Dynamics of Collective Litigation*

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Abstract

In collective litigation, outcomes of previous settlements are used by other potential plaintiffs when choosing their own litigation strategies. We study how collective litigation forms and explore actions that the defendant can take to affect in this process. We propose a model of litigation in which a defendant faces the stochastic arrival of plaintiffs over time and where the defendant is privately informed about the scope of the harm she has caused. While settling with each individual plaintiff is a very effective tool that allows the defendant to avoid litigation, one bad apple may spoil the barrel: when some negotiations are doomed to fail, it “cascades back”, making it possible that earlier (and otherwise frictionless) negotiations also fail. When secret settlements are feasible, equilibrium payoffs look no different than when only secret settlements are allowed. Our results highlight potential social benefits from restricting secrecy.

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1 Introduction

Collective litigation, i.e. a case in which multiple plaintiffs sue a single defendant, is an important part of modern legal systems. The most known form of collective litigation – class action cases in years 2006 and 2007 in the US were worth at least $33 billion in total [Fitzpatrick, 2010]. A single collective litigation suit could be worth several billion dollars. For example, the infamous “Fen-Phen” case, in which American Home Products Corporation was accused of causing heart valve damage to consumers of Fen-Phen dietary pills, resulted in $3.75 billion being paid [White, 2004]. However, individual values of class action claims are typically small. According to Fitzpatrick [2010], in 2006 and 2007 US class action cases majority of plaintiffs obtained compensation of less than $6.

A prominent rationale for collective litigation is that it enables legal action in cases in which individual harms are small but widespread over a large population [Anderson and Trask, 2012, Mulheron, 2004]. Forming a collective, however, has its own challenges. For instance, in the context of product liability disputes, individual plaintiffs typically have insufficient information about the scope of the harm to accurately determine whether or not it is worth litigating. As a result, plaintiffs use the defendant’s observed behavior to make inferences about the number of similar claims in the future, which is linked to the likelihood of a successful litigation. A key aspect of this challenge is that its severity is (at least partially) endogenous: the defendant can implement strategies, such as using secret settlements, that confound learning and exacerbate the uncertainty faced by the plaintiffs.

In this paper, we study the dynamics of settlement negotiations between a privately-informed defendant and several potential plaintiffs who arrive over time. We focus on cases in which no plaintiff expects to be successful in an individual trial or where individual litigation is prohibitively costly. Our main interest is on the extent to which collective litigation actually facilitates the aggregation of small but widespread harms. With this objective in mind, we investigate the defendant’s ability to influence the number of plaintiffs that file the case and the size of the collective that goes to trial.

The main results of our analysis are as follows. First, the ability to settle with each individual plaintiff is a very effective tool for the defendant. Each plaintiff knows that any subsequent arrival will settle the case, therefore it is never worth to file the case on the first place. This is, without any frictions in the settlement negotiations, collectives are unlikely to be formed and individual harms are never aggregated. Interestingly, one bad apple may spoil the barrel: when some negotiations are doomed to fail, it “cascades back”, making it possible that earlier (and otherwise frictionless) negotiations also fail. This occurs because settlement offers become a signaling device through which a defendant can trustfully convey her private information. In this context, we show that whether secret settlements are used is not as relevant as whether they are allowed. Indeed, even if the defendant typically uses public settlements, as long as she has the option to settle secretly, negotiations and filing decisions look no different than when only secret settlements are feasible. Moreover, the availability of secret settlements can be harmful for both the defendant and the plaintiffs. On the other hand, it is never beneficial for both sides simultaneously, suggesting that restricting the possibility of secret settlements can increase social welfare.

In our model the plaintiffs can arrive over three periods, which represent different
stages of the collective litigation. Early plaintiffs do not have any observations about the scope of the harm and decide to file the case based solely on their prior beliefs; intermediate plaintiffs make inferences from previous behavior but remain uncertain about the outcome of the case; late plaintiffs arrive close to a deadline and can accurately predict the mapping between their actions and the outcome of the trial. The scope of the harm, which corresponds to the probability that a plaintiff arrives in a given period, can be low or high and its realization is the defendant’s private information. We refer to this probability as the defendant’s type. After arriving, plaintiffs decide whether to file the case at some cost or to drop it. If a case is filed, with some probability the defendant can approach the plaintiff and propose a settlement agreement (negotiations are said to be frictionless when this probability equals one). The outcome of the trial depends on the number of active litigants at the end of the process.

First, we study the case in which the occurrence of any settlement is publicly observed. When future settlement negotiations are sufficiently likely to exogenously fail a given negotiation becomes a signaling game. Two types of signaling arise in our model: intra-period, corresponding to the inference made by a plaintiff from the offer made to him, and inter-period, corresponding to the inference made by a plaintiff from having or not observed previous settlements. In any separating equilibrium, the offer by a high-type defendant is always accepted while the low-type’s offer is rejected with some strictly positive probability. Moreover, observing a settlement makes the second-period plaintiff more inclined to file the case.

Then, we consider the case in which it is feasible for the defendant to hide the occurrence of a settlement (and the arrival of the plaintiff involved) from other potential litigants. When secret settlements are allowed, equilibrium filing decisions of later plaintiffs period cannot depend on whether a settlement was observed in the earlier periods. This is, the inter-period signaling disappears. The intuition behind this result is simple. Suppose that public settlements increased the likelihood of litigation in future periods. Then, the low-type defendant would be (at least weakly) more inclined to use them, since she is less likely to face future arrivals. However, if that was the case, future plaintiffs should be less (rather than more) likely to file the case upon observing past settlements.

Although the secrecy of a settlement is irrelevant in equilibrium, whether they are allowed is indeed important. When secret settlements are allowed, the plaintiffs’ filing decisions are driven by the prior probability that the scope of the harm is high. When this prior probability is relatively low, the defendant (regardless of her type) benefits from introduction of secret settlements. First, because it decreases the number of filed cases in late periods. Additionally, due to the fact that outcomes of negotiations are influenced by the expectation of future filing decisions, secret settlements facilitate

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1Throughout the paper we assume that the terms of a settlement (i.e. the amount of the offer) are not publicly observed.

2There exists a literature that studies causes of failure in negotiations. See Spier [2007] for a review. Without aiming to be exhaustive, negotiations may fail because plaintiffs are socially motivated or overconfident about their prospects in the trial (see Vasserman and Yildiz [2019]). Additionally, plaintiffs could also be privately informed, such that some types of plaintiffs reject the offer made by the defendant. On the other hand, challenges of logistical nature may impede that the defendant approaches some plaintiffs in time. In this paper, we are agnostic about the ultimate cause of such failures.
successful settlements in the first period. On the other hand, if the prior probability is relatively high, the defendant loses with the introduction of secret settlements as filings in later periods are more likely. However, such additional litigation is mostly driven by plaintiffs filing the case when the scope of the harm is actually low. Hence, the total payoff of the plaintiffs may also decrease as a result of the availability of secret settlements.

The main contribution of this paper is to highlight challenges that collective litigations face when defendants are privately informed. In this context, we add to the literature on litigation in dynamic environments by studying the consequences of the defendant’s manipulation of plaintiff’s learning about the scope of the harm. Daughety and Reinganum [2010], Deffains and Langlais [2011], and Bernhardt and Lee [2014] propose models with symmetric information. On the other hand, Che [1996] and Daughety and Reinganum [2011] study private information on the side of the plaintiffs.

Closer to this paper, there are models of sequential settlement negotiation with a privately informed defendant [Daughety and Reinganum, 1999, 2002]. A common feature of these models is the existence of an exogenous “publicity effect”: litigation in early periods automatically increases the probability of litigation in later periods. A fundamental difference in our approach is the endogeneity of this publicity effect: the extent to which the outcome of a settlement negotiation affects subsequent potential plaintiffs tightly depends on the equilibrium strategies. We show that when the publicity effect is endogenous, it must be the same for public and secret settlements, i.e. filing decisions cannot depend on whether previous settlements are public or secret. Moreover, the publicity effect does not need to be the same when only public settlements are allowed. Finally, our model also differs in its description of settlement negotiations as a signaling game.

The channel through which the secrecy influences the likelihood of settlement in our model is novel. We show that whenever the presence of secret settlements does influence the negotiation with early plaintiffs it is only through its effect on filing decision of the late plaintiffs. As a result, secret settlements either decrease the likelihood of the negotiation failing for both early and late plaintiffs, or increase the likelihood of the negotiation failing for the late plaintiffs without impacting the negotiation with early plaintiffs.

The implications of secret settlements have been studied in other environments. In line with our conclusions, it has been suggested that a firm (which corresponds to the defendant in our setting) may be better off if secret settlements were unavailable [Daughety and Reinganum, 2005]. We complement this analysis by focusing on ways in which the secret settlements influence the litigation process itself, instead of market interactions between the firm and its customers.

The paper is organized as follows. Section 2 introduces the model, Section 3 provides the main results, Section 4 concludes. The proofs of the propositions are placed in the appendix.
2 Model

We model the collective litigation as a three-period game between a defendant and three potential plaintiffs.

In each period $t \in \{1, 2, 3\}$ a potential plaintiff suffers a harm with probability $\lambda_i$, where $i$ represents the (random) scope of an accident caused by the defendant. The accident has a high scope ($i = H$) with a commonly-known probability $\mu$ and a low scope ($i = L$) with complementary probability. We assume $\lambda_H > \lambda_L$, that is, an accident of high scope is more likely to result in a harm. The potential plaintiffs do not observe the scope of the harm, but the defendant does. Therefore, we refer to the scope of the harm as the defendant’s type.

After arrival, the plaintiff decides whether to file the case or not. Filing the case results in a cost $c$ for the plaintiff. After the case is filed, the defendant makes a take-it-or-leave-it settlement offer ($S_t$). The offer has two dimensions: the monetary transfer $s_t \in \mathbb{R}$ and the secrecy regime $\zeta_t \in \{0, 1\}$ (where $\zeta_t = 0$ denotes a secret settlement). After observing the offer, the plaintiff decides whether to accept it and settle the case ($a_t = 1$) or to reject it and litigate the case ($a_t = 0$).

Additionally, we assume that with probability $\eta$ the plaintiff is behavioral. A behavioral plaintiff is one whose actions are fully pre-determined: he always files the case and rejects any settlement offer. In contrast, with probability $1 - \eta$ the plaintiff is strategic and makes decisions in order to maximize his expected payoff. The role of behavioral plaintiffs is to introduce some friction in the negotiation process. It can be justified by different preferences of the plaintiffs: for example, being vengeful and deriving additional utility from participating in a trial. Alternatively, the friction can arise because some plaintiffs are represented by lawyers who could benefit from representing the entire class, or simply because the defendant fails to identify some plaintiffs in time.

The sequence of events within each period $t$ is presented on Figure 1.

![Figure 1: Timing](image)

The outcome of the litigation depends on the amount of plaintiffs that litigate.\footnote{Similar results are obtained, if all the plaintiffs behave strategically when filing the case, as long as some of them reject any settlement offers.}
We focus on the simplest situation: there is a minimal amount of plaintiffs required for the collective litigation to be successful. The closest collective litigation form to this scenario is a class action, when some amount of representative plaintiffs must be gathered to file the case. When the litigation is successful, the defendant is required to transfer the compensation \( w > c \) to each of the participants. Otherwise, the collective litigation fails and no transfer is realized.

We focus on the most interesting scenario, in which the minimal amount of participants is set to 2. We assume that \( \lambda_H w > c > \lambda_L w \), that is, if the scope of accident is known to be low the second period plaintiff would never start a collective litigation, but he may consider it if the scope of the harm is high.

Overall, the payoff of the defendant is given by \(-\left( \sum_t a_t s_t + 1_{k>1}kw \right)\) and the payoff of the period \( t \) plaintiff is given by \( a_t s_t + (1-a_t)1_{k>1}w \), where \( 1_{k>1} \) is an indicator taking value 1 if there is more than 1 litigant at the end of the game, and 0 otherwise.

Plaintiffs form beliefs about the scope of the accident using the Bayes' rule. We denote the probability that the plaintiff arriving in period \( t \) and observing some public history \( h_t \) assigns to the scope of the harm being high by \( \mu_{t,h_t} \). Public histories are composed by two variables: the number of previous litigants \( (k_t) \) and the number of public settlements by period \( t \) \((n_t)\). In other words, a plaintiff does not observe the terms of previous settlements, but only the number of publicly settled cases. Additionally, plaintiffs also update beliefs after observing an offer from the defendant. We denote by \( \mu_{t,h_t}(S_t) \) the probability that the \( t \)-th period plaintiff gives to the scope of the harm being high after observing a history \( h_t \) and an offer \( S_t \).

The solution concept used through the analysis is Perfect Bayesian Equilibrium (PBE) satisfying the D1 criterion \cite{Banks and Sobel 1987}. A complete description of the equilibrium conditions is included in the appendix. Hereinafter, we simply refer to any pair of strategies and beliefs satisfying such conditions as an equilibrium.

### 3 Analysis

Along with the baseline model we consider two simplified versions. Firstly, we study the game under symmetric information. Additionally, we consider a situation with asymmetric information in which all the settlements are public. Then, we move to the more general case, where secret settlements are allowed.

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4 In the model we allow for the plaintiffs to settle the case even after a minimal amount of the representative plaintiffs has been already reached, which corresponds to an opt-in rule. In the United States an opt-out rule is used instead, that is, once the class action case is filed subsequent plaintiffs are automatically participating in the litigation. However, in terms of payoffs, the choice of participation rule is irrelevant in our model.

5 The model is easily extendable for the case in which the payoff from the litigation is described by a strictly increasing triplet \((w_1, w_2, w_3)\). However, whether the negotiation at some period \( t \) with history \( h_t \) result in a separating or a pooling equilibrium depends on the particular choice of the triplet.

6 If \( k = 1 \) the model is a simple sequence of ultimatum games. If \( k = 3 \) only the decision of the first period plaintiff is relevant, hence there is no incentive to manipulate the information for the defendant.
3.1 Symmetric information model

We start the analysis by considering a symmetric information scenario, in which both the plaintiffs and the defendant observe the scope of the harm and hence the probability of arrival (λ) of future plaintiffs.

In any period, a strategic plaintiff always files the case if there already is at least one other litigant. He realizes that the case will be certainly successful if he joins the litigation, and the costs of filing the case will be covered. After the case is filed, the negotiation between the plaintiff and the defendant is a simple ultimatum bargaining game. The defendant proposes a settlement transfer equal to w, which is always accepted in the equilibrium. The scope of the harm is known, so, the selected secrecy regime is irrelevant.

If a strategic plaintiff does not observe any past litigants, he files the case only if the proportion of behavioral plaintiffs is sufficiently high. He realizes that if he litigates any future strategic plaintiff will necessarily settle the case; hence the collective litigation can be successful only if at least one behavioral plaintiff arrives. We denote this probability by ρt. Naturally, ρ3 = 0 and ρt = λη + (1−λη)ρt+1 for t ∈ {1,2}. Once the case is filed, the defendant proposes a settlement offer which exactly covers the expected payoff of the plaintiff, that is, ρtw, and the case is settled in the equilibrium.

The equilibrium of the symmetric information model is summarized in Proposition 1.

Proposition 1. If kₜ > 0 a strategic plaintiff files the case independently of the period. After a case is filed, the defendant makes an offer sₜ,k=1 = w, which is always accepted by the strategic plaintiff. If kₜ = 0 a strategic plaintiff files the case if and only if ρₜ ≥ c/w.

Proposition 1 shows that when the information is symmetric the litigation is completely driven by behavioral plaintiffs. Importantly, it implies that if all plaintiffs are strategic (that is η = 0) no case is ever filed independently of how high are λ and w. Each plaintiff realizes that even if he files the case and decides to litigate, future plaintiffs will free-ride on his decision by settling the case. Hence, it is always optimal to drop the case. Moreover, if η is low the probability of successful collective litigation is small. Hence, a strategic plaintiff never files the case unless there are previous litigants. The cut-off values for η are provided in Corollary 1.

Corollary 1. If η < (1−c/w)/λ, then no strategic plaintiff files the case unless kₜ > 0. If η < (1−c/w)/λ, then a first-period strategic plaintiff always files the case, but a second-period strategic plaintiff files the case only if k₂ = 1. If η > c/(wλ) then a strategic plaintiff in periods 1 and 2 always files the case.

It is worth observing that in standard individual litigation models the settlement can be seen as a positive outcome. It allows the plaintiff to be compensated for the harm by the defendant without incurring litigation costs for both parties and the state. However, in the context of collective litigation this assertion is not necessarily...

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7To be precise, if k₃ = 2 there are two possible outcomes of the sub-game in the final period. One in which the final plaintiff settles the case, and one in which he litigates the case. As they result in the same payoffs for all the players, we ignore the latter possibility.
correct. Indeed, if all the plaintiffs were represented by a single lawyer who negotiates settlement terms on their behalf (as it may happen in class action), it could be socially beneficial to settle the case. But, in our model, the defendant settles the case with each plaintiff separately and is capable of preventing collective litigation. In fact, the mere capability of the defendant of paying-off future plaintiffs is enough to prevent any case from being filed.\footnote{This issue is discussed in more detail in Che and Spier [2008].}

### 3.2 Asymmetric information with public settlements

Before considering the possibility of information manipulation, we study the case in which the information is asymmetric but all the settlements are public. We make two additional assumptions on the value of the parameters. We assume that \( \eta \geq c/(\lambda_H w) \), that is, there exist beliefs sufficiently high for a strategic plaintiff to file the case in the second period even if no previous litigants have been observed. Additionally, let \( \Delta \lambda \equiv \lambda_H - \lambda_L \) denote the difference between arrival rates. Through most of the subsection we assume that \( \lambda_H + \lambda_L \leq 4/3 \), we discuss the consequences of relaxing this assumption at the end of the section.

When \( k_t > 0 \), the analysis is exactly the same as when \( \lambda \) is commonly known, because the success of the collective does not depend on future arrivals. However, in the absence of previous litigants, the results from the symmetric information model do not longer hold. Consider a strategic plaintiff who arrives in periods 1 or 2. He realizes that if he decides to litigate, his success will depend on the arrival of a behavioral plaintiff. Let \( \rho_i^t \) denote the probability that at least one behavioral plaintiff arrives in the future when the defendant is type \( i \). Naturally, \( \rho_i^1 = 0 \) and \( \rho_i^t = \lambda_i \eta + (1 - \lambda_i \eta)\rho_{i,t+1}^t \) for \( t \in \{1,2\} \). The difference between these conditional probabilities is denoted by \( \Delta \rho_t \equiv \rho_H^t - \rho_L^t \). Since the probability of arrival of a new behavioral plaintiff is higher when the scope of the accident is high, the case is filed only when he assigns sufficiently large probability to the scope of the harm being high.

When the case is filed, the interest of the defendant is to settle with a high probability at a low offer. The offers a strategic plaintiff is willing to accept depend on his belief about the state of the world. In particular, the minimum acceptable offer decreases as the belief that the scope of the harm is high decreases. As a result, the defendant always has an incentive to present herself as a low type.

However, in equilibrium, a plaintiff can infer the scope of the harm from the offer made by the defendant. When the scope of the harm is low, the defendant expects future arrivals with low probability. Thus, her expected cost of failing in achieving a settlement is small compared to the defendant of a high type. As a result, a low-type defendant is more willing to risk a rejection of her offer. In equilibrium, the defendant of each type makes an offer that exactly compensates the expected payoff of the plaintiff for the realized type. The plaintiff always accepts the offer coming from the high-type defendant, but he rejects the offer coming from the low-type defendant with some strictly positive probability.

Filing decision depends on the plaintiffs’ prior. A strategic plaintiff can anticipate the outcome of the negotiation, he realizes that he will be always compensated for his expected payoff under litigation. Hence, if a strategic plaintiff does not hold a strong belief that the scope of the harm is high, he will decide to drop the case, unless there
are previous litigants.

Formally, the equilibrium is described in details in Proposition 2. In order to simplify the exposition we describe the decision of the strategic plaintiff in terms of the likelihood ratio \( l_{t,h,t} \equiv \mu_{t,h,t}/(1 - \mu_{t,h,t}) \). All closed form expressions for the thresholds mentioned in the proposition are included in the appendix.

**Proposition 2.** Assume that \( \lambda_H + \lambda_L \leq 4/3 \) and that only public settlements are available. Equilibrium behavior is as follows:

- If \( k_t > 0 \) then
  - A strategic plaintiff files the case at \( t \)
  - After the case is filed, the defendant makes an offer \( s_{t,k} = w \)
  - The offer is always accepted by the strategic plaintiff

- If \( k_t = 0 \) then
  - A strategic plaintiff never files the case in period 3
  - A strategic plaintiff files the case in \( t \in \{1, 2\} \) if and only if \( l_{t,h,t} \) is high enough, i.e \( l_{t,h,t} \geq \bar{l}_t \) for some threshold \( \bar{l}_t \)
  - After the case is filed the defendant makes an offer \( s^H_t,0 = \rho_t w \)
  - The offer \( s^H_t,0 \) is always accepted by the strategic plaintiff, but the offer \( s^L_t,0 \) is rejected with probability \( p_{1,0} > 0 \)

The probabilities of rejecting the low offer in the second period is given by \( p_{2,0} = \frac{\Delta \rho_2}{(\Delta \rho_2 + \lambda_H)} \). On the other hand, in the first period \( p_{1,0} = \tilde{p}_{1,0} \) if the prior \( l \) above some threshold \( \bar{l} \), and \( p_{1,0} = \bar{p}_{1,0} \) otherwise. Note that \( p_{1,0} \) depends on whether the players expect the case to filed in the second period. To be precise, when \( f_{2,0} = 0 \) rejecting the low offer in the first period is given by:

\[
\bar{p}_{1,0} = \frac{\Delta \rho_1}{\Delta \rho_1 + 2\lambda_H(1 - \lambda_H \eta)},
\]

and when \( f_{2,0} = 1 \) it is given by:

\[
\tilde{p}_{1,0} = \frac{\Delta \rho_1}{\Delta \rho_1 + 2\lambda_H(1 - \lambda_H \eta) - \lambda_H^2 \eta(1 - \eta)} > \bar{p}_{1,0}.
\]

There is simple intuition behind this result: if ensuring a settlement in the first period limits litigation in the second period even small probability of rejecting small offers is enough to ensure that the high type defendant will choose settling the case with certainty at a high offer. However, if the settlement in the first period does not change the likelihood of case being filed in the second period a larger probability of rejecting small offers is required to deter high type defendant from making them.

Our results on the negotiation outcome relay on an intuition that a failure to ensure a settlement is more harmful for the high type defendant than for the low type defendant. Hence, a separating equilibrium of the negotiation can be sustained by the plaintiff rejecting low offers with some probability. Indeed, as long as \( \lambda_H + \lambda_L \leq 4/3 \) this intuition is correct. However, once this assumption is relaxed it may be the case
that at least in the first period ensuring a settlement is more valuable for the low type than for the high type defendant. As a result during the first period negotiation only one pooling offer is proposed to a plaintiff, this offer is always accepted. More detailed discussion of the equilibria when $\lambda_H + \lambda_L > 4/3$ can be found in the appendix.

### 3.3 Asymmetric information with endogenous secrecy regime

In this subsection we allow for the secrecy regime to be endogenously determined. That is, while making an offer the defendant chooses not only a transfer size ($s_t$) but also decides whether the potential settlement will be public ($\zeta_t = 1$) or secret ($\zeta_t = 0$). Private settlements prevent future plaintiffs from observing past arrivals. To be precise, the plaintiffs are not able to distinguish between a history in which no litigant arrived in some previous period, from a history in which there was an arrival but the case was secretly settled.

In a three period setting presence of secret settlements can impact only the first period negotiation and the second period filing decision. In the final period there is no more uncertainty and the plaintiff always knows if the case should be filed. As a result it is irrelevant for the defendant whether the case is settled publicly or secretly in the second period. We begin the analysis assuming $\lambda_H + \lambda_L \leq 4/3$.

The first relevant observation is that when secret settlements are available the filing decision of the second period plaintiff cannot depend on whether a previous settlement is observed or not. To illustrate why this must be the case, suppose that the strategic plaintiff in the second period starts the litigation if and only if he observes a previous settlement. Intuitively, the high-type defendant would then prefer to always ensure a secret settlement and prevent any future litigation. On the contrary, the low-type defendant would propose a public settlement. As she faces low probability of plaintiff any subsequent plaintiff arriving, the possibility of the case being filed in the second period is not very costly for her. Therefore, she would prefer to signal her type to the first-period plaintiff and ensure a certain settlement through choosing a public settlement. However, if only the low-type defendant settles the case publicly in the first period, the second-period plaintiff would never file the case after observing a public settlement. Analogous reasoning can be applied to show that it cannot be that the second period plaintiff does not start litigation only if he observes past settlement.

However, there still is a connection between equilibrium behavior of the negotiation in the first period, and the equilibrium filing decision in the second period. First, analogously to public settlements scenario, the negotiation with the low type defendant are more likely to fail in the first period if the parties expect plaintiff in the second period to always file the case. Moreover, unlike in the public settlements scenario, the filing decision of the second period plaintiff depends on his expectation of the negotiation process in the first period. To be precise, the plaintiff in the second period is more keen to file the case despite observing no arrivals when the negotiation are likely to fail in the first period than when the negotiation are likely to succeed. To understand this effect observe that when the second period plaintiff observes no previous litigants or settlements, he believes it can result from three scenarios: no arrival in the first period, secret settlement with the high type defendant, or secret settlement with the low type defendant. If it is difficult for the low type defendant to reach a settlement, the last scenario is unlikely. Hence, the second period plaintiff holds a stronger belief
that the defendant is of a high type, and is more likely to start the litigation on his own.

Combining this two results we conclude that there exist equilibria of two types. A “high litigation equilibrium”, in which the second period plaintiff always files the case and the negotiation in the first period fail often, and a “low litigation equilibrium”, in which the second period strategic plaintiff never starts the litigation and the negotiation in the first period are likely to end in a settlement agreement. Moreover, due to feedback effect of probability of negotiation failing on the decision to file the case the two equilibrium types can coexist.

The equilibria are described in details in Proposition 3. Figure 2 presents the comparison of a second-period strategic plaintiff’s decision in two versions of the model. The upper part of the figure represents the decision of a second-period strategic plaintiff when all the settlements are public, and the lower part of the figure represent this decision when the secret settlements are available.

Proposition 3. If \( \lambda_L + \lambda_H \leq \frac{4}{3} \) and \( l < \tilde{l} \) there exists a low litigation in equilibrium, in which \( f_{2,0} = 0 \). In the first period when the scope of the harm is high the negotiation with strategic plaintiff in the first period are settled at \( s_{1,0}^H \) and when the scope of the harm is low the negotiation with the strategic plaintiff in the first period fails with probability \( \tilde{p}_{1,0} \), and is settled at \( s_{1,0}^L \) otherwise.

If \( l > \tilde{l} \) then there exists a high litigation equilibrium, in which \( f_{2,0} = 1 \). In the first period when the scope of the harm is high the negotiation with strategic plaintiff in the first period are settled at \( s_{1,0}^H \) and when the scope of the harm is low the negotiation fails with probability \( \tilde{p}_{1,0} \), and is settled at \( s_{1,0}^L \) otherwise.

If \( l \in [\tilde{l}, \tilde{l}] \) both equilibria coexist.

The effects of introducing endogenous secrecy regime on the players payoff depend on whether it results in the equilibrium being high or low litigation. Naturally, in the low litigation equilibrium the defendant is better-off and in the plaintiffs are worse-off. The defendant limits the probability of the negotiation failing in the first period, and in expectation faces less litigants in the subsequent period. The plaintiff in the first period is unaffected by the change in secrecy regime, but the second period plaintiff has less information and does not file some cases she would if the settlements are public. Also the third period plaintiff is affected: the negotiation fails less often in the earlier periods of the game, hence, the final period plaintiff is less likely to face previous litigants and ensure profitable settlement or successful trial.
On the contrary, in the high litigation equilibrium the defendant would be better-off committing to settling the case publicly. Then, a second-period strategic plaintiff could always distinguish a history in which there was a previous arrival from a history in which no arrival happened, and he will file the case only in the first scenario. However, if the privacy regime is endogenous when the defendant faces a strategic plaintiff in the first period, it is tempting for her to settle the case privately. Hence, a plaintiff in the second period cannot distinguish between the histories with sufficient precision and he always files the case. Similarly to the low-litigation equilibrium in a high-litigation equilibrium a first-period plaintiff is not affected by endogenizing the secrecy regime but a second-period strategic plaintiff is loosing when secret settlements are allowed. He receives less information through observing the history, and more often files the case when the scope of the harm is low. However, it generates a positive externality on the third period plaintiff. Since the case is filed more often, it is also litigated more often, and the final period plaintiff is more likely to ensure a profitable settlement or a successful trial. As a result, the total expected payoff of the plaintiffs can be higher in a high-litigation equilibrium than when only public settlements are allowed.

Our model does not allow for making predictions on whether secret or public settlements will be chosen – there are multiple equilibria in terms of likelihood of settling the case privately. To be precise, any pair of probabilities of making a public settlement offer by each type of the defendant in period 1 can be sustained as an element of some equilibrium, as long as the decision of the second-period plaintiff is unaffected by the choice of the secrecy regime in the first period. In particular, a decision to always settle the case secretly can always be supported as an element of the equilibrium. It implies that introducing the possibility of settling the case privately is equivalent in terms of payoffs to allowing only secret settlements. This result is summarized in Corollary 2.

**Corollary 2.** *Any equilibrium of the game with endogenous secrecy regime is payoff-equivalent to the equilibrium of the game in which only secret settlements are available.*

Proposition 3 does not take into account possible equilibria, in which the decision to file the case in the second period is random. Whenever both high and low litigation equilibria exist there exist also “intermediate” equilibria. In such equilibria the first period negotiation the defendant mixes between settling the case privately and publicly so that the second period plaintiff becomes indifferent between starting litigation and dropping the case and randomizes between the two. The property that larger probability of the negotiation failing in the first period is associated with larger probability of the case being filed in the second period is retained. The result is summarized in Corollary 3.

**Corollary 3.** *If both the high and the low litigation equilibrium exist, then any probability of starting the litigation by the second period plaintiff \( r \in [0,1] \) can be supported in some equilibrium. An associated probability of the low offer being rejected in the first period negotiation \( p_1(r) \) is increasing in \( r \).*

Finally, when we relax the assumption that \( \lambda_H + \lambda_L \leq \frac{4}{3} \), similarly to the public settlements case, in the first period the plaintiff may not be able to separate between the high- and low-type defendant. Then the defendant makes a single pooling offer that is always accepted by the plaintiff. The results for the second period extend. Second period plaintiff needs to start the litigation or drop the case independently.
from observing past settlement, and the second period negotiation behave as in public settlements case.

4 Conclusion

We study the dynamics of settlement negotiations between a privately-informed defendant and several potential plaintiffs arriving over time. We show that the possibility of break-down in some negotiations affect the possibility of successful settlements in earlier (and otherwise frictionless) negotiations. Additionally, we study the effects of private settlements in this context. We show that introducing a possibility of settling the case privately, is equivalent in terms of payoffs to only secret settlements being present. The defendant gains on availability of private settlements when the plaintiffs hold a low prior about the arrival rate, but loses on it in the opposing scenario. Moreover, there are situations in which both the defendant and the plaintiffs are worse off when secret settlements are introduced. Our results highlight potential benefits from restricting the use of secret settlements.

Some extensions are left for future research. First, we assume in the model that all the costs of litigation are sunk at the moment of filing the case. In reality, at least part of the cost could be saved if the case is not litigated. We conjecture that in this situation semi-separating equilibria, in which the high type defendant pretends to be of a low type with positive probability exist instead. As a result, the case can be endogenously litigated even when the scope of the harm is large. Second, we allow only for a simple information structure, in which the terms of settlement always remain hidden. In reality, the litigants’ choice includes a possibility of settling the case publicly and revealing the terms of settlement to the public. We conjecture that even if such an option is available, it is never used at the equilibrium. Revealing that the case was settled for a low transfer could be beneficial for the defendant, as it could signal low scope of the harm and prevent future litigants from filing. However, when the scope of the harm is small the defendant is already unlikely to face future litigation. Hence, settling the case at a low offer and revealing the terms of the settlement would be especially tempting when the scope of the harm is high. At the equilibrium the plaintiffs can predict this pattern of behavior, and would reject any offer of settling the case publicly at a low transfer. Finally, in our analysis we ignore the role of attorneys. In fact, our model suggests that the attorneys may play much more relevant role in collective litigation than in individual litigation. In particular, apart from providing their services and expertise, they may limit the ability of the defendant to exploit the plaintiffs through sequential settlement by joining the cases and handling the negotiation on behalf multiple litigants.
References


A Solution Concept

Definition 1. We call a pair of strategy profiles of a plaintiff in each period and the defendant of each type, and a belief profile of a plaintiff in each period consists a Perfect Bayesian Equilibrium if:

(i) for all periods $t \leq T$, and all possible realisations of $h_t$, the $t$-th period optimally decides on filing the case given his beliefs taking the strategies of agents in the future periods as given;

(ii) for all periods $t \leq T$, all possible realisations of $h_t$ and all possible offers $S_t$ the $t$-th period plaintiff optimally decides on accepting the offer given his beliefs taking the strategies of agents in the future as given;

(iii) for all periods $t \leq T$, all possible realisations of $h_t$ the defendant of both types optimally decides on making an offer $S_t$ taking the behavior of the plaintiff in the period $t$, and the behavior of all the agents in the future as given;

(iv) for all periods $t \leq T$, and all realisations of $h_t$ which occur with a positive probability on the equilibrium path, the beliefs of the $t$-th period plaintiff at the moment of filing the case follow the Bayes’ rule;

(v) for all periods $t \leq T$, all realisations of $h_t$ which occur with a positive probability on the equilibrium path, and all offers $S_t$ which are made with a positive probability on the equilibrium path the beliefs of the $t$-th period plaintiff at the moment of deciding on accepting the offer follow the Bayes’ rule.

Note that in our setting there can exist a large multiplicity of PBE. It is due to the fact that conditions (i) and (ii) require the plaintiff to act optimally at any public history (or public history and offer pairs) given her beliefs, however, conditions (iv) and (v) pin down the beliefs of the plaintiffs only at these public histories (public history and offer pairs) which happen with a positive probability. Hence, a large variety of behaviors of the plaintiffs can be justified as long as they are not happening on the equilibrium path. However, the out-of-equilibrium path behavior of the plaintiffs can drive the equilibrium behavior of the defendant, which in turn determines which events happen with a positive probability. For example, one can always build an equilibrium in which any sufficiently small offer is never made. It is enough that the plaintiff believes that this particular offer is only made by the high type defendant. Then he will always reject it. But as a result the defendant will never make this offer. Since, the offer is made with 0 probability the beliefs of the plaintiff satisfy the requirement (iv). In principle, similar problem can arise when the plaintiff decides to file the case. However, it matters only in one specific case, namely, the defendant may be driven to never propose public settlement agreements, by a belief of the second period plaintiff that this agreements are only proposed by a high type defendant.

To limit the multiplicity of equilibria we restrict the beliefs that the plaintiff can hold when observing some event with 0 probability of happening on the equilibrium path following commonly used D1 criterion [Banks and Sobel, 1987]. The D1 criterion tends to select equilibria which are on one hand separating (that is the low-type defendant and the high-type defendant make different offers at any history), and on
the other hand are characterized by a high probability of settlement being achieved (to be precise the highest probability of settlement being achieved which allows for separation of types). However, in our setting separation, at least for some histories, may not be possible. In this situation the D1 criterion selects pooling equilibria, in which the settlement offer is always accepted. However, it generally does not select a single offer at which the agreement is reached.

The D1 criterion requires for the plaintiffs not to believe that any given out-of-equilibrium offer \( s \) comes from a defendant of type \( i \), if the set of strategies of the plaintiff for which the defendant of type \( i \) would be better off making the offer \( s \) rather than receiving her equilibrium payoff is a strict subset of the strategies of the plaintiff for which the defendant of type \( j \neq i \) is better off making the offer \( s \) rather than receiving her equilibrium payoff. We call this restriction **deleting a defendant of type \( i \) from an offer \( S_{ht} \).** In our setting, the original definition of Banks and Sobel [1987] can be restated as follows.

**Definition 2.** Take any Perfect Bayesian Equilibrium and consider negotiation in period \( t \) for some history \( h_t \). Take some offer \( S_{ht} \) which is never made on the equilibrium path, and a probability of rejecting the offer which makes the defendant of type \( i \) indifferent between receiving her equilibrium payoff and making an offer \( S_{ht} \): \( p^i(S_{ht}) \). A defendant of type \( i \) is said to be deleted for an offer \( S_{ht} \) if either (i) or (ii) holds:

(i) \( p^i(S_{ht}) < p^j(S_{ht}) \) for some \( j \),

(ii) the defendant of type \( i \) always strictly prefers her equilibrium payoff to certainly settling at an offer \( S_{ht} \), and the defendant of type \( j \) weakly prefers certainly settling at an offer \( S_{ht} \) to her equilibrium payoff.

Having established the notion of deleting a defendant of some type from making a certain offer one can define the D1 criterion as follows,

**Definition 3.** A Perfect Bayesian Equilibrium is said to satisfy the D1 criterion if the following holds:

(i) for any history \( h_t \) and an offer \( S_{ht} \) which is never made for a given history \( \mu_{ht}(S_i) > 0 \) (\( \mu_{ht}(S_i) < 1 \)) only if the defendant of type \( H \) (type \( L \)) is not deleted for the offer \( S_{ht} \);

(ii) for any history \( h_t \) which is never observed at the equilibrium path \( \mu_{ht} > 0 \) (\( \mu_{ht} < 1 \)) only if there exists a sequence of offers \( S_{ht}, ..., S_{ht-1} \), such that the defendant of type \( H \) (type \( L \)) is not deleted for any of the offers, and the offers can generate the history \( h_t \) for some behavior of the plaintiffs in the previous periods.

**B Proofs**

Proof of Proposition 1.

In the final period there is no uncertainty, and the negotiation, whenever the plaintiff files the case, is a simple ultimatum bargaining game. That is, if \( k = 0 \) an offer \( s_3 = 0 \) is made and accepted by a strategic plaintiff. If \( k > 0 \) an offer \( s_3 = w \) is made and accepted by a strategic plaintiff. Since \( 0 < c < w \), the case is filed if and only if \( k > 0 \).
Using backwards induction, if $k_2 = 0$, the plaintiff in the second period expects a payoff of $\lambda \eta w$ from litigation. Hence, if the case is filed, in the equilibrium the defendant makes an offer $\lambda \eta w$ and a strategic plaintiff accepts it. What follows is that the strategic plaintiff files the case if and only if $\eta \geq \frac{c}{\lambda w}$, that is $\rho_2 \geq \frac{c}{w}$.

Analogs reasoning applies in period 1.

Proof of Proposition 2

Proposition 2 is proved by backward induction in lemmas 1–4.

Lemma 1. In period 3 a strategic plaintiff files the case if and only if $k_3 > 0$. If he files the case, it is always settled for $w$.

Since the game in the final period is a simple ultimatum bargaining game the proof is omitted.

Lemma 2. In period 2, if $k_2 = 1$ a strategic plaintiff always files the case and settles it for $w$.

Lemma 2 is the direct consequence of a fact that if there are two participants of the litigation the litigation is necessarily successful and yields a known payoff of $w$ to the plaintiff.

Lemma 3. In period 2, if $k_2 = 0$ in any PBE satisfying D1 criterion:

(i) the defendant of type $i$ makes an offer $s^i_{2,0} = \lambda \eta w$, 

(ii) the plaintiff’s beliefs satisfy $\mu(s^L_{2,0}) = 0$, and $\mu(s) = 1$ for any $s \in (s^L_{2,0}, s^H_{2,0}]$.

(iii) the plaintiff accepts any offer $s \geq s^H_{2,0}$, rejects any offer $s \in (-\infty, s^H_{2,0}) - \{s^L_{2,0}\}$, and rejects an offer $s^L_{2,0}$ with probability $p_{2,0} = \frac{\Delta \rho_2}{\Delta \rho_2 + \lambda H}$.

Lemma 3 is proved in claims 1–3

Claim 1. The described equilibrium is a PBE satisfying the D1 criterion.

Proof. Simple inspection shows that the equilibrium is indeed a PBE: the plaintiff’s beliefs are consistent, and the plaintiff is best responding to his beliefs. Given the response of the plaintiff, there is no profitable deviation for the defendant. In order to show that the equilibrium satisfies the D1 criterion it is enough to prove that the high type is not deleted for any strategy $s \in (s^L_{2,0}, s^H_{2,0})$. That is, a plaintiff can assign a positive probability for the scope of harm being high if an offer $s \in (s^L_{2,0}, s^H_{2,0})$ is observed.

Take any such an offer $s$, then the high type is weakly better off making it if it is rejected with probability at most $p^H(s) \equiv \frac{p_{2,0}(w \lambda_H (1+\eta) - s^L_{2,0}) - (s - s^L_{2,0})}{w (1+\lambda_H (1+\eta) - s)}$. The low type is strictly better off making this offer if it is rejected with probability at most $p^L(s) \equiv \frac{p_{2,0}(w \lambda_L (1+\eta) - s^L_{2,0}) - (s - s^L_{2,0})}{w (1+\lambda_L (1+\eta) - s)}$. Since $p^H(s) \geq p^L(S)$ the equilibrium satisfies the D1 criterion.

Claim 2. There is no PBE satisfying the D1 criterion in which the high-type defendant makes an offer $s < s^H_{2,0}$ with positive probability.
Proof. Take some PBE in which some offer \( s < s^H_{2,0} \) is made with a positive probability by the high-type defendant. Then, it must be the case that this offer is accepted with some positive probability \( 1 - p(s) \). Since it is always the best-response of the plaintiff to accept any offer \( s > s^H_{2,0} \), otherwise the high-type defendant would have a profitable deviation of offering \( s^H_{2,0} + \varepsilon \) and ensuring settlement. Since \( p(s) < 1 \) it must be the case that the plaintiff assigns a positive probability to \( s \) being made by the low-type defendant. Hence, in the equilibrium, the offer \( s \) has to indeed be made with a positive probability also by the low-type defendant. Observe that there can exist only one such an offer. Suppose there are more, and denote any two of them by \( s_1 \) and \( s_2 \). Then it must be the case that both the high type and the low type must be indifferent in between making the offers, that is:

\[
(1 - p(s_1))s_{1,0} + p(s_1)w(1 + \eta)\lambda_i = (1 - p(s_2))s_2 + p(s_2)w(1 + \eta)\lambda_i \quad \text{for } i = H, L, (3)
\]

which yields a contradiction.

Take some offer \( s' = s^L_{2,0} + \varepsilon \) which is not made on the equilibrium path. Then the high-type defendant is better off making the offer \( s' \) than under her equilibrium payoff if and only if it is rejected with probability at most \( p^H(s') = \frac{(1-p(s))s+p(s)(\lambda_H(1+\eta)w)-s'}{\lambda_H(1+\eta)w-s'} \).

The low type is better off making the offer \( s' \) than under her equilibrium payoff if and only if it is rejected with probability at most \( p^L(s') = \frac{(1-p(s))s+p(s)(\lambda_L(1+\eta)w)-s'}{\lambda_L(1+\eta)w-s'} \) since \( p_L(s') < p^H(s') \), if the equilibrium satisfies the D1 criterion, then \( p_{2,h_2}(s') = 0 \). But then the offer \( s' \) is accepted by the plaintiff with probability 1 and the defendant has a profitable deviation.

Claim 3. The described equilibrium is the unique PBE satisfying D1 criterion.

Proof. A consequence of Claim 2 is that the high type always makes an offer \( s^H_{2,0} \) in any PBE satisfying D1. Moreover, since the unique best response of a plaintiff is to always accept any offer \( s > s_{2,0}^H \), the offer \( s_{2,0}^H \) must also always be accepted on the equilibrium path. Otherwise the defendant of a high type would have a profitable deviation of making an offer \( s_{2,0}^H + \varepsilon \).

Observe that the low type cannot make any offer \( s \in (s_{2,0}^L, s_{2,0}^H) \) on the equilibrium path. Otherwise, the equilibrium beliefs of the plaintiff would be \( \mu_{2,h_2}(s) = 0 \) and it would be always accepted. Hence the high-type defendant would have a profitable deviation of making an offer \( s \). Any offer \( s > s_{2,0}^H \) cannot be an element of the equilibrium path, since the defendant would have a profitable deviation of making an offer \( s \in (s_{2,0}^L, s_{2,0}^H) \). An equilibrium in which the low-type defendant makes an offer \( s_{2,0}^H \) cannot satisfy the D1 criterion. The proof follows exactly the proof of Claim 2 and is omitted.
and \( \lim_{s \to s_{2,0}^L} p^L(s) = p \). Thus, there exists \( s \) small enough such that \( p^L(s) < p^H(s) \). Therefore \( \mu_{2,h_2}(s) = 0 \) and the offer \( s \) is always accepted by the plaintiff. Hence, the defendant has a profitable deviation of making the offer \( s \).

Note that the proof applies also for any equilibrium in which the low-type defendant makes an offer \( s < s_{2,0}^L \).

**Lemma 4.** In period 1 in any equilibrium satisfying the D1 criterion:

(i) the defendant of type \( i \) makes an offer \( s_{1,0}^i = \rho^i_1 w \),

(ii) the plaintiff’s beliefs satisfy \( \mu(s_{1,0}^i) = 0 \), and \( \mu(s) = 1 \) for any \( s \in (s_{2,0}^L, s_{2,0}^H) \),

(iii) the plaintiff accepts any offer \( s \geq s^H \), rejects any offer \( s \in (-\infty, s_{2,0}^H) - \{s_{1,0}^i\} \), and rejects an offer \( s_{2,0}^i \) with probability \( p_{1,0} = \frac{\Delta \rho_1}{\Delta \rho_1 + 2\lambda \eta (1 + \eta)} \), if \( \mu < \bar{l} \), and \( p_{1,0} = \frac{\Delta \rho_1}{\Delta \rho_1 + 2\lambda \eta (1 + \eta)} \) otherwise.

We establish the existence of the equilibrium in claims [4] and [5]. Observe that in the described equilibrium the defendant of each type makes an offer exactly compensating the expected payoff of a first-period plaintiff conditional on the realized state of the world, and the high type is exactly indifferent between making an offer \( s_{1,0}^H \) and \( s_{1,0}^L \). Hence, the proof that the described equilibrium is the unique equilibrium satisfying the D1 criterion exactly follows claims [2] and [3]. Therefore, it is omitted.

**Claim 4.** The continuation value of the game for the defendant of type \( i \), given that there is \( k \in \{0,1\} \) plaintiffs litigating by the end of period 1 and a plaintiff in period 1 filed the case is given by: \(-\kappa^i_k(\mu)\) such that:

\[
\kappa^i_k(\mu) = \begin{cases} 
2(1 + \eta) - \lambda \eta^2 & \text{if } k = 1 \\
\lambda^i_2 \eta (1 + \eta) w & \text{if } k = 0 \text{ and } \frac{\mu}{1 - \mu} \leq \bar{l} \\
\lambda^i_2 \eta w \left[ 2 + \frac{\Delta \lambda_1 (1 - \eta)}{\Delta \lambda_1 \eta + \lambda_1 \eta} \mathbb{1}_{\lambda_1 = \lambda_L} \right] & \text{if } k = 0 \text{ and } \frac{\mu}{1 - \mu} > \bar{l} 
\end{cases}
\]  

(4)

**Proof.** Following lemmas [1] and [2] observe that if \( k_1 = 1 \) then a strategic plaintiff files the case in periods 2 and 3 and settles it at \( w \) and a behavioral plaintiff always litigates the case. Hence, the continuation value of the game for the defendant is given by: \(-2(1 + \eta) - \lambda \eta^2 \lambda_i w \).

If \( k = 0 \) there are two cases. Either the plaintiff in the second period files the case, or he does not. If he does not file the case the litigation is driven fully by the behavioral plaintiff. Hence, the continuation value of the game is given by: \(-\lambda^2 \eta (1 + \eta) w \).

If the plaintiff in the second period files the case, then, following Lemma [3] conditional on the arrival of the plaintiff, in the second period the defendant makes an offer exactly compensating the expected payoff of the plaintiff conditional on the scope of the harm. Moreover, the offer made by the low type defendant is rejected with positive probability \( p_{2,0} \). Hence, the continuation value of the game is given by: \(-\lambda^2 \eta w \left[ 2 + \frac{\Delta \lambda_1 (1 - \eta)}{\Delta \lambda_1 \eta + \lambda_1 \eta} \mathbb{1}_{\lambda_1 = \lambda_L} \right] \).

To finish the proof recall that a strategic plaintiff in the second period files the case conditional on \( k_2 = 0 \) if and only if \( l_{2,h_2} \geq \bar{l}_2 \). The beliefs of the plaintiff in period 2 if \( h_2 = (0,1) \) are given by \( l_{2,h_2} = \bar{l}_2 \lambda^2 / \lambda^2_L \). Hence \( l_{2,h_2} = (0,1) \bar{l}_2 \) if and only if \( l \geq \bar{l} \).

**Claim 5.** The described equilibrium is a PBE satisfying D1 criterion.
Proof. We start by analyzing the case when $\mu < \frac{\lambda - \rho_{2}}{\Delta \rho_{2}}$. We firstly show that the proposed strategy profile can be indeed sustained as a PBE. Set the following interim belief profile $\mu_{i,h_{1}}(s) = 1_{s \neq s_{t,0}^{L}}$.

A strategic plaintiff accepts an offer $s_{1,0}^{i}$ from type $i$ if

$$s_{1,0}^{i} \geq [\lambda_{i} \eta + (1 - \lambda_{i}) \lambda_{i} \eta] w = (2 - \lambda_{i}) \lambda_{i} \eta w$$

Thus, the unique best response for the plaintiff is to reject the offer whenever $s_{1,0} \in (s_{1,0}^{L}, s_{1,0}^{H})$. Also note that for $s_{1,0} \in \{s_{1,0}^{L}, s_{1,0}^{H}\}$ the plaintiff is indifferent between accepting or rejecting the offer. Hence, the plaintiff has no profitable deviation.

Note that $p_{1,0}$ is such that the high-type defendant is indifferent between offering $s_{1,0}^{L}$ or $s_{1,0}^{H}$:

$$p_{1,0} \left[ (2 + \eta - \lambda_{H} \eta^{2}) \lambda_{H} w \right] + (1 - p_{1,0}) \left[ s_{1,0}^{L} \eta + \lambda_{H}^{2} \eta(1 + \eta) w \right] = s_{1,0}^{H} \eta(1 + \eta) w$$

$$\iff p_{1,0} \left[ (2 + \eta) \lambda_{H} w - \lambda_{H}^{2} \eta^{2} w - s_{1,0}^{L} \lambda_{H} \eta(1 + \eta) w \right] = s_{1,0}^{H} - s_{1,0}^{L}$$

Using $s_{1,0}^{1} = (2 - \lambda_{i}) \lambda_{i} \eta w$ we get

$$p_{1,0} \left[ (2 + \eta) \lambda_{H} w - \lambda_{H}^{2} \eta^{2} w - (2 - \lambda_{L}) \lambda_{L} \eta w - \lambda_{H}^{2} \eta(1 + \eta) w \right] = (2 - \lambda_{H} - \lambda_{L}) \Delta \lambda \eta w$$

$$\iff p_{1,0} = \frac{(2 - \lambda_{H} - \lambda_{L}) \Delta \lambda \eta}{2 \lambda_{H} \eta^{2} w + (2 - \lambda_{H} - \lambda_{L}) \Delta \lambda \eta} \in (0, 1)$$

Any other offer is either rejected or higher than the equilibrium offer. Hence, she does not have a profitable deviation.

Finally, we show that the low-type defendant does not have a profitable deviation either.

We have that in the proposed equilibrium the payoff for the low-type defendant equals to:

$$-p_{1,0} \left[ (2 + \eta) \lambda_{L} \eta^{2} \lambda_{L} w \right] - (1 - p_{1,0}) \left[ (2 - \lambda_{L}) \lambda_{L} \eta w + \lambda_{L}^{2} \eta(1 + \eta) w \right]$$

$$= -2p_{1,0} w \lambda_{L} (1 - \lambda_{L} \eta^{2}) - w \lambda_{L} \eta(2 + \lambda_{L} \eta)$$

A deviation to any $s_{1,0} \in (s_{1,0}^{L}, s_{1,0}^{H})$ delivers expected payoffs equal to $- \left[ (2 + \eta) - \lambda_{L} \eta^{2} \right] \lambda_{L} w$. Note that

$$-2p_{1,0} w \lambda_{L} (1 - \lambda_{L} \eta^{2}) - w \lambda_{L} \eta(2 + \lambda_{L} \eta) > - \left[ (2 + \eta) - \lambda_{L} \eta^{2} \right] \lambda_{L} w$$

$$\iff p_{1,0} < \frac{1 - \lambda_{L} \eta^{2}}{1 - \lambda_{L} \eta^{2}} = 1$$

which always holds.

Let $g(\lambda_{i})$ be the expected gain for type $\lambda_{i}$ from offering $s_{1,0}^{H}$ instead of $s_{1,0}^{L}$, taking $(s_{1,0}^{L}, s_{1,0}^{H}, p_{1,0})$ as given:

$$g(\lambda_{i}) = -s_{1,0}^{H} \lambda_{i} \eta(1 + \eta) w + p_{1,0} \left[ (2 + \eta) - \lambda_{i} \eta^{2} \right] \lambda_{i} w + (1 - p_{1,0}) \left[ s_{1,0}^{L} + \lambda_{i} \eta(1 + \eta) w \right]$$
We already argued that \( g(\lambda_H) = 0 \), hence \( g(\lambda_L) = g(\lambda_L) - g(\lambda_H) \). As a result

\[
g(\lambda_L) = -s_{1,0}^H - \lambda_L^2 \eta (1 + \eta) w + p_{1,0} [2(1 + \eta) - \lambda_L \eta^2] \lambda_L w + (1 - p_{1,0}) [s_{1,0}^L + \lambda_L^2 \eta (1 + \eta) w]
\]

\[
+ s_{1,0}^H + \lambda_H^2 \eta (1 + \eta) w - p_{1,0} [2(1 + \eta) - \lambda_H \eta^2] \lambda_H w - (1 - p_{1,0}) [s_{1,0}^L + \lambda_H^2 \eta (1 + \eta) w]
\]

\[
eq wp_{1,0} \Delta \lambda \eta (\lambda_H + \lambda_L)(2\eta + 1) - 2(1 + \eta)
\]

Therefore, \( g(\lambda_L) \leq 0 \) if and only if:

\[
(\lambda_L + \lambda_H)/2 \leq (1 + \eta)/(2\eta^2 + \eta)
\]

which is implied by \( \lambda_H + \lambda_L \leq \frac{4}{3} \) for any choice of \( \eta \).

Then, we show that if \( \mu \geq \frac{\phi - \rho^{\mathcal{I}_2}}{\Delta \rho^2} \) the described strategy profile is an element of a PBE.

Set the belief profile to \( \mu_1(s) = 1_{s \neq s_{1,0}} \). The plaintiff is indifferent between accepting or rejecting equilibrium offers. For any offer \( s_{1,0} \in (s_{1,0}^L, s_{1,0}^H) \) the plaintiff strictly prefers to reject. Hence, there is no profitable deviation for the plaintiff.

We choose \( p_{1,0} \) such that the high-type defendant is indifferent between offering \( s_{1,0}^H \) and \( s_{1,0}^L \):

\[
p_{1,0} [2(1 + \eta) - \lambda_H \eta^2] \lambda_H w + (1 - p_{1,0}) [s_{1,0}^L + 2\lambda_H^2 \eta w] = s_{1,0}^H + 2\lambda_H^2 \eta w
\]

\[
\iff p_{1,0} [2 + (2 - \lambda_H) \eta - \lambda_H \eta(1 + \eta)] \lambda_H w - s_{1,0}^L \lambda_H w - s_{1,0}^L = s_{1,0}^H - s_{1,0}^L.
\]

Using \( s_{1,0} = (2 - \lambda) \lambda \eta \) we get

\[
p_{1,0} = \frac{s_{1,0}^H - s_{1,0}^L}{2 - \lambda_H \eta (1 + \eta)} \frac{s_{1,0}^H - s_{1,0}^L}{s_{1,0}^H - s_{1,0}^L} = \frac{(2 - \lambda_H - \lambda_L) \Delta \lambda \eta}{\lambda_H [2 - \eta \lambda_H (1 + \eta)] + (2 - \lambda_H - \lambda_L) \Delta \lambda \eta}.
\]

Finally, we check that the low-type defendant has no incentive to deviate. As in the previous part of the proof let \( g(\lambda) \) be the expected gain for type \( \lambda \) of offering \( s_{1,0}^H \) instead of \( s_{1,0}^L \), taking \( (s_{1,0}^L, s_{1,0}^H, p_{1,0}) \) as given.

\[
g(\lambda) = -s_{1,0}^H - \lambda_L^2 \eta w \left[ 2 + \frac{\Delta \lambda (1 - \eta)}{\Delta \lambda \eta + \lambda_H} I[\lambda = \lambda_L] \right]
\]

\[
+ p_{1,0} [2(1 + \eta) - \lambda_L \eta^2] \lambda_L w + (1 - p_{1,0}) [s_{1,0}^L + 2\lambda_L^2 \eta w \left[ 2 + \frac{\Delta \lambda (1 - \eta)}{\Delta \lambda \eta + \lambda_H} I[\lambda = \lambda_L] \right]]
\]

Since \( g(\lambda_H) = 0 \), we can write \( g(\lambda_L) = g(\lambda_L) - g(\lambda_H) \) to get the following expression:

\[
g(\lambda_L) = -s_{1,0}^H - \lambda_L^2 \eta w \left[ 2 + \frac{\Delta \lambda (1 - \eta)}{\Delta \lambda \eta + \lambda_H} \right]
\]

\[
+ p_{1,0} [2(1 + \eta) - \lambda_L \eta^2] \lambda_L w + (1 - p_{1,0}) \left[ s_{1,0}^L + 2\lambda_L^2 \eta w \left[ 2 + \frac{\Delta \lambda (1 - \eta)}{\Delta \lambda \eta + \lambda_H} \right] \right]
\]

\[
+ s_{1,0}^H + 2\lambda_H^2 \eta w - p_{1,0} [2(1 + \eta) - \lambda_H \eta^2] \lambda_H w - (1 - p_{1,0}) [s_{1,0}^L + 2\lambda_H^2 \eta w]
\]

\[
eq p_{1,0} \Delta \lambda \left[ -2(1 + \eta) - \lambda_L \eta \frac{1 - \eta}{\Delta \lambda \eta + \lambda_H} + (\lambda_H + \lambda_L)(2\eta + \eta^2) \right]
\]

\[20\]
Therefore, \( g(\lambda_L) \leq 0 \) if and only if

\[
\frac{\lambda_H + \lambda_L}{2} \leq \frac{1 + \eta}{2\eta + \eta^2} + \frac{\lambda_H^2(1 - \eta)}{2(2 + \eta)(\Delta \lambda \eta + \lambda_H)},
\]

which is implied by \( \lambda_H + \lambda_L \geq \frac{4}{3} \) for any choice of \( \eta \).

To finish the proof, we show that the proposed equilibrium satisfies the D1 criterion. To prove it, it is enough to show that the high-type defendant is not eliminated for any strategy \( s \in (s_{1,0}^L, s_{2,0}^H) \) under the D1 criterion.

Take any such an offer. Then the defendant of type \( i \) is better-off making the offer \( s \) rather than under her equilibrium payoff if and only if the offer \( s \) is rejected at most with probability

\[
p'(s) = p_{1,0} - \frac{\rho_i - \rho_{i,0} + s}{\kappa_i - \kappa_{i,0} + s_L}.
\]

Recall that the low type never has profitable deviation of proposing \( s_{1,0}^H \), and the high type never has a profitable deviation of proposing \( s_{2,0}^L \). Hence, it is always the case that \( \kappa_H^i - \kappa_{H,0} > \kappa_L^i - \kappa_{L,0} \). Hence \( p^H(s) > p^L(s) \) and the defendant of the high type is not eliminated for strategies \( s \in (s_{1,0}^L, s_{2,0}^H) \).

Proof of Proposition 3: Proposition 3 is proved in lemmas 5 – 7. The proof includes only the analysis of the negotiation in period 1 and the decision on filing the case in period 2, as other subgames follow exactly the proof of Proposition 2.

**Lemma 5.** In any equilibrium satisfying the D1-criterion during the negotiation in the first period:

(i) the defendant makes an offer including a transfer \( s_1 = \rho^L_1 w \).

(ii) A pair of probabilities \((q^H, q^L)\) with which the i-type defendant makes a public settlement offer can be supported as a part of some equilibrium if and only if the decision of the second period plaintiff is independent from observing a public settlement.

(iii) The plaintiff always accepts the offer with a transfer \( s_1^H \), and rejects the offer with a transfer \( s_1^L \) with some positive probability.

**Lemma 5** is proved in claims 6 – 10.

**Claim 6.** In any PBE satisfying the D1 criterion during the first-period negotiation the defendant of a high type makes an offer \( s_{1,0}^H \).

**Proof.** Firstly, observe that a strategic plaintiff always accepts any offer including a transfer \( s > s_{1,0}^H \) independently of the secrecy regime proposed. Hence, no offer \( s > s_{1,0}^H \) can be made in the equilibrium.

Take some candidate equilibrium in which the high type makes the offer \( S = (s, \zeta) \) where \( s < s_{1,0}^H \). Then it must be the case that this offer is not rejected with probability 1, but only with some probability \( p \). Hence, the low type must make an offer \( S \) with positive probability. Following the proof of Claim 2, recall that, for a given \( \zeta \), there exists at most one such an offer.

Then take some offer \( S' = (s', \zeta) \), which is not made on the equilibrium path, with \( s' = s_{1,0}^L + \epsilon \) and \( \zeta \) that is used in the offer \( S \). Recall from Claim 4 the values of the continuation game for the defendant \( \kappa_k \). Observe that if the second-period plaintiff
files the case after observing history \( h_2 = (0, \zeta) \), then \( \kappa_i^0 = \lambda_i^2 \eta w \left[ 2 + \frac{\Delta \lambda_i (1 - \eta)}{\Delta \lambda_i + \lambda_H \lambda_i} \right] \). and otherwise \( \kappa_i^1 = \lambda_i^2 \eta (1 + \eta) w \). Moreover \( \kappa_i^1 \) remains unchanged.

Hence, the \( i \)-type defendant is better-off making an offer \( S' \) if it is rejected with probability at most \( p'(S') = \frac{p(\kappa_i - \kappa_0) - s - s'}{\kappa_1 - \kappa_0 - s'} \). Recall from the proof of Claim 5 that \( \kappa_i^H - \kappa_0^H > \kappa_i^L - \kappa_0^L \). Hence, \( p^H(S') < p^L(S') \), and if the equilibrium satisfies the D1 criterion \( \mu_{i,h_i}(S') = 0 \). Therefore, the offer \( S' \) is always accepted by the plaintiff, and the defendant has a profitable deviation of making the offer \( S' \).

**Claim 7.** In any PBE satisfying the D1 criterion during the first-period negotiation the defendant of a low type makes an offer \( s_{1,0}^L \).

**Proof.** Claim 6 implies that there does not exist an equilibrium in which the low-type defendant makes an offer with a transfer \( s > s_{1,0}^L \). If \( s \in (s_{1,0}^L, s_{1,0}^H) \) in a candidate equilibrium, then the offer made by the low type is always accepted and the high type has a profitable deviation of making an offer \( s \). If \( s \geq s_{1,0}^H \) then the proof of Claim 6 applies, and there exists some offer \( S' \) with a transfer \( s' = s_{1,0}^L + \epsilon \), which is always accepted by the plaintiff. Thus, the defendant has a profitable deviation of making the offer \( S' \).

Suppose there exists an equilibrium, in which some offer \( s < s_{1,0}^L \) is made by the defendant of the low type. Then, it is always rejected by the plaintiff. Consider some offer \( S' \) with a transfer \( s' = s_{1,0}^L + \epsilon \). Then the plaintiff of the low type is better-off making this offer than under her equilibrium payoff if it is accepted with any positive probability. The plaintiff of the high type is better-off making the offer \( S' \) only if it is accepted with a probability higher than some threshold. Hence, if the equilibrium satisfies the D1 criterion, \( \mu_{1,h_1}(S') = 0 \), and the offer \( S' \) is always accepted. Therefore the defendant has a profitable deviation of making an offer \( S' \).

**Claim 8.** There does not exist a PBE satisfying the D1 criterion, in which the second-period plaintiff files the case upon observing \( h_2 = (0, 0) \) but not upon observing \( h_2 = (0, 1) \).

**Proof.** Take any such candidate equilibrium. Then, it must that the high-type defendant settles the case secretly with some positive probability. Hence, the high-type defendant has a profitable deviation of proposing a public settlement with probability 1.

**Claim 9.** There does not exist a PBE satisfying the D1 criterion, in which the case is settled publicly with some positive probability and the second-period plaintiff files the case upon observing \( h_2 = (0, 1) \), but not \( h_2 = (0, 0) \).

**Proof.** Take any such an equilibrium. Then, it must be that the high type proposes a public settlement with some positive probability. Hence, she has a profitable deviation of proposing a secret settlement with probability 1.

**Claim 10.** There does not exist a PBE satisfying the D1 criterion, in which the second-period plaintiff files the case upon observing \( h_2 = (0, 1) \), but not \( h_2 = (0, 0) \).

**Proof.** Claim 9 proves the case when the case is settled publicly with some positive probability.
Suppose there exists an equilibrium in which the case is always settled secretly in the first period, and the second-period strategic plaintiff files the case if he observes \(h_2 = (0, 1)\), but not \(h_2 = (0, 0)\).

Observe that in any such an equilibrium, the low offer during the first-period negotiation must be rejected with some probability \(p \geq p_{1,0} = \frac{\Delta \rho_1}{\Delta \rho_1 + 2 \lambda_H (1-\lambda_H \eta - \lambda_H (1-\eta))}\).

Denote by \(-\kappa^i_0(\zeta)\) the value of the continuation game for the defendant of type \(i\), if the case in period 1 is settled at a privacy regime \(\zeta\). Following the proof of Claim 4, \(\kappa^i_0(0) = \lambda^2_\eta (1+\eta) w\), and \(\kappa^i_0(1) = \lambda^2_\eta w \left[ 2 + \frac{\Delta \lambda_\eta (1-\eta)}{\Delta \rho_1 + \lambda_H} \right] \).

Consider an offer \(S' = (s^i, 1)\) with \(\zeta = 1\). Then, the high-type defendant is better-off making the offer \(S'\) than under her equilibrium if it is rejected with probability at most \(\rho^H = \frac{s^H_0 - s^H - \kappa^H_0(1) - \kappa^H_0(1)}{\kappa^H_0(1)}\). And the low-type defendant is better-off making the offer \(S'\) than under her equilibrium if it is rejected with probability at most \(\rho^L = \frac{s^L_0 - s^L_0 - \kappa^L_0(0) - \kappa^L_0(0)}{\kappa^L_0(0) - \kappa^L_0(0)}\).

We claim that for \(\varepsilon\) small enough it must be the case that \(p^H < p^L\) and \(p^L > 0\). Observe that \(p^L\) is increasing in \(p\), hence take the smallest possible \(p = p_{1,0}\). Knowing that if \(p = p_{1,0}\), the defendant of a high type is indifferent between making an offer \(S = (s^H_0, 0)\) and \(S = (s^L_0, 0)\), we can restate the expression for \(p^i\) where \(i = H, L\) in the following way:

\[
p_{1,0} \kappa^i_1 + (1 - p_{1,0})(\kappa^i_0(0) + s^i_1) = p^i \kappa^i_1 + (1 - p^i)(s^i + \kappa^i_0(0)) + (1 - p^i)(\kappa^i_0(1) - \kappa^i_0(0)).
\]

(9)

Hence, if \(\kappa^H_0(1) - \kappa^H_0(0) > \kappa^L_0(1) - \kappa^L_0(0)\), there exists an offer \(s'\) sufficiently close to \(s^i_1\) for which indeed \(p^L\) is strictly smaller than \(p^H\). Substituting for \(\kappa^i_k\)'s and simplifying we obtain:

\[
\lambda^2_H > \lambda^2_L + \lambda^2_L p_{2,0}.
\]

(10)

Substituting for \(p_{2,0}\) by \(\frac{\Delta \lambda_\eta}{\Delta \rho_1 + \lambda_H}\) and simplifying we obtain:

\[
(\lambda_H + \lambda_L)(\eta \Delta \lambda_\eta \lambda_H) > \lambda^2_L \eta,
\]

(11)

which must be always satisfied, since \(\lambda_H > \lambda_L \eta\) and \(\lambda_H + \lambda_L > \lambda_L\).

Thus, if the out-of-equilibrium beliefs follow the D1 criterion, there exists an offer \(S' = (s^i, s^1_0, \zeta = 1)\) such that \(\mu(S') = 0\). This offer is always accepted by the plaintiff and (by the fact that \(p^L > 0\)) the defendant of a low type has a profitable deviation of making the offer \(S'\).

\textbf{Lemma 6.} If the probability of rejection of the offer \(s^L_1\) during the first-period negotiation is given by \(p\), then in any PBE equilibrium satisfying the D1 criterion in which the decision on filing the case is taken in pure strategies, the second-period strategic plaintiff files the case upon observing \(k_2 = 0\) if and only if \(l \geq l(p) = \frac{l_2 \lambda_L (1-\eta \lambda_L - p \lambda_L (1-\eta))}{\lambda_H (1-\eta \lambda_H)}\).

Otherwise, there exists an equilibrium in which the second-period plaintiff always files the case.

\textbf{Proof.} Following Lemma [5] it must be that a decision of a second-period plaintiff is independent from the realization of \(n_2\).

Denote by \(q^i\) the probability with which the defendant of type \(i\) proposes a public
settlement in period 1. Then, if an equilibrium in which the second-period plaintiff never starts the litigation exists, there must exist a pair \((q^H, q^L) \in [0,1]^2\) satisfying the following two conditions:

\[
\tilde{l}_2 \leq l \frac{\lambda_H \left( \lambda_H (1 - \eta)(1 - q^H) + 1 - \lambda_H \right)}{\lambda_L \left( (1 - \eta)(1 - p)(1 - q^L) + 1 - \lambda_L \right)},
\]

\[
\tilde{l}_2 \leq l \frac{\lambda_H \lambda_L (1 - \eta)q^H}{\lambda_L, \lambda_L (1 - \eta)(1 - p)q^L}.
\]

Condition (12) ensures that a second-period strategic plaintiff does not file the case if he observes \(h_2 = (0,0)\), condition (13) ensures that a second-period strategic plaintiff does not file the case if he observes \(h_2 = (0,1)\).

Rearranging the conditions we obtain:

\[
q^L \geq \frac{\lambda^2_H q^H}{1 - p} \left( \frac{l \lambda^2_H q^H}{\tilde{l}_2 \lambda^2_L q^H} - \frac{l \lambda_H (1 - \eta \lambda_H)}{l \lambda_L (1 - \eta)} + \frac{1 - ((1 - \eta)p + \eta) \lambda_L - 1 - \lambda_L (1 - \eta)}{\lambda_L (1 - \eta)} \right),
\]

\[
q^L \leq \frac{1}{1 - p} \frac{l \lambda^2_H q^H}{\lambda_L (1 - \eta)}.
\]

From (14) and (15) we get that the set of \((q^H, q^L) \in [0,1]^2\) satisfying (12) and (13) is non-empty if and only if:

\[
\frac{1 - ((1 - \eta)p + \eta) \lambda_L}{\lambda_L (1 - \eta)} \leq \frac{l \lambda_H (1 - \eta \lambda_H)}{l \lambda_L (1 - \eta)}.
\]

Solving (16) for \(l\) the following condition is obtained:

\[
l \leq \tilde{l}_2 \frac{\lambda_L (1 - \lambda_L \eta - p \lambda_L (1 - \eta))}{\lambda_H (1 - \lambda_H \eta)} = \tilde{\tilde{l}}(p).
\]

Hence the equilibrium, in which a second-period strategic plaintiff never starts the litigation exists if and only if \(l \leq \tilde{\tilde{l}}(p)\).

The proof that the equilibrium in which a second-period plaintiff always files the case exists if and only if \(l > \tilde{\tilde{l}}(p)\) follows exactly the same steps, and requires only reversing the direction of inequalities.

**Lemma 7.** If \(l \leq \tilde{\tilde{l}}(\tilde{p}_{1,0})\) then there exists a PBE satisfying the D1-criterion, in which the second-period strategic plaintiff never starts the litigation, and the probability of rejecting the offer \(s^L_{1,0}\) during the first period negotiation is given by \(\tilde{p}_{1,0}\).

If \(l \geq \tilde{\tilde{l}}(\tilde{p}_{1,0})\) then there exists a PBE satisfying the D1-criterion, in which the second-period plaintiff always files the case, and the probability of rejecting the offer \(s^L_{1,0}\) during the first period negotiation is given by \(\tilde{p}_{1,0}\).

No other PBE satisfying the D1 criterion, in which the decision on filing the case is taken in pure strategies exists.
Proof. Observe that $\bar{p}_{1,0}$ is probability of rejecting the low offer during the first-period negotiation which makes the high-type defendant exactly indifferent between making the offer $s_{L,1}^1$ and $s_{H,1}^1$, conditional on the second-period plaintiff always filing the case. The proof that if a second-period plaintiff always files the case, then in any PBE satisfying the D1-criterion during the first-period negotiation the low offer is rejected with probability $\bar{p}_{1,0}$ follows the proof of Proposition 2. Hence, the existence condition is a corollary of Lemma 6.

Analogous reasoning applies for the equilibrium in which a second-period strategic plaintiff never starts the litigation.

C Equilibria of the game when $\lambda_H + \lambda_L > \frac{3}{4}$

In this section we briefly discuss the effects of allowing $\lambda_H$ and $\lambda_L$ to large. As long as these parameters are not accompanied by a large value of $\eta$ the analysis presented in the main body of the paper applies. However, if $\eta$ is large the negotiation in the first period of the game changes – to be precise separating equilibria of the negotiation cede to exist. It is a consequence of the fact that the settlement in the first period becomes more valuable for the defendant of the low type than for the defendant of the high type when $\eta$ grows large. Since separating equilibria require that the offers made by the low type are rejected with positive probability, whenever the low type values the settlement more than the high type, they cede to exist.

To better understand this result consider the following example: suppose $w = 1$, $H = 1$, $L = 0.5$ and $\eta$ is very close to one. For the high type defendant the litigation is almost inevitable regardless of the outcome of the negotiation in the first period, hence, she is willing to make an offer of at most 1. However, the low type can diminish the likelihood of the collective forming by settling in the first period. To be precise, if the case fails to be settled in the first period the low type defendant faces litigation with probability of approximately 0.75, and when the settlement is secured this probability diminishes to 0.25. As a result, the low type defendant is ready to pay up to 1.75 for achieving a settlement. To deter the high-type defendant from making low offers, at the equilibrium the plaintiff needs to reject them with a positive probability. However, since the settlement is more valuable for the low-type than for the high-type defendant, the low-type defendant is deterred from making them as well. As a result only pooling exist. This result is summarized in Proposition 4.

**Proposition 4.** If only public settlements are available and $\lambda_L + \lambda_H > \frac{3}{4}$ then there exists $\bar{\eta}$ s.t. for $\eta \leq \bar{\eta}$ Proposition 3 applies and for $\eta \geq \bar{\eta}$ the negotiation in the first period has only pooling equilibria in which any offer $s_{1,0} \in \left[\mu_{1,0} s_{1,0}^H + (1 - \mu_{1,0}) s_{1,0}^L, s_{1,0}^H \right]$ can be made and it is always accepted. For later periods Proposition 3 applies.

A similar result can be established when secret settlements are available. As we established in the body of the paper, although the availability of secret settlements can change the likelihood of the negotiations being successful, it is never enough to ensure that there exist separating equilibrium in which the negotiation is successfully all the time. Hence, analogous logic applies – the low type defendant is willing to pay more for ensuring a settlement of the case, but low offers need to be rejected with some probability to deter the high type from making them. As a result only equilibria exist. In case of endogenous privacy regime we state only existence result, since potentially
pooling and separating equilibria can locally coexist. To be precise, for some values of \( \eta \) the high-litigation equilibria can become pooling, while the low-litigation equilibria remain separating.

**Proposition 5.** If the secrecy regime is endogenous and \( \lambda_L + \lambda_H > \frac{3}{4} \) then there exists \( \bar{\eta} \) s.t. for \( \eta \leq \bar{\eta} \) Proposition 2 applies and for \( \eta \geq \bar{\eta} \) there exist pooling equilibria of the negotiation in the first period, in which any offer \( s_1 \in [\mu_{1,H} s_{1,0}^H + (1 - \mu_{1,H}) s_{1,0}^L, s_{1,0}^H] \) can be made and it is always accepted. For later periods Proposition 3 applies.

**Proof of Proposition 4.** Proposition 4 is proved in lemmas 8 – 12.

**Lemma 8.** If \( \lambda_H + \lambda_L > \frac{4}{3} \) and \( \eta \) sufficiently small, Proposition 2 applies.

**Proof.** To verify the lemma it is enough to recall the condition for existence of equilibrium described in Proposition 2 (7) and (8), and note that the RHS of both conditions continuously increases in \( \eta \) on \((0, 1)\), and it goes to infinity as \( \eta \) goes to 0.

**Lemma 9.** If \( \lambda_H + \lambda_L > \frac{4}{3} \) and \( \eta \) sufficiently large then only pooling PBE of the first period negotiation in which the defendant makes a single offer \( s_{1,0} \) independently of the type exist.

Lemma 10 is proved in claims 11 and 12.

**Claim 11.** If \( \lambda_H + \lambda_L > \frac{4}{3} \) and \( \eta \) sufficiently large there does not exist a PBE in which some offer is made by the low type defendant but not by the high type defendant during the first period negotiation.

**Proof.** First, observe that if \( \lambda_H + \lambda_L > \frac{4}{3} \) then (7) and (8) are violated for \( \eta = 1 \), hence, there exists a continuum of values of \( \eta \) for which they violated.

Second, observe that if (7) is violated an \( l < l \) (8) is violated an \( l \geq l \) then:

\[
\kappa_1^H(\mu) - \kappa_0^H(\mu) < \kappa_1^L(\mu) - \kappa_0^L(\mu). \tag{18}
\]

Denote the smallest offer made only by the low type defendant by \( s^* \). Note that \( s^* \) needs to be rejected with some positive probability \( p(s^*) > 0 \), as otherwise the high-type defendant would also make it. Hence, \( s^* \leq s_{1,0}^L \). However, \( s^* \) needs to be accepted with some positive probability, as otherwise the low type defendant would have a profitable deviation of making an offer \( s_{1,0}^H \), which needs to be accepted in any PBE. This result follows from the fact that \( s_{1,0}^H < \kappa_1^H(\mu) - \kappa_2^H(\mu) \), and hence, \( \kappa_0^L(\mu) + s_{1,0}^H < \kappa_1^L(\mu) \). Thus, \( s^* = s_{1,0}^L \).

Denote the smallest offer made by the high type defendant by \( s' \). If \( s' = s_{1,0}^H \), then the fact that the equilibrium does not exist follows directly from the proof of Claim 5. Moreover, \( s' > s_{1,0}^L \), \( s' \neq s_{1,0}^L \) by assumption, and any offer \( s < s_{1,0}^L \) is always rejected by the plaintiff, hence the high type defendant would prefer to make an offer \( s_{1,0}^H \) which is always accepted by the plaintiff.

In order for the high type defendant not to have a profitable deviation of making an offer \( s^* \) the probabilities of rejecting an offer \( s' \) and rejecting an offer \( s^* \) need to satisfy:

\[
p(s^*)\kappa_1^H(\mu) + (1 - p(s^*)) (\kappa_0^H(\mu) + s^*) \geq p(s')\kappa_1^H(\mu) + (1 - p(s')) (\kappa_0^H(\mu) + s'). \tag{19}
\]
Analogously, in order for the low type not to have a profitable deviation of always making an offer \( s' \) the probabilities of rejecting an offer \( s' \) and rejecting an offer \( s^* \) need to satisfy:

\[
p(s^*)\kappa^L_H(\mu) + (1 - p(s^*))((\kappa^L_0(\mu) + s^*) \leq p(s')\kappa^L_H(\mu) + (1 - p(s'))((\kappa^L_0(\mu) + s^*). \tag{20}
\]

Condition \((19)\) reduces to: \((p(s^*) - p(s'))(\kappa^L_H(\mu) - \kappa^L_0(\mu)) \geq (1 - p(s'))s' - (1 - p(s^*))s^*, \) and condition \((20)\) reduces to: \((p(s^*) - p(s'))(\kappa^L_H(\mu) - \kappa^L_0(\mu)) \leq (1 - p(s'))s' - (1 - p(s^*))s^*. \) Hence they can simultaneously hold only if \( \kappa^L_H(\mu) - \kappa^L_0(\mu) \geq \kappa^L_H(\mu) - \kappa^L_0(\mu), \) which yields a contradiction.

Claim 12. If \( \lambda_H + \lambda_L > \frac{4}{3} \) and \( \eta \) sufficiently large, there does not exist a PBE in which some offer is made by the high type defendant but not by the low type defendant during the first period negotiation.

**Proof.** Denote by \( s^* \) the largest offer that is made by the high type defendant but not by the low type defendant. In any PBE there is no offer made on the equilibrium path larger than \( s^* \), hence, \( s^* \leq s^*_L, \) Moreover \( s^* \geq s^*_L, \) since otherwise the offer \( s^* \) would be always rejected on the equilibrium path, and the high type defendant would have a profitable deviation of making an offer \( s^*_L, \) which is necessarily accepted. Hence, \( s^* = s^*_L. \)

Denote by \( s' \) the largest offer that is made by the low type defendant, and note that \( s' \geq s^*_L, \) If \( s' < s^*_L \) it would be necessarily rejected an the low type defendant would have a profitable deviation of making an offer \( s^*_L. \) In order for the high type defendant not to have a profitable deviation of making an offer \( s' \) the probability of rejection of the offer \( s' \) needs to satisfy:

\[
p(s') \geq \frac{s^* - s' - \kappa^L_0(\mu)}{\kappa^L_H(\mu) - s' - \kappa^L_0(\mu)}. \tag{21}
\]

Analogously, in order for the low type defendant not to have a profitable deviation of always making an offer \( s^* \) the probability of rejection of the offer \( s' \) needs to satisfy:

\[
p(s') \leq \frac{s^* - s' - \kappa^L_0(\mu)}{\kappa^L_H(\mu) - s' - \kappa^L_0(\mu)}. \tag{22}
\]

Condition \((21)\) and \((22)\) can simultaneously hold only if \( \kappa^L_0(\mu) - \kappa^L_0(\mu) = \kappa^L_H(\mu) - \kappa^L_0(\mu), \) which as shown in the proof of Claim 12 does not hold.

Claim 13. If \( \lambda_H + \lambda_L > \frac{4}{3} \) and \( \eta \) sufficiently high there does not exist a PBE in which both the high and low type defendant make multiple offers with positive probability.

**Proof.** For such a PBE to exist it would need to be that there exist at least two offers \( s, s' \) and corresponding probabilities of rejection of each \( p(s), p(s') \) such that both the low and the high type defendant are indifferent in between making those offers. However, this can only happen if \( \kappa^L_H(\mu) - \kappa^L_0(\mu) = \kappa^L_H(\mu) - \kappa^L_0(\mu), \) which was shown in the proof of Claim 12 not to hold.

Lemma 10. No PBE in which an offer \( s_1,0 \) is rejected with positive probability satisfies the D1 criterion.
Proof. Observe that an offer \(s_{1,0}\) is rejected with a positive probability only if \(s_{1,0} \leq \mu_{1,h_1}s_{1,0}^H + (1 - \mu_{1,h_1})s_{1,0}^L\). Moreover, in no PBE \(s_{1,0}\) is rejected with certainty, as then the defendant would have a profitable deviation of ensuring the settlement through making an offer \(s_{1,0}^L\). Hence \(s_{1,0} = \mu_{1,h_1}s_{1,0}^H + (1 - \mu_{1,h_1})s_{1,0}^L\).

To show that any PBE in which \(s_{1,0} = \mu_{1,h_1}s_{1,0}^H + (1 - \mu_{1,h_1})s_{1,0}^L\) and \(p(s_{1,0}) > 0\) fails the D1 criterion, it is enough to show that for an offer \(s' = s_{1,0} + \varepsilon\) for \(\varepsilon\) sufficiently small the high type defendant is eliminated under the D1 criterion. Hence, \(\mu(s') = 0\) and \(s'\) is always accepted on the equilibrium path. As a result, there is profitable deviation for the low type defendant of making and offer \(s'\).

To prove this result, first, observe that the low type defendant is better-off making an offer \(s'\) rather than \(s_{1,0}\) if and only if it is rejected with probability \(p \leq p^L\), for \(p^L L \equiv \frac{p(s_1)(\kappa^H(\mu) - \kappa(\mu)L_{1,0} - s_{1,0}) - \varepsilon}{\kappa^H(\mu) - \kappa^L(\mu) - s'}\). Second, observe that the high type defendant is better of making an offer \(s'\) rather than \(s_{1,0}\) if and only if it is rejected with some probability \(p \leq p^H\), for \(p^H L \equiv \frac{p(s_1)(\kappa^H(\mu) - \kappa(\mu)L_{1,0} - s_{1,0}) - \varepsilon}{\kappa^H(\mu) - \kappa^L(\mu) - s'}\). Both \(p^H\) and \(p^L\) are smaller than 1 for \(\varepsilon\) sufficiently small. Moreover, as \(\kappa^L(\mu) - \kappa^H(\mu) > \kappa^H(\mu) - \kappa^L(\mu)\), \(p^H < p^L\), hence the high type defendant is eliminated for strategy \(s'\) under the D1 criterion.

Lemma 11. If \(\lambda_H + \lambda_L > \frac{4}{3}\) and \(\eta\) sufficiently large there always exist pooling PBE satisfying the D1 criterion with any \(s_{1,0} \in [\mu_{1,h_1}s_{1,0}^H + (1 - \mu_{1,h_1})s_{1,0}^L, \text{and } p(s_{1,0}) = 0\).

Proof. First, we show that such PBE exist. Take offer \(s_{1,0}\) and set the beliefs of the first period plaintiff at \(\mu_{1,h_1}(s_{1,0}) = \mu_{1,h_1}\), \(\mu_{1,h_1}(s) = 1\) for all \(s \in (\mu s_{1,0}^L, s_{1,0})\), and set any beliefs for other offers.

Then, it is always the strategy of the plaintiff to reject any offer \(s < s_{1,0}\), and accept an offer \(s_{1,0}\) is his best response. Given this strategy, it is a best response of the defendant independently of their type to always make an offer \(s = s_{1,0}\). Hence, the belief \(\mu_{1,h_1}(s_{1,0}) = \mu_{1,h_1}\) follows the Bayes’ rule, and no other event is observed on the equilibrium path.

To verify that these equilibria satisfy the D1 criterion, it is enough to show that the high type defendant is not eliminated for offers \(s \in (s_{1,0}^L, s_{1,0})\), and the the plaintiff can hold a belief \(\mu(s) = 1\) in an equilibrium satisfying the D1 criterion. Take any \(s \in (s_{1,0}^L, s_{1,0})\). The high type defendant is better-off making an offer \(s\) than under her equilibrium payoff if the probability of rejection of an offer \(s\) satisfies \(p(s) \leq p^H \equiv \frac{\kappa^H(\mu) - \kappa^L(\mu) - s}{s_{1,0}^L} - s\), and the low type defendant is better-off making an offer \(s\) than under her equilibrium payoff if the probability of rejection of an offer \(s\) satisfies \(p(s) \leq p^L \equiv \frac{\kappa^H(\mu) - \kappa^L(\mu) - s}{s_{1,0}^L} - s\). Since \(\kappa^H(\mu) - \kappa^L(\mu) > \kappa^H(\mu) - \kappa^L(\mu)\) if \(\lambda_H + \lambda_L > \frac{4}{3}\) and \(\eta\) sufficiently large. Then \(p^L < p^H\) and the high type defendant is not eliminated for strategy \(s\) under the D1 criterion.

Lemma 12. No other PBE satisfying the D1 criterion exist.

Proof. The only two candidate equilibria types are a pooling equilibrium with \(s > s_{1,0}^H\), and a pooling equilibrium with \(s < \mu_{1,h_1}s_{1,0}^H + (1 - \mu_{1,h_1})s_{1,0}^L\). However, in both candidate equilibria types the defendant would have a profitable deviation of making an offer \(s_{1,0}^H + \varepsilon\), which is always accepted by the plaintiff.

Proof of Proposition 5 We prove only the existence of a pooling equilibrium in the first period negotiation when \(\eta\) is sufficiently large. The remaining elements of the
follow directly from the proofs of Propositions 3 and 4. Proposition 5 is proved in lemmas 13 and 14.

**Lemma 13.** If $\eta >$ sufficiently large then there exists an equilibrium in which:

1. the second period plaintiff’s filing decision is independent from realization of $n_2$,
2. the first period negotiation is always secretly settled at a transfer $s_1 \in [\mu_{1,h_1}s_1^H + (1 - \mu_{1,h_1})s_1^L]$.

**Proof.** Since on the equilibrium path $n_2 = 0$, $\mu_{2,h_2=(0,1)}$ is an out-of-equilibrium belief and it can be set to $\mu_{2,h_2=(0,0)}$. Hence, the decision of the second period plaintiff is independent of $n_2$ and (i) holds.

Assume during the first period negotiation $\mu_{1,h_1}(s_1, \zeta) = \mu_{1,h_1}$ independently of $\zeta$, and $\mu_{1,h_1}(s, \zeta) = 1$ for all $s \leq s_1$ independently of $\zeta$. And pick any beliefs for other $S$. Then it is the best response of the plaintiff to accept an offer $s_1$ and reject any offer $s < s_1$. Hence, it is the best response of the defendant to make an offer $s_1$.

**Lemma 14.** Equilibrium described in (13) satisfies the D1 criterion.

**Proof.** To show that the equilibrium does satisfy the D1 criterion, it is enough to show that the high type is not eliminated for making any offer $S = (s, \zeta)$ for $s < s_1$ under the D1 criterion.

First, observe that since the decision of the filing decision of the second period plaintiff is independent from $n_2$ if the choice of $\zeta$ is irrelevant. Second take any $s < s_1$ Then, the defendant of type $i$ is better-off making an offer $s$ if it is rejected with probability at most $p^i$:

$$p^i = \frac{s_1 - s}{\kappa^i_H(\mu) - \kappa^i_0(\mu) - s}. \quad (23)$$

The high type is eliminated from making an offer $s$ only if $p^H > p_H$, which is true only if $\kappa^H_H(\mu) - \kappa^H_0(\mu) > \kappa^H_L(\mu) - \kappa^H_0(\mu)$. However for that to be the case either (7) or (8) needs to hold. But for $\eta$ sufficiently large and $\lambda_H + \lambda_L > \frac{4}{3}$ both (7) and (8) are not satisfied. Hence, the high type is not eliminated from making an offer $S$ under the D1 criterion.