleoseismology of the Johnson Valley, Kickapoo, and Homestead Valley faults: Clustering of
924 earthquakes in the eastern California shear zone, *Bull. Seismol. Soc. Am.* 90(5), 1200-1236.

925

926 Rockwell, T.K., Dawson, T.E., Ben-Horin, J.Y., Seitz, G., 2015. A 21-event, 4,000-year history
927 of surface ruptures in the Anza seismic gap, San Jacinto Fault, and implications for long-term
928 earthquake production on a major plate boundary fault, *Pure and Applied Geophys.* 172(5),
929 1143-1165.

930

931 Rogers, G., Dragert, H., 2003. Episodic tremor and slip on the Cascadia subduction zone: The
932 chatter of silent slip, *Science* 300(5627), 1942–1943, doi:10.1126/science.1084783.

933

934 Rousseeuw, P.J., 1987. Silhouettes: a graphical aid to the interpretation and validation of cluster
935 analysis, *Journal of computational and Applied Mathematics* 20, 53-65.

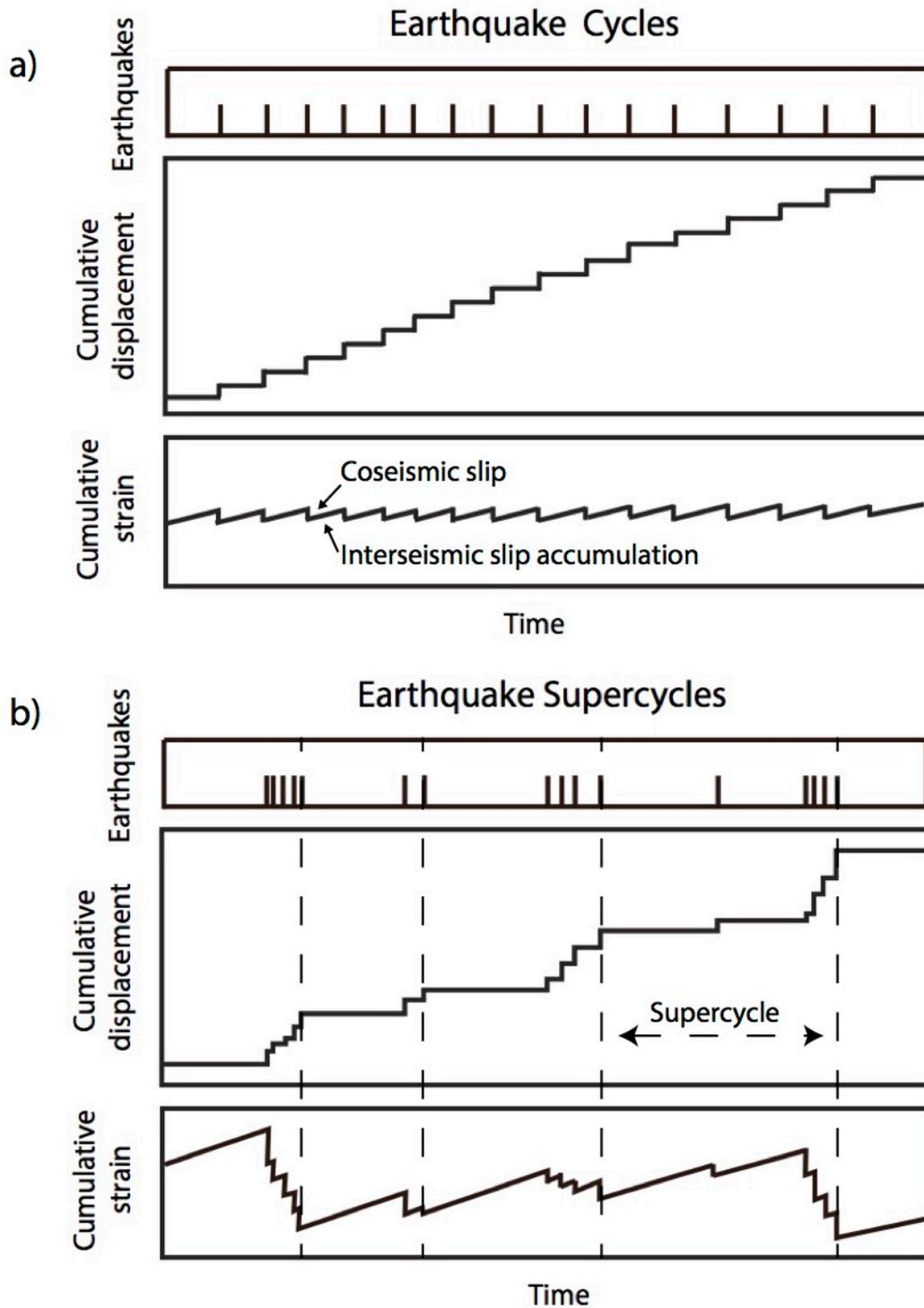
936

- 937 Rubin, C.M., Horton, B.P., Sieh, K., Pilarczyk, J.E., Daly, P., Ismail, N., Parnell, A.C., 2017.
938 Highly variable recurrence of tsunamis in the 7,400 years before the 2004 Indian Ocean
939 tsunami, *Nature Communications* 8, 16019.
- 940
- 941 Rundle, J. B., Jackson, D. D. 1977. Numerical simulation of earthquake sequences, *Bull.*
942 *Seismol. Soc. Am.* 67(5), 1363-1377.
- 943
- 944 Rundle, J.B., Rundle, P.B., Donnellan, A., Li, P., Klein, W., Morein, G., Turcotte, D.L., Grant,
945 L., 2006. Stress transfer in earthquakes, hazard estimation and ensemble forecasting: inferences
946 from numerical simulations, *Tectonophysics* 413(1–2), 109–125.
- 947
- 948 Satake K., 2015. Geological and historical evidence of irregular recurrent earthquakes in Japan,
949 *Phil. Trans. R. Soc. A* 373, 20140375, <http://dx.doi.org/10.1098/rsta.2014.0375>.
- 950
- 951 Savage, J. C., Burford, R. O., 1973. Geodetic determination of relative plate motion in central
952 California, *J. Geophys. Res.* 78, 832–845.
- 953
- 954 Scharer, K.M., Biasi, G.P. and Weldon, R.J., 2011. A reevaluation of the Pallett Creek
955 earthquake chronology based on new AMS radiocarbon dates, San Andreas fault,
956 California, *Journal of Geophysical Research: Solid Earth* 116(B12).
- 957
- 958 Schwartz, D. P., Coppersmith, K. J., 1984. Fault behavior and characteristic earthquakes:
959 Examples from the Wasatch and San Andreas fault zones, *J. Geophys. Res.* 89, 5681–5698.
- 960
- 961 Shimazaki, K., Nakata, T., 1980. Time-predictable recurrence model of large earthquakes,
962 *Geophys. Res. Lett.* 7, 279–282.
- 963
- 964 Sibson, R.H., 1992. Implications of fault-valve behavior for rupture nucleation and
965 recurrence, *Tectonophysics* 211(1-4), 283-293.

- 966 Sieh, K., Stuiver, M. and Brillinger, D. A., 1989. More precise chronology of earthquakes
967 produced by the San Andreas Fault in Southern California, *J. Geophys. Res.* 94, 603–623.
- 968 Sieh K., Natawidjaja, D.H., Meltzner, A.J., Shen, C.C., Cheng, H., Li, K.S., Suwargadi, B.W.,
969 Galetzka, J., Philiposian, B., Edwards, R.L., 2008. Earthquake supercycles inferred from sea-
970 level changes recorded in the corals of West Sumatra, *Science* 322, 1674–167.
- 971 Smalley, R.F., Chatelain, J.-L., Turcotte, D.L., Prevot, R., 1987. A fractal approach to the
972 clustering of earthquakes: applications to the seismicity of the New Hebrides, *Bull. Seismol. Soc.*
973 *Am.* 77(4), 1368-1381.
- 974 Stein, S., Engeln, J.F., DeMets, C., Gordon, R.G., Woods, D., Lundgren, P., Argus, D., Stein, C.,
975 Wiens, D.A., 1986. The Nazca-South America convergence rate and the recurrence of the Great
976 1960 Chilean Earthquake, *Geophysical Research Letters* 13(8), 713-716.
- 977 Stein, S., Okal, E.A., 2007. Ultralong period seismic study of the December 2004 Indian Ocean
978 earthquake and implications for regional tectonics and the subduction process, *Bulletin of the*
979 *Seismological Society of America* 97(1A), S279-S295.
- 980 Stein, S., Liu, M., Calais, E., Li, Q., 2009. Mid-continent earthquakes as a complex system,
981 *Seismol. Res. Lett.* 80(4), 551–553.
- 982 Stein, S., Wyssession, M., 2009. An introduction to seismology, earthquakes, and earth structure,
983 John Wiley & Sons.
- 984 Stein, S., Geller, R.J. and Liu, M., 2012. Why earthquake hazard maps often fail and what to do
985 about it, *Tectonophysics* 562, 1-25.
- 986 Stein, S., Stein, J.L., 2013. Shallow versus deep uncertainties in natural hazard assessments, *EOS*
987 94, 4, 133-140.
- 988 Stein, S., Liu, M., Camelbeeck, T., Merino, M., Landgraf, A., Hintersberger, E., Kübler, S.,
989 2017a. Challenges in assessing seismic hazard in intraplate Europe, *Geol. Soc. London Special*
990 *Publications* 432(1), 13-28.

- 991 Stein, S., Salditch, L., Brooks, E., Spencer, B., Campbell, M., 2017b. Is the coast toast?
992 Exploring Cascadia earthquake probabilities, *GSA Today* 27, 6-7.
- 993 Sykes, L.R., Nishenko, S.P., 1984. Probabilities of occurrence of large plate rupturing
994 earthquakes for the San Andreas, San Jacinto, and Imperial faults, California, 1983–2003, *J.*
995 *Geophys. Res.: Solid Earth* 89(B7), 5905-5927.
- 996
- 997 Thatcher, W., 1990. Order and diversity in the modes of circum-Pacific earthquake recurrence, *J.*
998 *Geophys. Res.* 95, 2609–2624.
- 999
- 1000 Tibshirani, R., Walther, G. and Hastie, T., 2001. Estimating the number of clusters in a data set
1001 via the gap statistic, *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*
1002 63(2), 411-423.
- 1003 Turcotte, D.L., 1997. *Fractals and Chaos in Geology and Geophysics*, Cambridge University
1004 Press.
- 1005 Tuttle, M.P., Al-Shukri, H., Mahdi, H., 2006. Very large earthquakes centered southwest of the
1006 New Madrid seismic zone 5,000–7,000 years ago, *Seismol. Res. Lett.* 77(6), 755–770.
- 1007 Vere-Jones, D., 1970. Stochastic models for earthquake occurrence, *Journal of the Royal*
1008 *Statistical Society Series B* 32(1), 1-62.
- 1009 Wallace, R. E., 1987. Grouping and migration of surface faulting and variation in slip rates on
1010 faults in the Great Basin province, *Bull. Seismol. Soc. Am.* 77, 868–877.
- 1011 Ward, J.H., 1963. Hierarchical grouping to optimize an objective function, *Journal of the*
1012 *American Statistical Association* 58(301), 236-234, doi: 10.1080/01621459.1963.10500845.
- 1013 Ward, S.N., 1992. An application of synthetic seismicity in earthquake statistics: The Middle
1014 America Trench, *Journal of Geophysical Research: Solid Earth* 97(B5), 6675-6682.

- 1015 Weldon, R., Scharer, K., Fumal, T., Biasi, G., 2004. Wrightwood and the earthquake cycle: What
1016 a long recurrence record tells us about how faults work, *GSA Today* 14(9), 4-10, doi:
1017 10.1130/1052-51732004)014<4:WATECW>2.0.CO;2.
- 1018 Weldon, R. J., Fumal, T. E., Biasi, G. P., Scharer, K. M., 2005. Past and future earthquakes on
1019 the San Andreas Fault, *Science* 308, 966-967, doi: 10.1126/science.1111707.
1020
- 1021 Wechsler, N., Rockwell, T.K., Klinger, Y., Stepancikova, P., Kanari, M., Marco, S., Agnon, A.,
1022 2014. A paleoseismic record of earthquakes for the Dead Sea Transform fault between the first
1023 and seventh centuries C.E.: Nonperiodic behavior of a plate boundary fault, *Bull. Seis. Soc. Am.*
1024 104(3), doi: 10.1785/0120130304.
1025
- 1026 WGCEP, Working Group on California Earthquake Probabilities, 2003. Earthquake
1027 Probabilities in the San Francisco Bay Region, U.S. Geologic Survey, Denver,
1028 USGS Open File Report 03-214.
1029
1030
1031



1032

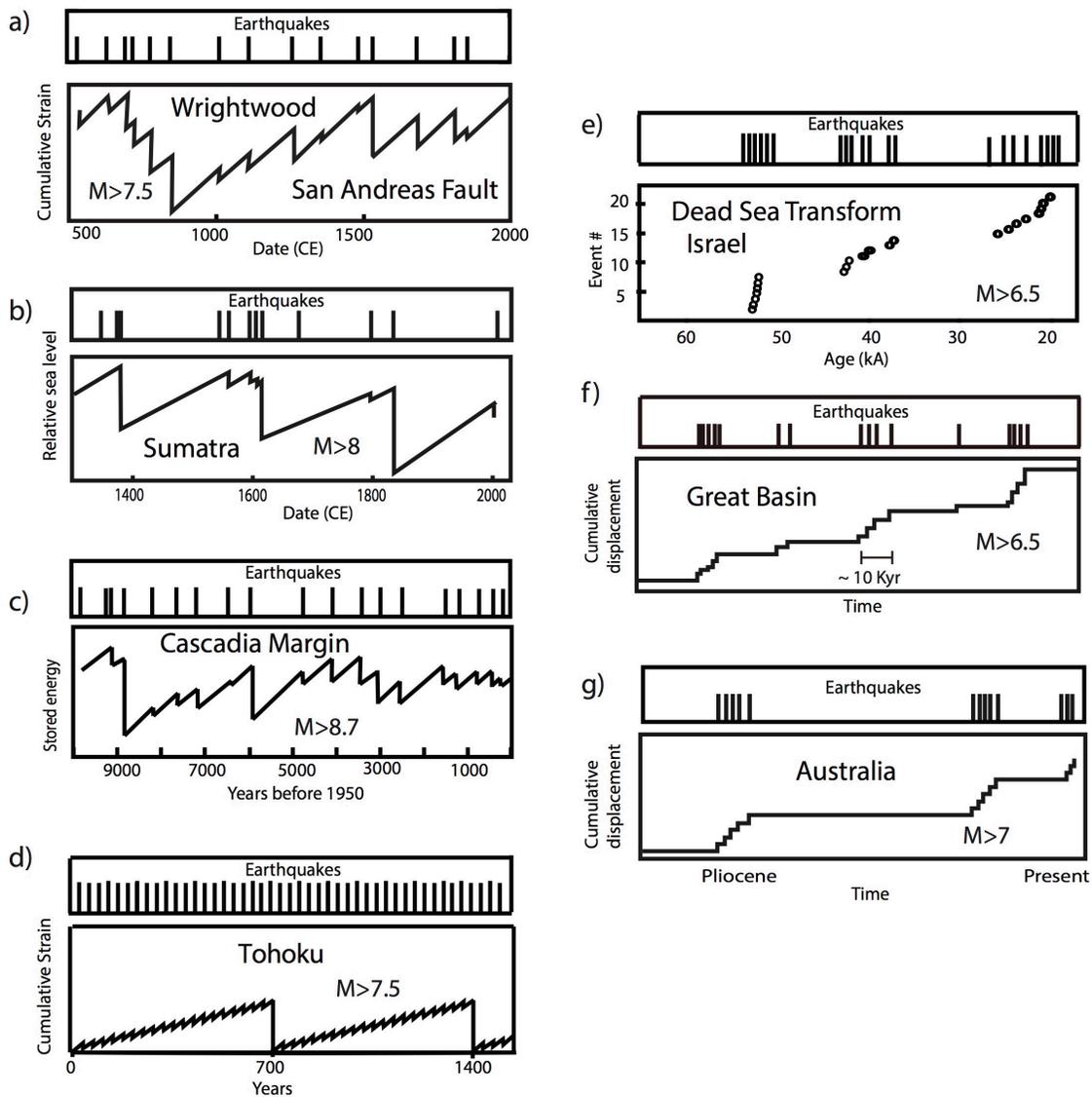
1033 Figure 1: Schematic comparison of the histories of earthquake occurrence, cumulative

1034 displacement, and cumulative strain for a fault without supercycles (a) and a fault with

1035 supercycles (b). Adapted from Wallace (1987) and Friedrich et al. (2003).

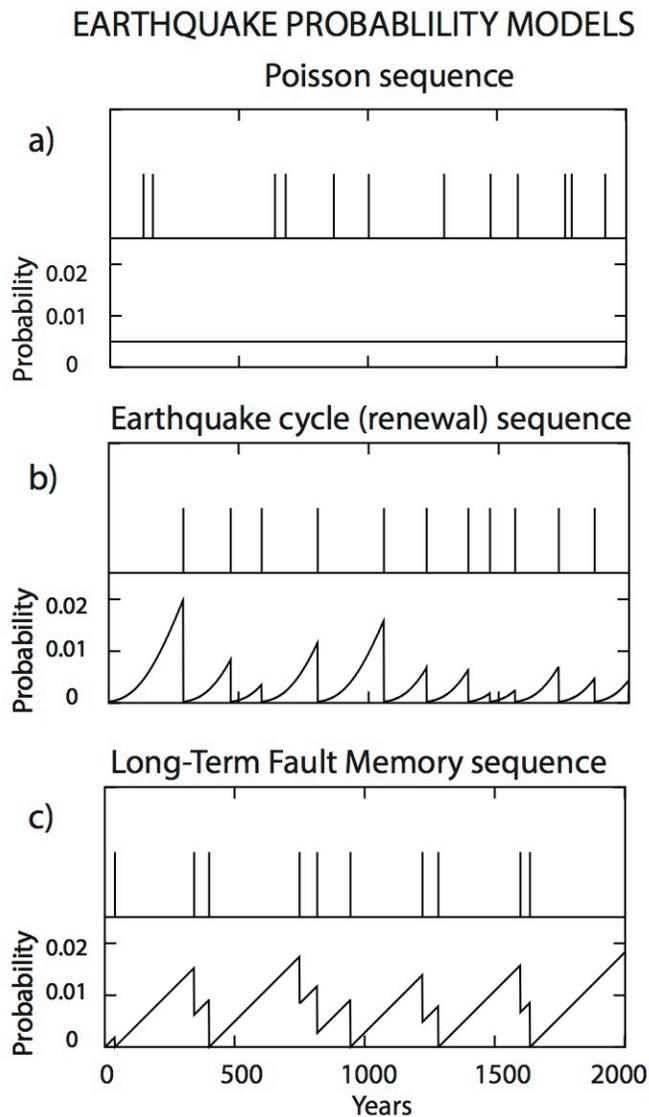
1036

EARTHQUAKE SUPERCYCLES



1037

1038 Figure 2: Examples of reported supercycles. a) Strain accumulation and release inferred from
 1039 paleoseismic data across the San Andreas fault (Weldon et al., 2004). b) Supercycles on the
 1040 Sumatra megathrust inferred from corals (Sieh et al., 2008). c) Long-term energy cycling
 1041 inferred from turbidites on the Cascadia megathrust (Goldfinger et al., 2013). d) Schematic
 1042 earthquake history for the Japan Trench off Tohoku (Satake, 2015). e) Earthquake history on the
 1043 Dead Sea transform (Agnon, 2014). f) Schematic earthquake history for faults and groups of
 1044 faults in the Western U.S.'s Great Basin (Wallace, 1987). g) Schematic earthquake history for
 1045 faults in Australia (Clark et al., 2012).

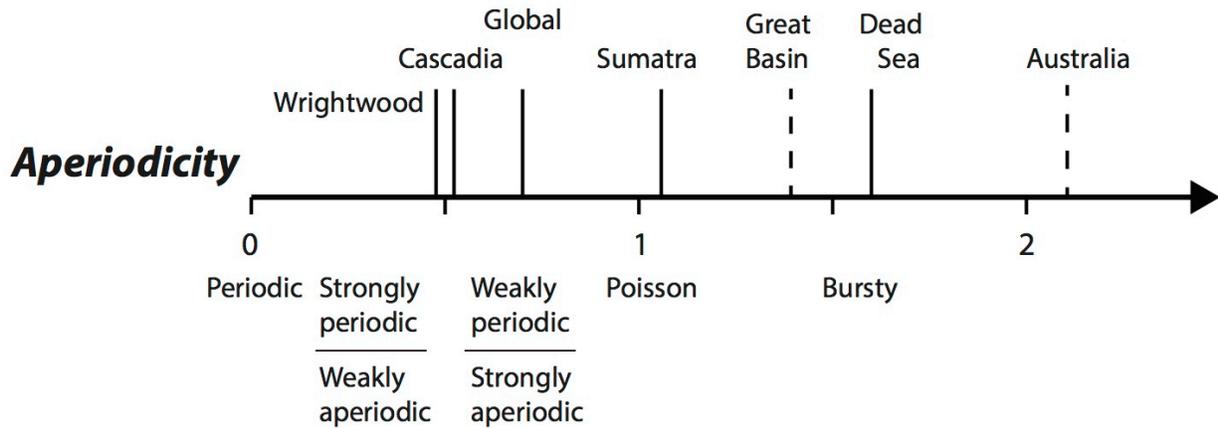


1046

1047 Figure 3: Comparison of earthquake recurrence models. a) Poisson process, in which the
 1048 probability of a large earthquake is constant with time, so the fault has no memory and any
 1049 clusters resulting from short intervals between events arise purely by chance. b) Earthquake
 1050 cycle, in which the probability of a large earthquake increases with time until one occurs, at
 1051 which point the probability drops to zero. The fault has only “short-term memory” because the
 1052 probability of a large earthquake depends only on the time since the past one. c) Modified
 1053 earthquake cycle in which after an earthquake the probability decreases, but not necessarily to
 1054 zero. The fault has “long-term memory” because the probability depends on the earthquake
 1055 history over previous cycles.

1056

1057



1058

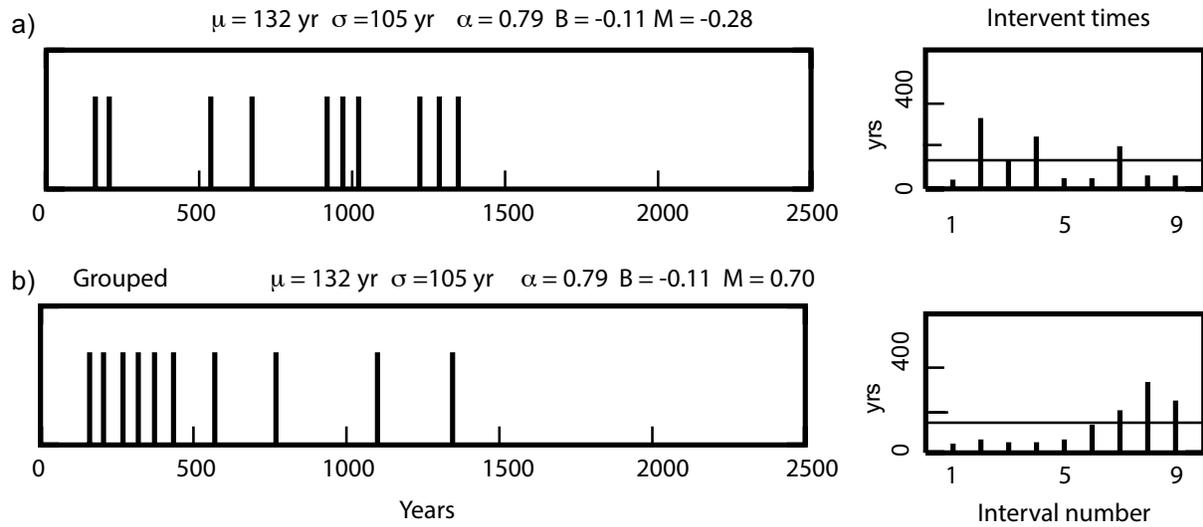
1059

1060 Figure 4: Illustration of characterizing earthquake sequences by their aperiodicity, which
 1061 measures the extent that a sequence differs from perfectly periodic. Values are shown for
 1062 examples in Figure 2. Solid bars show sequences with dates and dashed bars show schematic
 1063 sequences with approximate aperiodicities. Also shown is the result from Goes' (1996) global
 1064 compilation.

1065

1066

SEQUENCES WITH SAME APERIODICITY



1067

1068

1069

1070 Figure 5: Comparison of two sequences with the same aperiodicity. a) Sequence with strong

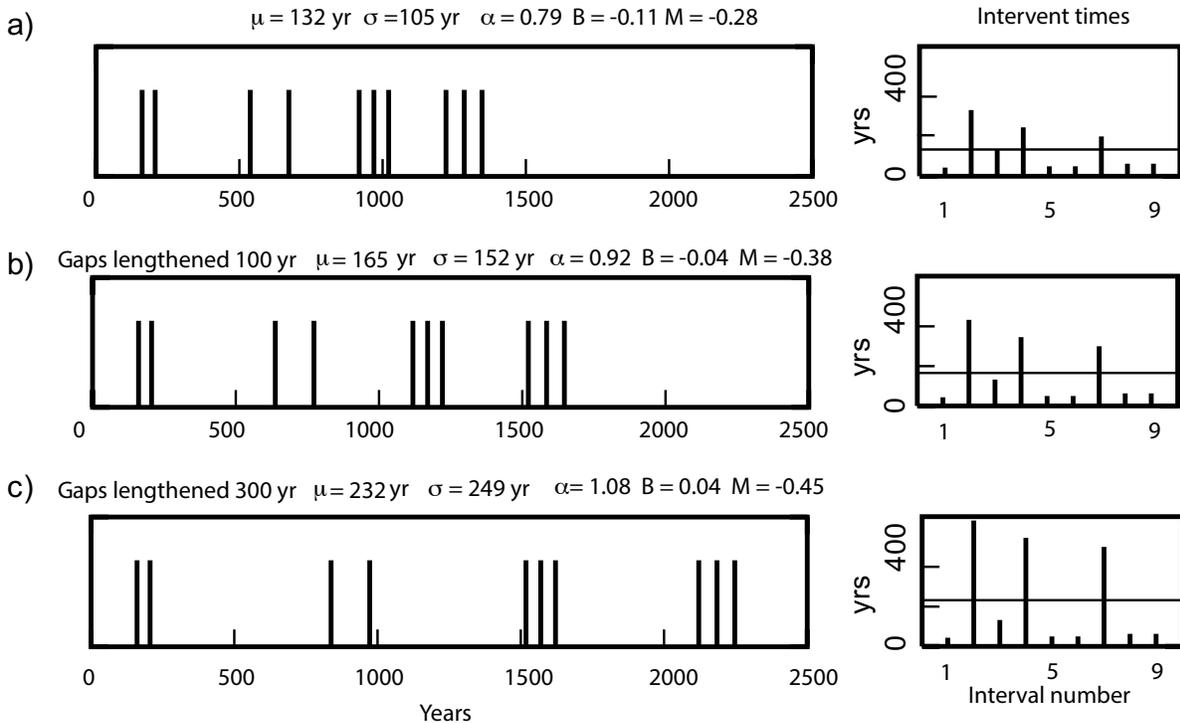
1071 aperiodicity ($\alpha = 0.79$) showing clustering. (b) Same sequence with the short-interval events

1072 grouped together, which does not show clustering.

1073

1074

APERIODICITY AND CLUSTERS



1075

1076

1077 Figure 6: Illustration of the fact that "quasiperiodic" ($\alpha < 1$) sequences can be quite clustered. a)

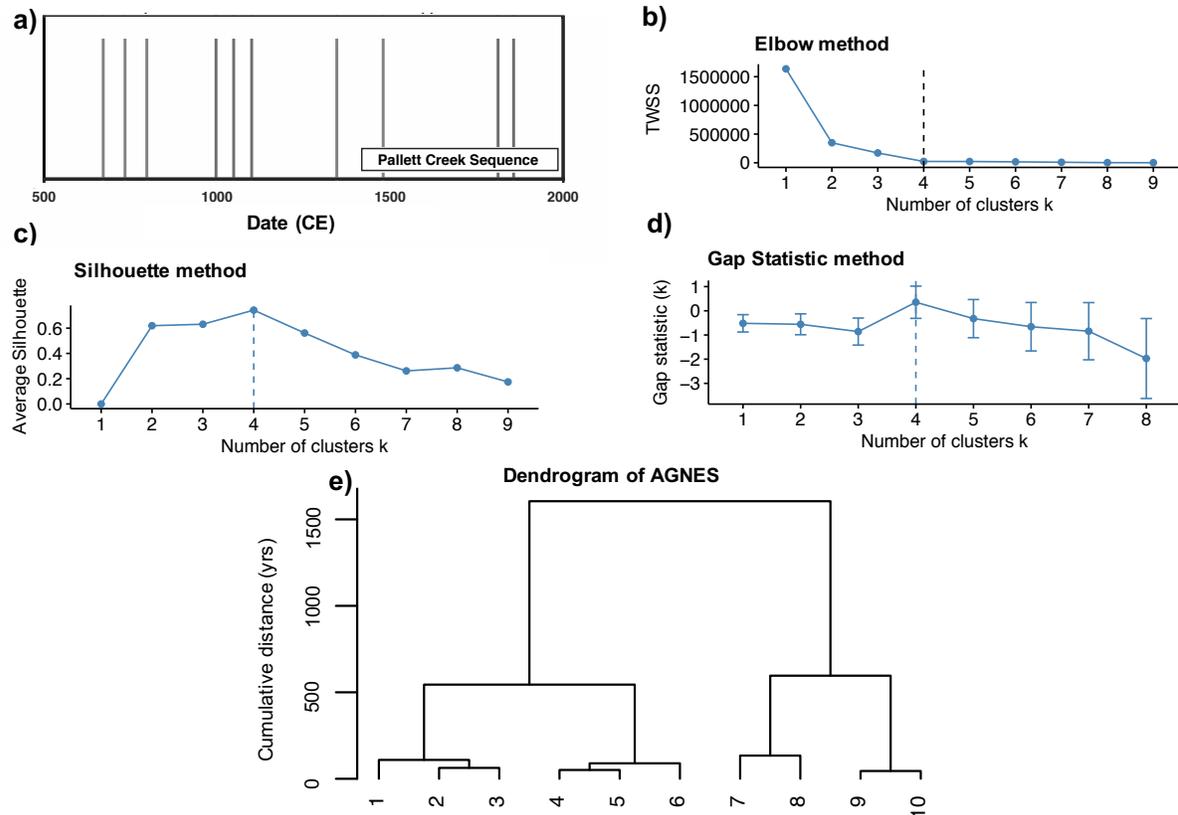
1078 Initial sequence. b) Same sequence with major gaps lengthened by 100 years. c) Same sequence

1079 with major gaps lengthened by 300 years. Only c) has aperiodicity above the nominal burstiness

1080 criterion of $\alpha > 1$.

1081

1082



1083

1084

1085 Figure 7: Results of different methods to determine the number of clusters in a) Pallett Creek
 1086 record of Sieh et al., 1989, with event order corresponding to the figure in part e). b) Elbow
 1087 method where number of clusters is the largest k before increasing k creates only minor
 1088 improvements of TWSS. c) Silhouette method where maximum value indicates number of
 1089 clusters. d) Gap statistic method where maximum value indicates number of clusters. e)
 1090 Hierarchical clustering method using agglomerative nesting (AGNES) with Ward's method;
 1091 vertical axis shows the cumulative length of time between cluster centers being merged at each
 1092 step.

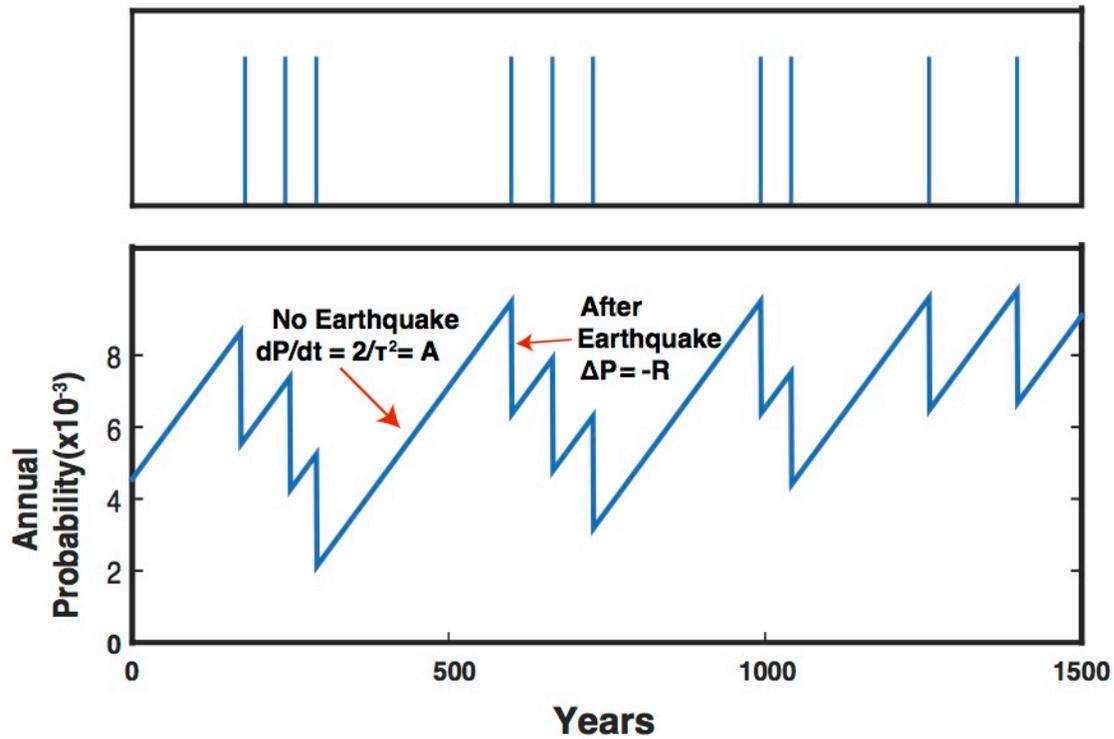
1093

1094

1095

1096

Long-Term Fault Memory Model



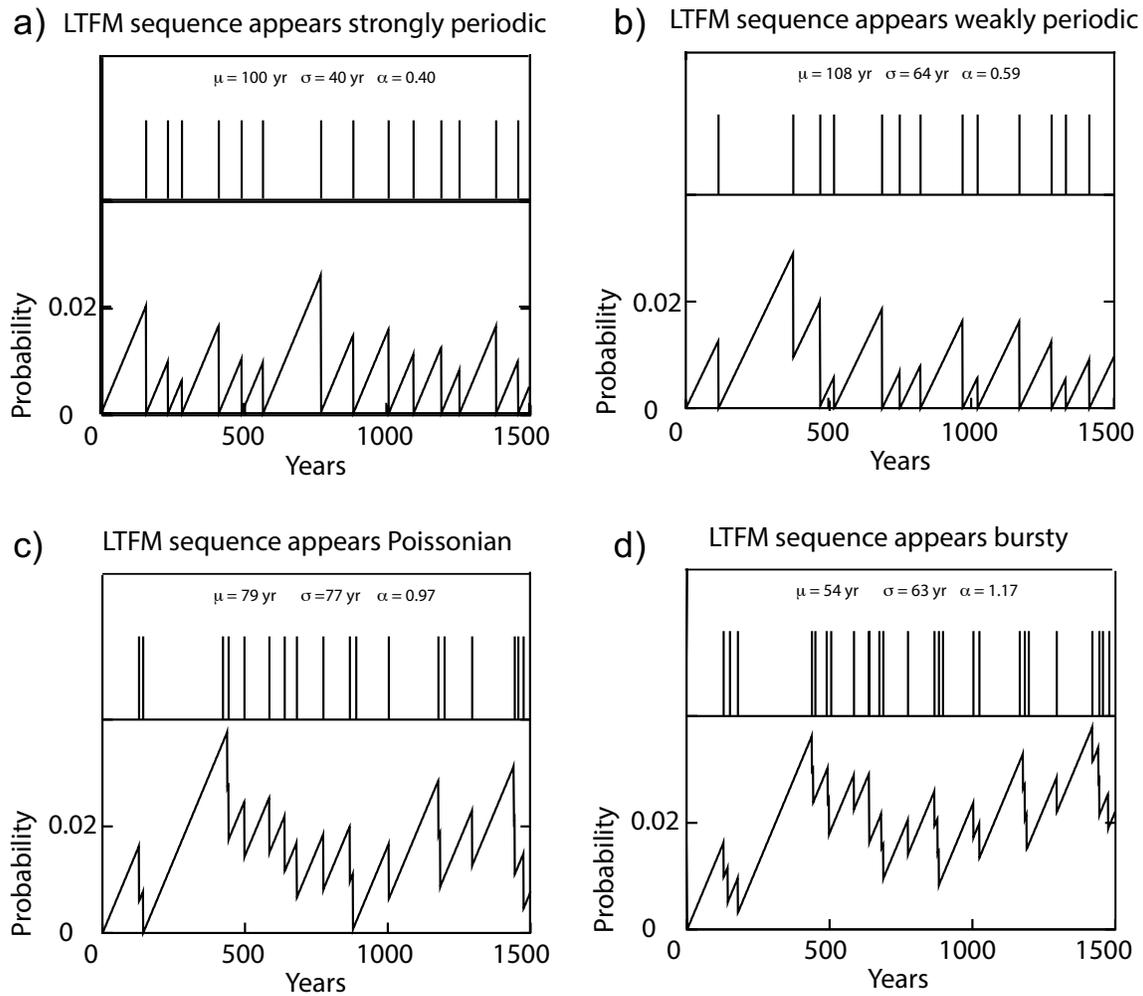
1097

1098 Figure 8: Long-Term Fault Memory model. (Top) Simulated earthquake history. (Bottom)

1099 Earthquake probability versus time.

1100

1101



1102

1103

1104

1105

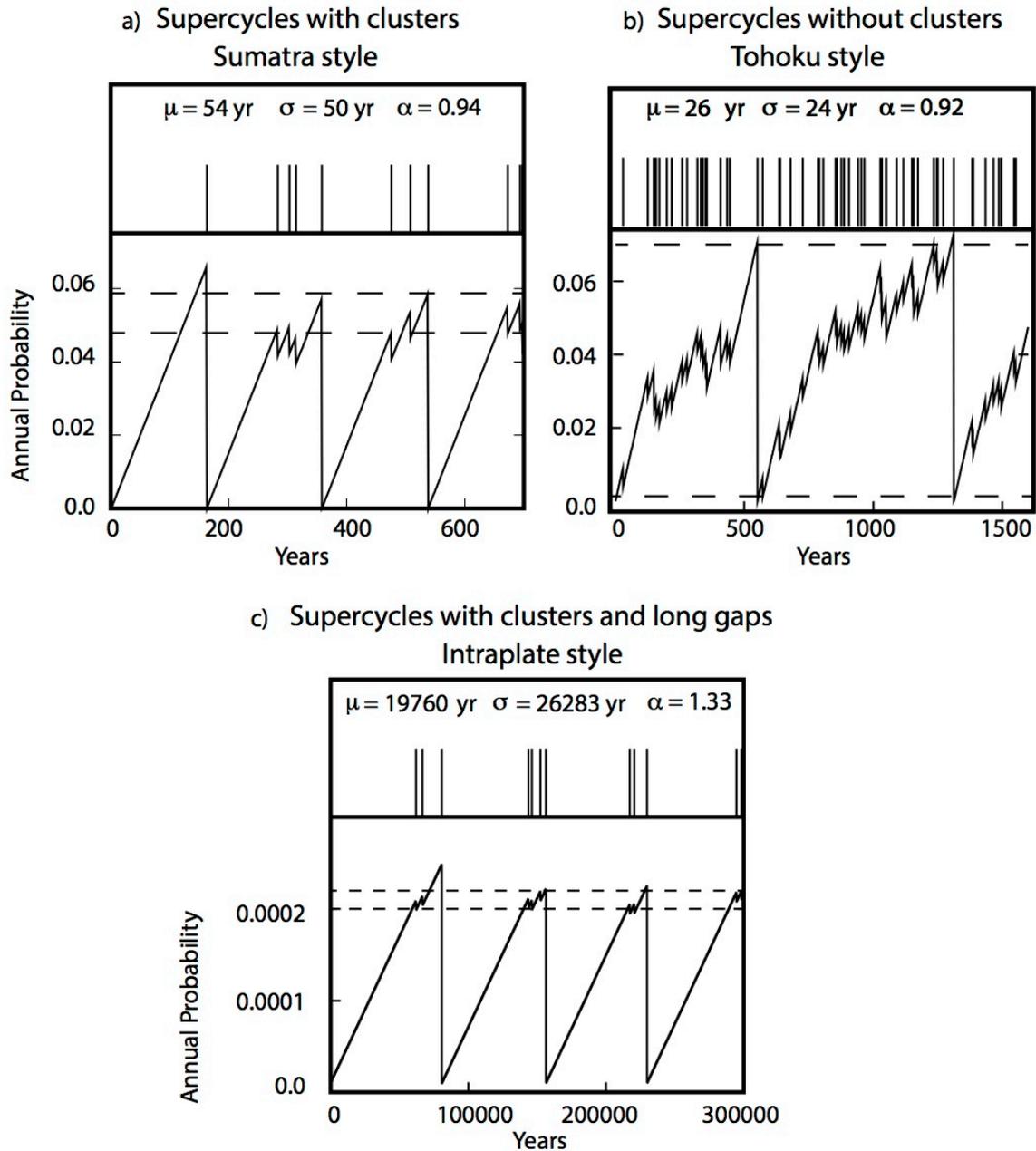
1106

1107

Figure 9: Sequences produced by the LTFM model can appear a) strongly periodic, b) weakly periodic, c) Poissonian, or d) bursty, depending on the model parameters. The four sequences shown have the same probability accumulation rate but different release parameters, so the aperiodicity increases as R decreases.

1108

LTFM with variable strain release

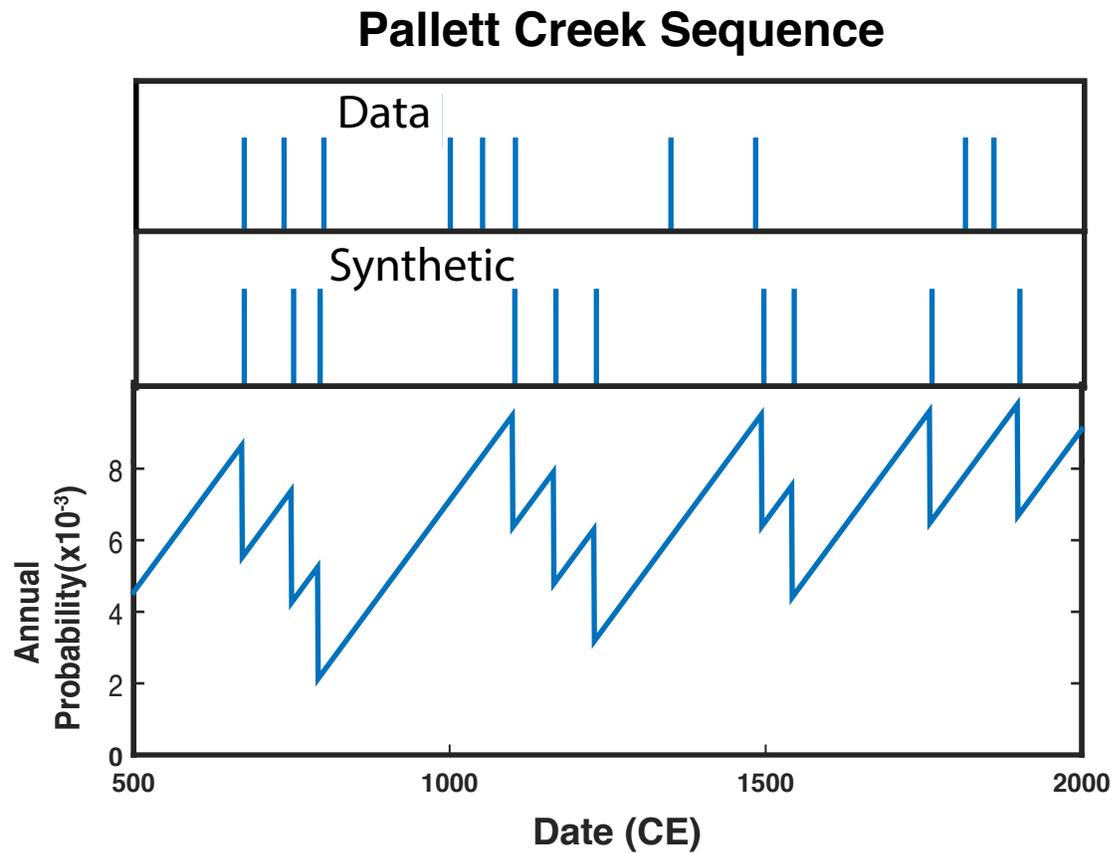


1109

1110

1111 Figure 10: Using two probability thresholds (dashed lines) and probability drops to describe rare
 1112 larger events and more frequent smaller events allows LTFM to simulate a wide range of
 1113 observed supercycle behavior.

1114



1115

1116

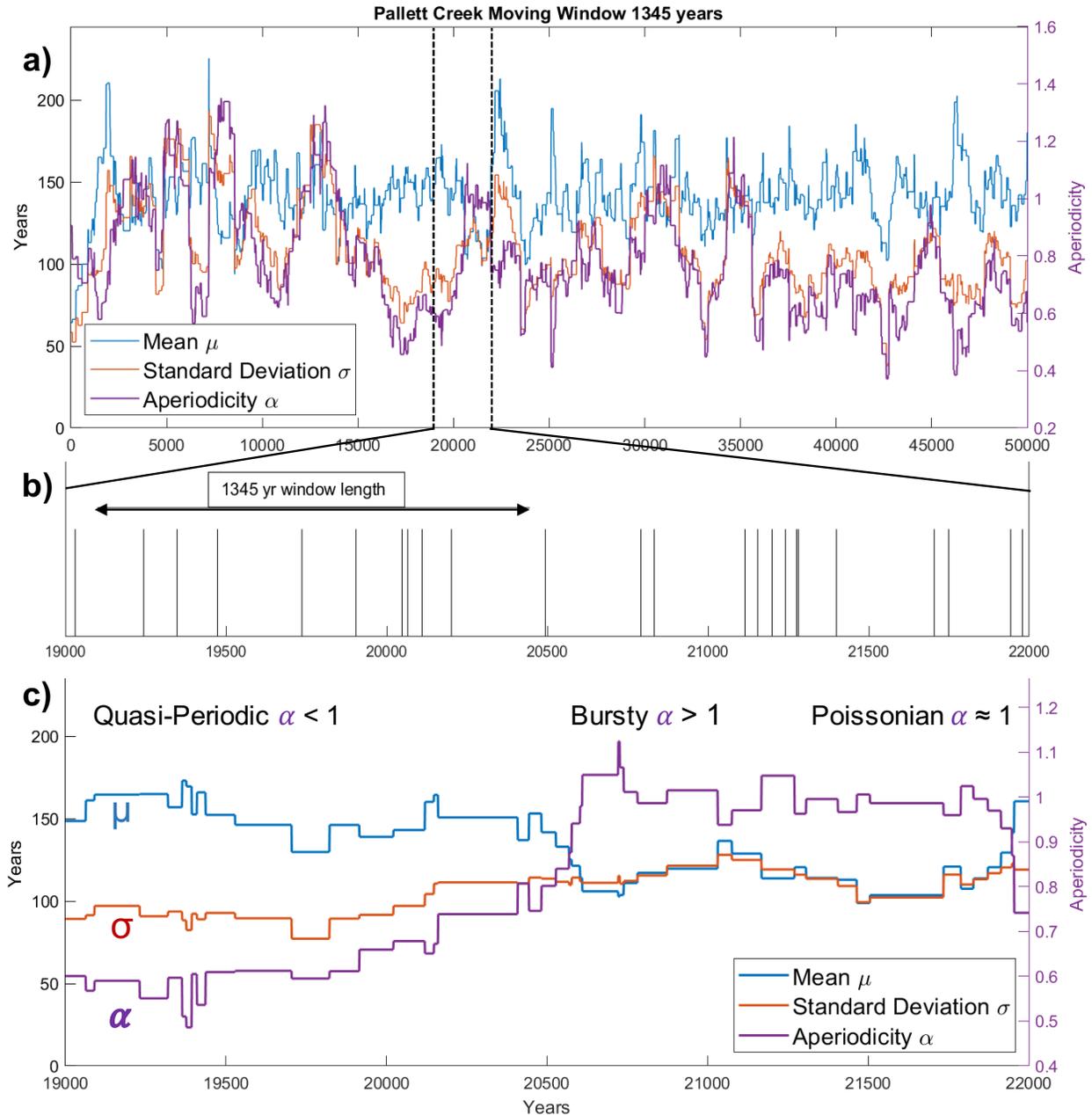
1117

1118 Figure 11: LTFM simulation for Pallett Creek, California. Top: Paleoseismic record (Sieh et al.,

1119 1989). Center and bottom: Simulation giving clustering similar to that observed. The event

1120 timing differs between the simulation and the observed record due to the model's stochastic

1121 nature.

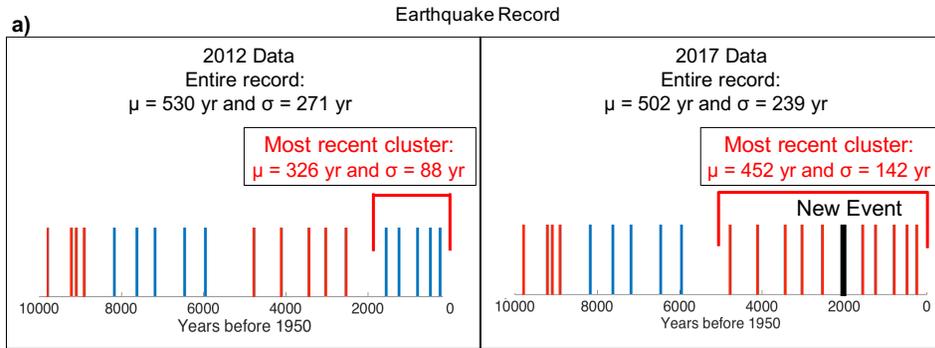


1122
1123

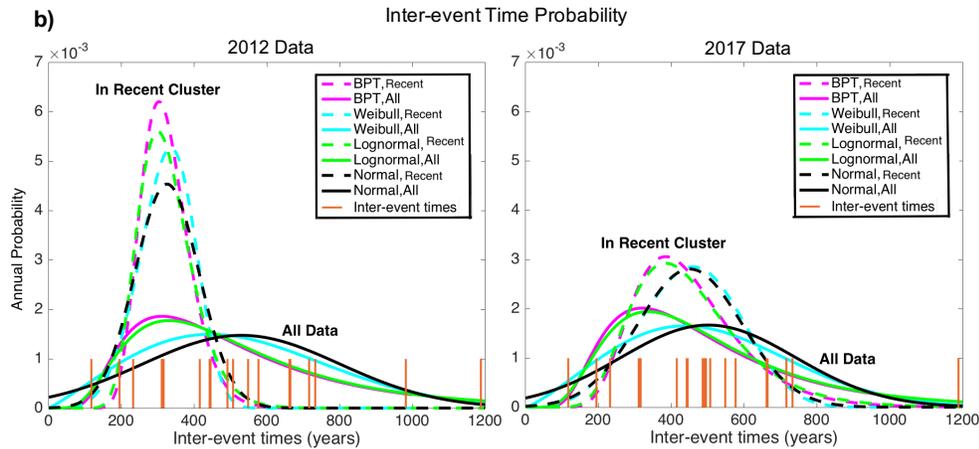
1124 Figure 12: a) 50,000 year LTFM simulation using Pallett Creek parameters. The mean and
1125 standard deviation of recurrence times are averaged over a moving 1345-year window,
1126 corresponding to a paleoseismic record. b, c) 3,000 year section of simulation above between
1127 dashed lines in a). The aperiodicity shows that the simulated paleoseismic record sometimes
1128 appears strongly periodic ($\alpha < 0.5$), while at other times it looks weakly periodic ($0.5 < \alpha < 1$),
1129 Poissonian ($\alpha \approx 1$), or bursty ($\alpha > 1$).

1130

1131

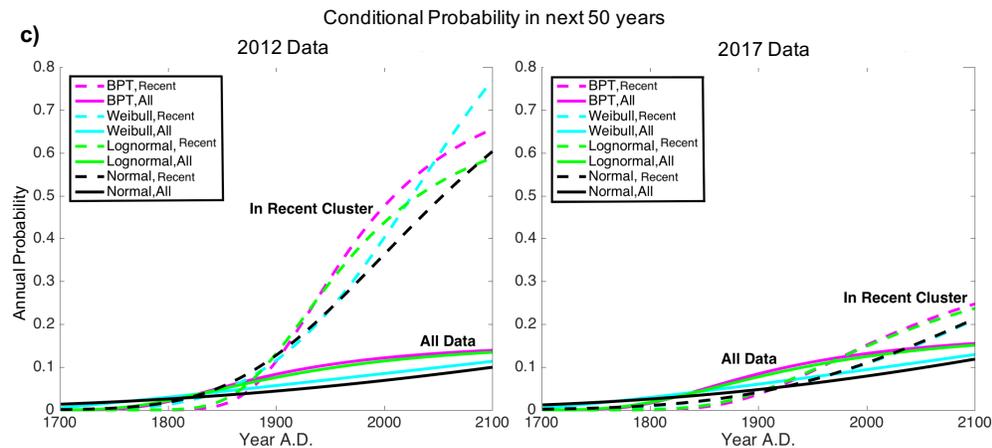


1132



1133

1134



1135 Figure 13: Illustration of earthquake probability issues for Cascadia due to a) differing
 1136 paleoseismic records of Goldfinger et al., 2012 and Goldfinger et al., 2017, with its newly
 1137 discovered event. Alternating red and blue events highlight the different clusters individual
 1138 events are assigned to. b) Various probability density functions for inter-event times with
 1139 parameters derived from the two chronologies in a). Orange sticks show the actual inter-event
 1140 times in the corresponding records. Dashed lines use parameters of just the most recent cluster,
 1141 corresponding to the assumption that the system is still in the recent cluster. Solid lines use the
 1142 parameters of the entire record, corresponding to the assumption that the recent cluster has
 1143 ended. c) Various conditional probabilities of an earthquake occurring in the next 50 years, using
 1144 the same line designations in b). The largest difference in b) and c) arises from the recent cluster
 1145 assumption, not in the specific density function assumed.