Fiscal Sustainability in Aging Economies *

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Abstract

How will an aging population affect fiscal policy and government debt? I answer this question using a dynamic overlapping generations model in which fiscal policy is determined endogenously by voting, and in which sovereign default is allowed. Contrary to existing results derived under risk-free debt, I show that population aging leads to a decline in government debt. The reason is that, ceteris paribus, the risk of default on sovereign debt increases as the population ages. The consequent rise in interest rates reduces the government's incentive to borrow and ensures that debt remains sustainable, i.e., the probability of observing a default in equilibrium stays low and constant. However, debt sustainability comes at the cost of recurrent tax hikes and severe cuts to entitlements for the elderly. An international lending facility that allows the government to borrow at a low fix rate eventually exacerbates the welfare costs of population aging. In contrast, increasing the minimum retirement age improves the welfare of all future generations.

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1 Introduction

This paper examines the effects of an aging population on fiscal policy. Ongoing demographic changes are striking, especially among advanced economies: old-age dependency ratios (the proportion of individuals age 65 and above to those ages 20-64) will increase dramatically over the coming few decades (see Figure 1).

**Figure 1: Old-Age Dependency Ratios in the G7**

![Graph showing old-age dependency ratios in the G7 countries from 1940 to 2100.](image)


What will be the effects of this demographic shift on the evolution of fiscal policy and government debt? An obvious concern that is prominent in the policy discourse (e.g., IMF, 2019) is that an aging population will lead to more debt accumulation as a result of higher spending on entitlement programs for the elderly. This prediction is consistent with projections by policy-making institutions (e.g., European Commission, 2018b), and results derived from theoretical models (e.g., Song et al., 2012). However, these analyses abstract from the possibility that the government could default on its debt repayment obligations.

I argue that the risk of sovereign default reverses the typical prediction for the evolution of government debt. The reason is that lenders impose fiscal discipline to an aging economy through interest rates. In my analysis, this market discipline leads to a declining debt-to-GDP ratio and ensures debt sustainability.

I reach these conclusions in a quantitative model of sovereign debt in the tradition of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008). Those authors analyze representative-household models. In contrast, I consider overlapping generations (OLG) of workers and retirees with stochastic lifetimes à la Gertler (1999). I model entitlements as promised transfers to the elderly, financed by sovereign debt and distortionary taxes on labor
As in Song et al. (2012), I assume that agents vote on fiscal policy according to a probabilistic voting scheme. Retirees become more politically powerful as their share in the electorate grows. Critically, I allow for the possibility that the government can renge, at a cost, on its debt and entitlement obligations.

Population aging leads to a key tension regarding how to manage the debt. On the one hand, evolving demographics would lead to debt accumulation. For the same level of per capita entitlements, an economy with more old people must finance a larger amount of spending while relying on a smaller pool of taxpayers (the workers). On the other hand, the possibility of sovereign default generates market forces that would lead to a decline in debt. For any given level of debt and per capita entitlements, lenders demand higher interest rates because the risk of default is higher, i.e., the interest rate schedule on government debt shifts out. There are two main reasons for this shift. First, the political cost of cutting entitlements rises as the electorate ages. Second, a smaller tax base requires higher rates of distortionary taxes to raise the same amount of tax revenue. So, other things equal, the government of an economy with a higher share of old people has more of an incentive to default on its debt.¹

I evaluate quantitatively how the forces discussed above interact in a version of my model calibrated to the Italian economy.² Specifically, I simulate the evolution of fiscal policy up to 2050 using projections of the Italian population from the United Nations. My results can be summarized as follows. First, as the population ages, there is an increase in total spending for entitlements as a share of GDP. Second, the debt-to-GDP ratio declines. Third, the debt remains sustainable: the likelihood of sovereign defaults is low and constant in the simulations, averaging less than 2% per year.³ Fourth, the government undertakes frequent fiscal adjustments involving both taxes and per capita spending on the elderly. The latter falls by almost 50% over the simulation period, through a combination of repudiations of current promises (i.e., defaults) and reductions of future promises compared to current ones (i.e., reforms). Both defaults on and reforms of entitlements happen with an average frequency of about 15% per year over the simulation period.

These results are driven by three main elements: (i) demographics, through the effects on tax revenues and spending in the government budget constraint; (ii) politics, through the rise in political power of the elderly; and (iii) market discipline, through the shift in the interest rate schedule for government debt. To shed more light on the contribution of each of these forces, I run two counterfactual exercises. First, I allow for demographic changes over the

¹Reducing debt in anticipation of future fiscal pressure is consistent with tax-smoothing theories (see Yared, 2019, and the references therein). Distortionary taxes are a feature that my model shares with this literature. However, I show that they are not enough to generate a decline in the debt-to-GDP ratio in my political setting with no commitment to future policies. Market discipline is necessary.

²Italy is an ideal target for my analysis. On the one hand, it faces one of the most severe demographic transitions in the world. On the other hand, it presents a very fragile current fiscal situation. Appendix E presents Italian fiscal statistics and discusses the reasons to target Italy rather than Japan, the world leader in terms of population aging.

³Defining debt sustainability in terms of probability of default has been advocated in D’Erasmo et al. (2016).
simulation period, but I keep the political power of the different voter groups and the interest rate schedule fixed at their 2017 levels. Second, I allow for both demographic and political forces to operate, but mute market discipline. These exercises show, first, that demographics are the main drivers of the fall in per capita entitlements; second, that politics (and to a lesser extent market discipline) is responsible for the tax hikes; and third, that market discipline is the key factor governing debt dynamics: absent the shift in the interest rate schedule, debt-to-GDP would rise and the frequency of sovereign defaults would be significantly higher.

The adjustments on taxes and per capita transfers derived in my baseline results entail sizable reductions in welfare for future generations vis-à-vis current ones. I calculate that the consumption of a worker in 2050 should be permanently increased by about 7% to achieve the same level of expected lifetime utility of a worker in 2017. The permanent rise in consumption that equates the expected lifetime utility of a retiree in 2050 to the one of a retiree in 2017 is roughly 80%.4

The large cuts in per capita entitlement and the implied reductions in retirees’ welfare may appear surprising in a model where fiscal policy is chosen through voting, with the elderly growing more powerful over time. In fact, these results are consistent with the probabilistic voting scheme I assume, where politically optimal fiscal policy is found by maximizing an appropriately weighted sum of the lifetime utility that voters derive from fiscal policy choices. This maximization is subject to the government budget constraint, i.e., only budget-feasible fiscal policies are considered. Therefore, as the population ages, the set of available policies changes. In particular, the feasible levels of individual entitlements decrease over time, resulting in declining per capita spending for the elderly.

In my baseline model, agents have rational expectations. Voters internalize the future fiscal implications of an aging population and vote accordingly. This means, for instance, that they refrain from choosing levels of future entitlements that would result in more reneging than in my baseline results. Moreover, in accordance with market discipline, a rational electorate decreases the economy’s reliance on debt to reduce the likelihood of costly future sovereign defaults. I highlight the importance of rational expectations for my baseline results with a counterfactual exercise in which some voters have myopic expectations about future fiscal policy. Specifically, they expect tax rates and per capita entitlements to remain at the current level forever. I find that for fractions of irrational voters exceeding 10% of the electorate, the debt-to-GDP ratio rises over the demographic shift. Moreover, the frequency of both debt and entitlement defaults is significantly higher than in the baseline scenario where all agents have rational expectations.

In the last section of the paper, I evaluate the welfare effects of two policy interventions. First, I consider an international lending facility from which the government can borrow pay-
ing a fixed and low spread over the risk-free rate in exchange for commitment to fiscal policies that enhance debt sustainability. This arrangement resembles the Primary Market Support Facility of the European Stability Mechanism. The rationale is to provide financial help in case of unexpected shocks that raise interest rates but do not fundamentally impair the solvency of the country. The fiscal relief offered under the lending scheme helps avoid sovereign defaults and lowers the interest rate schedule. However, the facility reduces market discipline because it relaxes the link between interest rates and the risk of default. I find that the introduction of the lending mechanism initially leads to spending hikes and tax cuts, financed with deficits. The debt-to-GDP ratio rises quickly, reaching levels substantially higher than the ones observed in the baseline scenario. Eventually, sovereign defaults become more frequent and larger fiscal adjustments on taxes and entitlements are necessary. These outcomes exacerbate the welfare losses for future generations associated with an aging population. These results provide evidence for the arguments stressing the importance of preserving market discipline when designing crisis resolution mechanisms in the euro area (see, e.g., Corsetti et al., 2015, and Andritzky et al., 2019).

Finally, I examine the effects of a gradual increase in the minimum retirement age. This reform eases the pressure on public finances induced by the demographic shift. Intuitively, milder entitlement cuts and tax hikes are necessary as the population ages, translating into lower welfare reductions for future generations.

**Related Literature.** My paper is related to several strands of the literature. First, it builds on quantitative models of strategic sovereign default, in the tradition of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008) (see Aguiar et al., 2016, for a review of this literature). In particular, my emphasis is on fiscal policy determination, as in Cuadra et al. (2010), and on politico-economic incentives stemming from differences across households in the economy.\(^5\) By considering a new dimension of heterogeneity, namely, individual preferences for fiscal policy over the life cycle, my setting allows to study the effects of a demographic transition on sovereign risk.

My paper also examines the impact on default risk of a novel set of (defaultable) government obligations: entitlements. I find that the propensity to default on debt largely increases in the amount of promised entitlements.\(^6\) Moreover, these spending obligations in my setting

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\(^5\)Several papers have introduced heterogeneity and/or politico-economic considerations in sovereign default models. The focus has been on: exogenous political turnover under either polarization (Cuadra and Sapirza, 2008, Önder and Sunel, 2019), or different government patience (Hatchondo et al., 2009); endogenous political turnover under private benefits from diversion of public funds (Chatterjee and Eyigungor, 2019); redistributive motives stemming from heterogeneity in households’ income (D’Erasmo and Mendoza, 2018, Andreasen et al., 2019); or heterogeneity in preferences for public goods aggregated with probabilistic voting (Scholl, 2017).

\(^6\)Myers (2017) studies the interplay between debt and pensions in a deterministic model with defaultable debt and multi-period pension promises. He focuses on sub-game perfect equilibria and derives analytical results showing that pension obligations can act as a commitment device for the repayment of risky debt in his setting. My model displays a similar “entitlement-disciplining-default effect” only in very limited regions of the state space.
can be interpreted as an arguably realistic micro-foundation of the lower bound on consumption that Bocola and Dovis (2019) and Bocola et al. (2019) introduce in the utility function of the government. Indeed, these authors present this new element as a reduced-form strategy to capture public spending components that are hard for the government to modify. In my analysis, these are the entitlements. Those authors show the importance of introducing the consumption lower bound to avoid counterfactual predictions of their models vis-à-vis data from advanced European economies. I find that my model displays the cyclical behavior of debt and spreads that are similar to the ones they find in the version of their model with the consumption lower bound.

There is a branch of the political economy literature that examines government debt dynamics while touching upon the effects of population aging. Heterogeneous discounting, as presented in Yared (2019), is one theory considered in this field of research. The main insight is that an increase in the share of old impatient voters implies more present-biased choices—in particular, higher deficits—when policies are chosen sequentially. This leads to rising debt levels. Also relevant is the theory developed by Song et al. (2012). Their two-period OLG model features the same political game that I assume. Debt accumulation in their setting is constrained by young generations’ concern about future public good provision. Therefore, a decline in the share of young voters leads to a rise in debt. In sum, prominent political economy theories predict debt increasing with population aging when debt is risk-free. My contribution is to show that the possibility of sovereign default leads to the opposite prediction.7

Another stream of the political economy literature considers the effects of an aging population on the sustainability of social security (e.g., Gonzalez-Eiras and Niepelt, 2008 and Song, 2011) or growth (e.g., Gonzalez-Eiras and Niepelt, 2012).8 These are dynamic OLG models similar to mine in terms of demographics and politics. However, they assume a balanced budget for the government. Therefore, they do not provide results on the dynamics of sovereign debt. They also abstract from entitlements. Bouton et al. (2016) is the first paper to study the political economy of total government obligations in terms of both debt and entitlements. However, their analysis is static and hence not focused on the dynamic effects induced by an aging population.9

The implications of the shift toward older populations have also been studied from a macroeconomic perspective. One stream of the literature focuses on the implications of aging for

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7Tabellini (1991) is a seminal paper with defaultable debt as an instrument for intergenerational redistribution. Interestingly, an increase in the fertility rate in that model yields ambiguous theoretical results on the size of the feasible debt region. This underscores the importance of quantitative analyses aimed at balancing the various forces that a demographic changes exert on government debt dynamics.

8Casamatta and Batté (2016) provide a review.

9Cukierman and Meltzer (1989) is a classic model featuring both sovereign borrowing and social security. However, no explicit promises for future old-age transfers are made, and the heterogeneity driving debt and deficit is in terms of wealth-determined bequests, rather than age.
growth, real interest rates, and inflation.\textsuperscript{10} Some other papers, instead, look at fiscal sustainability under the demographic transition in specific countries, using quantitative general equilibrium models.\textsuperscript{11} Those are typically heterogeneous-agents, OLG environments that are richer than mine in terms of demographic and institutional details. Some of them assume idiosyncratic shocks for the households, but none feature aggregate uncertainty. Another important difference with respect to my work is that fiscal policy in those papers is determined through fiscal rules. In particular, the typical assumptions concerning sovereign debt are a balanced budget (i.e., no sovereign debt), debt being the residual in the budget constraint of the government, or debt growing at some fixed rate (usually, the growth rate of output). My contribution to this literature is to study fiscal sustainability under the demographic shift in a model where fiscal policy and debt are chosen endogenously, and debt is defaultable.\textsuperscript{12}

Lastly, by providing a quantitative assessment of the impact of population aging on fiscal variables, my paper relates to various projection exercises performed by academic scholars and policy-makers (e.g., Auerbach, 2012; Amaglobeli and Shi, 2016, at the IMF; MEF, 2017, by the Italian Ministry of Economy and Finance; European Commission, 2018a; US Social Security Administration, 2019; CBO, 2019). With few exceptions, the focus of those works is on the dynamics of spending. When government debt is analyzed, it is considered as a residual in the government budget constraint, rather than a choice variable, and it is projected to grow substantially as the population ages. The main innovations in my work are twofold: first, fiscal policy and sovereign debt are determined endogenously in general equilibrium in my model: fiscal choices must satisfy the government budget constraint and can be adjusted in response to aging; second, sovereign debt is defaultable. The latter is the key assumption behind the decline in the debt-to-GDP ratio that my model predicts contrary to existing projections.

\textbf{Layout.} The reminder of the paper is organized as follows. Section 2 describes the model.

\textsuperscript{10}For the effects on growth and interest rates, see Lee (2016) for a survey, Favero and Galasso (2015) and Aksoy et al. (2019) for empirical evidence, and Sánchez-Romero (2013), Carvalho et al. (2016), Gagnon et al. (2016), and Aksoy et al. (2019) for studies with a quantitative component based on general equilibrium models. Different rationalizations of the generally negative association between aging and inflation have been proposed by Bullard et al. (2012) and Katagiri et al. (2019).

\textsuperscript{11}Examples include Kotlikoff et al. (2007) and Nishiyama (2015) for the US; Díaz-Giménez and Díaz-Saavedra (2009) for Spain; Kudrna et al. (2019) for Australia. More attention has been devoted to Japan (see İmrohoroğlu et al., 2016, Ma and Tran, 2017, and the references therein). Nishiyama and Smetters (2014) provide a review of the computational methods in the OLG literature. The welfare implications of different retirement financing options (tax-and-transfer versus saving-for-retirement systems) are examined in Birkeland and Prescott (2007). Their setting is a closed economy with safe government debt as an asset for private savings. In one comparative static exercise on demographics for the US economy, they find debt-to-output ratios becoming as high as 450%.

\textsuperscript{12}A branch of the literature studies debt sustainability in terms of fiscal space and debt limits with sovereign risk (e.g., Bi, 2012, Ghosh et al., 2013). D’Erasmo et al. (2016) propose and analyze alternative definitions of public debt sustainability, one of which is based on the likelihood of default in models of risky debt, as in my work. Blanchard (2019) assesses public debt sustainability in terms of the difference between growth rates and interest rates. None of these works is (directly) concerned about the effects of population aging.
Section 3 presents its calibration and its fit to Italian data. Section 4 conducts a numerical characterization of the solution. Section 5 shows the baseline results on the evolution of fiscal variables under population aging. It also presents counterfactual exercises that shed more light on the role of the various forces in the model. Section 6 describes the effects of the introduction of a lending mechanism with conditionality and the increase in the minimum retirement age. Finally, Section 7 offers some conclusions and suggestions for future research.

2 Model

This section lays out the model. Section 2.1 outlines the sequential problem of each agent. Section 2.2 presents the recursive formulation of the model and the equilibrium definition.

2.1 Environment

Consider a small open economy inhabited by a representative firm, households, a government, and investors. Time is discrete and indexed by \( t = 0, 1, 2, \ldots \). The economy is subject to a single aggregate shock to productivity. The problem of each economic agent is presented next.

2.1.1 Firms

The supply side of the economy consists of a representative firm producing output, \( Y_t \), from aggregate labor, \( L_t \), with the linear technology:

\[
Y_t = A_t L_t.
\]

\( A_t \) is productivity. It is stochastic and evolves according to a first-order Markov process with transition matrix \( p(A_{t+1}|A_t) \). The firm faces a competitive labor market. These assumptions imply that the equilibrium features a wage equal to \( A_t \) and zero profits.

2.1.2 Demographics and Household’s Problem

Households are divided into overlapping generations of young (\( Y \)) and old (\( O \)) of the Blanchard-Yaari type, as in Gertler (1999). Namely, individuals live for a stochastic number of periods, with the young facing a probability \( \gamma^p_t \) of becoming old at the end of each period \( t \), and the old facing a probability \( \gamma^D_t \) of dying at the end of each period \( t \). New young agents are born from current young individuals at rate \( \gamma^B_t \) at the end of each period \( t \). Total population grows at the rate \( n_t \) implied by the various \( \gamma_t \)’s (see Appendix D.3). The share of individuals in group \( J \) in the population is denoted by \( s^J_t \), with \( J \in \{Y, O\} \). Demographic variables vary over time (hence the subscript \( t \)), and their paths are assumed to be known to all agents in the model.

Old individuals do not face any economic decision: they are retired, so they do not supply labor, and they simply consume a transfer \( p_t \) they receive from the government each period.
Under a given sequence of transfers \( \{p_{t+j}\}_{j=0}^{\infty} \), their present discounted value of lifetime utility in period \( t \) is given by:

\[
\sum_{j=0}^{\infty} \beta^j \prod_{i=0}^{j-1} (1 - \gamma_{t+i}^D) u(c_{t+j}, l_{t+j}^O)
\]

s.t. \( c_{t+j} = p_{t+j} \) and \( l_{t+j}^O = 0 \) \( \forall j \),

where \( u(c, l) \) is the flow utility from consumption \( c \) and labor \( l \) (the latter being equal to zero for old individuals) and \( \beta \) is the discount factor. The term \( \prod_{i=0}^{j} (1 - \gamma_{t+i}^D) \), which is assumed to equal 1 for \( j = 0 \), adjusts the discount factor for the probability of dying in any future period.

Young agents work, consume, and pay taxes to the government on their labor income. Under a sequence of wages and tax rates \( \{w_{t+j}, \tau_{t+j}\}_{j=0}^{\infty} \) and old-age transfers \( \{p_{t+j}\}_{j=0}^{\infty} \), all of which the young take as given, the problem of a worker in period \( t \) can be written as:

\[
\max_{\{c_{t+j}, l_{t+j}^O\}_{j=0}^{\infty}} u(c_{t+1}, l_{t+1}^O) + \beta \left( \gamma_{t+1}^R u(c_{t+1}, l_{t+1}^O) + (1 - \gamma_{t+1}^R) u(c_{t+2}, l_{t+2}^Y) \right) + \beta^2 \left( \gamma_{t+2}^R (1 - \gamma_{t+1}^D) + (1 - \gamma_{t+1}^R) \gamma_{t+1}^Y u(c_{t+2}, l_{t+2}^O) + (1 - \gamma_{t+1}^R)(1 - \gamma_{t+1}^R) u(c_{t+3}, l_{t+3}^Y) \right) + \ldots
\]

s.t. \( c_{t+j} = (1 - \tau_{t+j}) w_{t+j} l_{t+j}^Y \) \( \forall j = 0, 1, 2, \ldots \),

\[c_{t+i} = p_{t+i} \quad \text{and} \quad l_{t+i}^O = 0 \quad \forall i = 1, 2, \ldots\]

In fact, the solution to (2) can be recovered from a sequence of static consumption-labor decisions taken optimally period by period.

Finally, note that there is no borrowing or lending by young and old agents in this economy. In particular, and importantly for my analysis, individuals do not privately save for retirement. This assumption is crucial to keep the model tractable, as I explain in Appendix B. In fact, the absence of private savings for retirement is in line with the empirical evidence I present in Appendix D.4 on the very limited role that income from private sources plays for Italian retirees.

### 2.1.3 Government

Fiscal policy is determined by a government. The state of the economy is summarized by the vector \( S_t = (b_t, A_t, e_t) \), where \( b_t \) is debt due at \( t \) (in per capita terms), \( A_t \) is the current level of productivity, and \( e_t \) are the per capita entitlements promised to the old individuals alive at \( t \). Given \( S_t \), the government chooses the current period labor income tax rate, \( \tau_t \), the entitlements to promise to next period’s retirees, \( e_{t+1} \), and the amount of new (per capita) debt to issue, \( b_{t+1} \).\(^\text{13}\) Importantly, the government also decides on whether to honor or break the promises on debt and entitlements. I denote these choices with the indicators \( d_i^B \) and \( d_i^E \), equal to one in case of default on, respectively, debt and entitlements.

\(^{13}\)I assume one-period debt. Long-term debt could be introduced, as in Chatterjee and Eyigungor (2012) and several other works following them. Yet, tractability considerations would restrict the maturity to be fixed over time, as it is common in most of the literature. I leave this extension as a possible direction for future work.
**Political objective function.** The government chooses fiscal policy sequentially in each period by maximizing the following objective function (subject to a budget constraint described shortly):

\[
\sum_J s^J_t \omega^J U^J(g_t),
\]

where \( s^J_t \) is the share individuals in age-group \( J \) in the population; \( \omega^J \) is a parameter capturing the political bias of the government toward group \( J \) in the society; and \( U^J(g_t) \) is the lifetime indirect utility derived by an individual in group \( J \) from the fiscal package \( g_t \), where \( g_t \) is a vector encompassing all the fiscal variables in the model. More specifically, \( U^J(g_t) \) is the sum of two terms: (i) the flow utility agents derive from the variables in \( g_t \) that directly affect their time \( t \)'s consumption and labor supply; and (ii) a continuation value that is the expected present discounted sum of the utility derived from future optimal policies (the evaluation is at the optimal policies given the maintained assumption of rational expectations). I provide a precise recursive formulation of the indirect lifetime utility of young and old individuals in equations (8) and (9) in Section 2.2. Essentially, they correspond to (1) for the old and (2) for the young, augmented with the entitlement default costs described shortly.

This political mechanism of fiscal policy determination can be micro-founded as the result of a probabilistic voting game à la Lindbeck and Weibull (1987), extended to a dynamic setting with Markov-perfect equilibria as in Song et al. (2012). I provide the precise derivation of (3) in the context of this voting game in Appendix A. The key intuition is that individuals vote according to the utility they derive from the fiscal packages proposed, and some other orthogonal, group-specific considerations that ultimately determine the \( \omega^J \)'s. The optimal strategy to win elections is then to choose the fiscal package that maximizes (3), i.e., the weighted sum of the lifetime utilities of the voters, with weights that reflect the total power of the groups in terms of both size \( (s^J_t) \) and political power above the sheer size \( (\omega^J) \).

**Default on debt.** The government can either repay the amount of debt \( b_t \) that is due at \( t \) or default on it, with a haircut of 100%. I assume three costs to be paid if debt obligations are not honored. The first two follow the sovereign default literature: there is exclusion from financial markets for a stochastic number of periods (with re-entry probability \( \chi \)); and productivity drops exogenously, from \( A_t \) to \( h(A_t) \leq A_t \), with \( h(\cdot) \) increasing and convex.

I also introduce an third default cost, with the purpose of capturing the harm that comes from defaulting on government debt when a fraction of it is held domestically. This cost is absent in standard sovereign default models, where the analysis is restricted to external debt. However, there are at least two reasons that make domestic debt particularly important in my setting: first, domestic debt is sizable in Italy, varying between 60 and 70% in the last two decades since the introduction of the Euro. Abstracting from this important source of funding and obligations would give an incomplete picture of fiscal sustainability. Second, reneging
on domestic debt entails important costs that would be missed in an environment with only external debt. Some authors have highlighted how sovereign defaults can severely impair the balance sheets of domestic financial institutions (Gennaioli et al., 2014, Bocola, 2016). In my setting, the relevant additional costs are political: governments might have lower incentives to default on debt held, at least partially or indirectly, by its citizens.

There are two possible ways to incorporate the political cost of defaulting on domestic debt in my model. The first one is a reduced-form approach: a term $k^H(\delta b_t)$ would be subtracted from the political objective function (3) in case of default. The assumption here is that a fraction $\delta$ of total government debt $b_t$ corresponds to domestic debt, and there is a political cost $k^H$, increasing in $\delta b_t$, that the government pays in case of default. The second option would be to allow voters to directly invest in domestic sovereign debt. This alternative would effectively micro-found the cost within the political game. Tractability considerations prevent me to directly allow workers and retirees to hold domestic debt, but in Appendix B I show how to introduce a third group of voters consisting of wealthy dynasties that invest in domestic debt. Proposition 1 in Appendix B also derives the conditions on $k^H$ and the third group of domestic investors that make the two approaches fully equivalent in terms of model’s solution. In the reminder of this section, I will continue to present the model adopting the more intuitive reduced-form approach. I will, instead, follow the micro-founded approach under the assumptions of Proposition 1 when I move to the calibration and the quantitative analysis.

**Default on entitlements.** The government can honor its entitlement promises by giving out a transfer $p_t = e_t$, or it can break them, by transferring $p_t < e_t$. I assume that cutting entitlements is politically costly. In reality, entitlements typically enjoy some degree of judicial protection (Bouton et al., 2016). Moreover, they are an extremely sensitive issue in political terms: historically, changes in both current and future entitlements have always encountered strong resistance ex-ante and public outcries ex-post.

For these reasons, and to make spending promises effectively exerting pressure for the government in the model, I assume that both defaulting on and reforming entitlements entails a utility cost for those affected by the cut. That is denoted by $k(x; e_t)$, with $k$ increasing in the difference between $x$ and $e_t$. In case of a default (i.e., $p_t < e_t$), the cost $k(p_t; e_t)$ reduces the flow utility of current retirees. A reform (i.e., $e_{t+1} < e_t$) can instead potentially affect both current young and old individuals, according to their life events. Therefore, $k(e_{t+1}; e_t)$ enters the utility of both groups. It is discounted by the probability that each individual faces of being impacted by the reform: $(1 - \gamma_t^D)$ for the old (i.e., the probability of surviving from $t$ to $t+1$), and $\gamma_t^K$ for the young (i.e., the probability of retiring between $t$ and $t+1$). By affecting the flow utility of agents, these $k$ costs enter their lifetime utilities (equations (1) and (2)) and hence the political objective function (3). The recursive formulation of the problem I present in Section 2.2 makes this dependence explicit.
**Government budget constraint.** The maximization of the political objective function (3) is subject to the budget constraint of the government. When debt is repaid, the constraint is:

$$
\tau_t s_t^Y A_t l^*(\tau_t, A_t) + q_t b_{t+1} \geq \frac{b_t}{1 + n_t} + s_t^O p_t.
$$

(4)

The first term on the left-hand side is labor-income tax revenues, where the equilibrium condition for the wage ($w_t = A_t$) has been used and $l^*(\tau_t, A_t)$ is the optimal labor supply of workers at the current tax rate and wage. The second term represents revenues from debt issuance, where $q_t$ is the price offered to the government by lenders. On the right-hand side of the inequality there are debt repayments, $b_t$ (scaled by $1 + n_t$ to adjust for population growth between $t - 1$ and $t$), and total transfers to retirees. If entitlements are honored, $p_t = e_t$.\(^{14}\)

The budget constraint under debt debt default is:

$$
\tau_t s_t^Y h(A_t) l^*(\tau_t, h(A_t)) \geq s_t^O p_t.
$$

(5)

This budget constraint reflects the economic default costs already discussed (market exclusion and the productivity drop from $A_t$ to $h(A_t) \leq A_t$). Again, $p_t = e_t$ if entitlements are honored.

**Government problem.** Finally, the problem of the government at $t$ is to choose $g_t$ to maximize (3), subject to the budget constraint consistent with the debt and entitlements default decisions (either (4) or (5)).

### 2.1.4 Investors

The market for sovereign debt features a large number of foreign investors that are deep-pocketed and risk-neutral.\(^{15}\) They have an opportunity cost of funds given by the world risk-free rate, $r$. The equilibrium price they offer for the debt issued by the government at $t$ is:

$$
q_t = \frac{1}{1 + r} \Pr\{\text{Debt repaid at } t + 1 \mid \text{Information at } t\}.
$$

This price is consistent with no-arbitrage for the lenders (in expectation). It also makes apparent the one-to-one relationship between the equilibrium debt price and sovereign risk, the latter being defined as $\Pr\{\text{Debt default at } t + 1 \mid \text{Info at } t\} = 1 - \Pr\{\text{Debt repaid at } t + 1 \mid \text{Info at } t\}$.\(^{12}\)

### 2.2 Recursive Formulation and Equilibrium

This section casts the problem of the government in recursive form and then formally defines an equilibrium of the model. I restrict my analysis to Markov recursive equilibria. The timing

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\(^{14}\)In the quantitative analysis, I add a term to the government budget constraint, calibrated to match the missing terms in my environment vis-à-vis the data (notably, non-labor income tax revenues and expenditures for programs other than old-age entitlements). Appendix D.2 provides the detailed mapping of model’s variables to the data.

\(^{15}\)The version of the model that I present in Appendix B, where a third group of voters holds domestic debt, features both domestic and foreign investors. The assumptions I make on the former ensure that the marginal pricer remains a foreigner.
of events is the following: first, the current $A_t$ is realized and, in case of market exclusion in $t-1$, the draw determining reentry takes place; then, elections occur and the debt price is posted accordingly to the fiscal package chosen by the winning candidate; fiscal policy actions are then carried out and individuals work and consume; finally, demographic transitions happen.

### 2.2.1 Government's Problem in Recursive Form

When the economy has market access in the current period, the recursive formulation of the problem of the government takes the following form:

$$ V_t(b, A, e) = \max_{d_B \in \{0, 1\}} (1 - d_B) V^R_t(b, A, e) + d_B V^D_t(b, A, e). $$

Demographic variables are effectively state variables of the problem, though special ones, since agents have perfect foresight on them. To ease the notation, I do not explicitly include them in the vector of states, but I instead index by $t$ any function that depends on them. In the above expression, $V^R_t$ is the value function under debt repayment, whereas $V^D_t$ the one under default. In case the economy has not regained market access after a previous default, there is no $d_B$ choice to be made, and $V^D_t$ is the relevant value function.

Formally, the value function under debt repayment is:
their expected continuation utility at the optimal \( t + 1 \) policies, denoted by starred functions. Moreover, old individuals discount the future also factoring in the possibility of not surviving to next period, which happens with probability \( \gamma^D_t \).

The indirect lifetime utility function for a young individual is:

\[
U^Y_t(\tau, b', e'; A, e) = u((1 - \tau)A^{I*}(\tau, A), l^{*}(\tau, A)) - \beta \gamma^R_t k(e'; e) + \beta \mathbb{E}_A[A] \left[ (1 - \gamma^R_t)U^Y_{t+1} \left( \tau^{*}_{t+1}, b''_{t+1}, e''_{t+1}; A', e' \right) + \gamma^R_t U^O_{t+1} \left( p^{*}_{t+1}, b'_{t+1}, e'_{t+1}; A', e' \right) \right],
\]

where the dependence of optimal starred policies from \( S' \) is dropped to ease the notation. This formulation already encompasses the optimal solution of the intra-temporal decision of the households, namely the \( e^* = (1 - \tau)A^{I*}(\tau, A) \) and \( l^* = l^{*}(\tau, A) \). The term \( \beta \gamma^R_t k(e'; e) \) is the cost of reforming entitlements described in Section 2.1.3. Note that there is no cost from defaulting on current spending promises, since young individuals are not entitled to the old-age transfers. The assumption of rational expectations and internalization of future demographic transitions hold also for the young, which are reflected in the expected continuation value.

The other value function in the government’s problem (6) is the one under default on debt:

\[
V^D_t(b, A, e) = \max_{\tau \in [0, 1], p \in [0, e], e' \in \mathcal{E}} s^O_t \omega^O U^O_t(p, 0, e'; h(A), e) + s^Y_t \omega^Y U^Y_t(\tau, 0, e'; h(A), e) - k^H_t(\delta b)
\]

s.t. \( \tau s^Y_t h(A) l^{*}(\tau, h(A)) \geq s^O_t p \).

The lifetime utilities are analogous to (8) and (9), with the only difference of continuation values now featuring optimal future policies consistent with the probability of remaining (exogenously) excluded from debt markets next period. \( V^D_t(b, A, e) \) features all the penalties for default described in Section 2.1.3, including \( k^H_t \), the cost related to the default on domestic debt due in the current period, \( \delta b \).

The equilibrium pricing schedule in recursive form is given by:

\[
q_t(b', A', e') = \frac{1}{1 + r} \mathbb{E}_A[A] \left[ 1 - d^{B^t}_{t+1}(b', A', e') \right],
\]

where \( d^{B^t}_{t+1}(b', A', e') \) is the indicator for the optimal choice of debt default in \( t + 1 \). A remark on time indexes, which recall denote the dependence on time-varying demographic variables. The function \( q \) depends on demographics only through \( d^{B^t}_{t+1}(b', A', e') \), which is indexed by \( t + 1 \). Therefore, the index on \( q \) could have been \( t + 1 \). However, perfect foresight over demographics effectively makes the actual time index immaterial. I therefore chose the more intuitive \( t \).

### 2.2.2 Recursive Equilibrium

Given a path for demographic variables, \( \{s^{Y}_t, s^{O}_t, n_t, \gamma^D_t, \gamma^R_t\} \), a recursive Markov equilibrium in this model is defined as a collection of value functions, \( \{V_t, V^R_t, V^D_t\} \), associated policy functions, \( \{\tau^*_t, p^*_t, b''_t, e''_t, d^{B^t}_t, d^{E^t}_t\} \), and pricing functions, \( \{q_t\} \), such that, at each \( t \): (i) taking \( q_t \) as given, the value and policy functions solve the political problem (6); and (ii) the debt pricing schedule \( q_t \) is consistent with the lenders’ problem, i.e., it satisfies (11).
3 Calibration And Fit To The Data

The model is solved numerically and calibrated to Italian data. The case of Italy is particularly interesting, given the magnitude of its aging phenomenon and its fragile fiscal situation. Figure 1 already provided evidence of the former. Appendix E presents more details on the latter, and elaborates further on the choice of targeting Italy in comparison to Japan, another country rapidly aging but with arguably more fiscal space to cope with it.

In the rest of this section, I first present the functional form assumptions and the model’s calibration, and then I analyze the fit of the results with respect to the data. I describe the solution and simulation algorithms in details in Appendix F. In a nutshell, the solution relies on global methods, given the non-linearity of the default decisions. It combines elements of value function iteration (for the terminal period) and transition dynamic exercises (for all the other previous periods). I defer a description of the main features of the simulation algorithm to Section 5.

3.1 Functional Forms

The functional forms adopted in my work follow the sovereign default literature, whenever appropriate. The flow utility is of the GHH type (Greenwood et al., 1988):

$$u(c, l) = \frac{1}{1 - \gamma} \left( c - \frac{l^{1+\frac{1}{\phi}}}{1 + \frac{1}{\phi}} \right)^{1-\gamma},$$

(12)

where \(l\) is labor supplied and \(\phi\) is the Frisch elasticity. For retirees, whose \(l = 0\), \(u(c, l)\) collapses to a standard utility function with constant relative risk aversion parameter \(\gamma\).

The productivity is modeled as an AR(1) process in logarithms:

$$\ln(A_t) = \rho \ln(A_{t-1}) + \sigma \varepsilon_t,$$

(13)

with \(\varepsilon_t\) being i.i.d. standard Normal errors.

The productivity after the exogenous drop in case of a debt default takes the form proposed in Chatterjee and Eyigungor (2012):

$$h(A) = A (1 - \max\{0; d_0 + d_1 A\}).$$

(14)

I assume the cost of reneging on and reforming entitlements takes the form:

$$k(x; e) = 1(x < e) [a_0 + a (u(e, 0) - u(x, 0))],$$

(15)

where \(x\) is either current transfers \(p\) in case of defaults \((p < e)\) or future entitlement promises \(e'\) in case of reforms \((e' < e)\). I assume \(k\) to be increasing in utility losses (i.e., \(a > 0\)). I use the flow utility \(u\) to value the harm due to promises broken or reformed in utility terms. This makes \(k\) convex in \(x\), which seems another natural requirement for such a cost. Finally, the positive
fixed cost $\alpha_0$ helps avoiding small default and reform episodes, which would complicate the model’s quantitative solution while being unrealistic.

Finally, I calibrate the cost of default on domestic debt following Proposition 1 in Appendix B. This proposition establishes the conditions under which the problem I presented in Section 2 is fully equivalent to one where a third group of voters, denoted by $H$, holds the domestic debt. Despite being equivalent to the former from an economic standpoint, the latter formulation of the problem is helpful for calibration purposes, given that it provides a precise and simple functional form for $k^H_t$:

$$k^H_t(\delta b) = \omega^H \frac{\delta b}{1 + n_t}.$$ 

Intuitively, the cost of default is increasing in the amount of domestic debt, $\delta b$. It is linear because wealthy families are required to be risk-neutral, as the foreign investors are, in order to guarantee that (11) is the correct equilibrium pricing schedule in the economy. Finally, as it is the case for $\omega^Y$ and $\omega^O$, $\omega^H$ can also be interpreted as the additional relative political power of voters’ group $H$ vis-à-vis the other groups.

### 3.2 Calibration

A period in the model corresponds to one year. The calibration is performed in three steps. First, the demographic variables are derived consistently with the historical and projected data, as I describe in Section 3.2.1. Then, I set some economic and political parameters at values that are either conventional in the literature, or consistent with the data. Finally, the remaining parameters are jointly calibrated by matching some relevant targets in the data. Section 3.2.2 expands on the last two steps. Appendix D provides more details on data sources and methods.

#### 3.2.1 Demographic Variables

Demographic variables in the model include the share of people in each group (the $s^i_t$), and the rates at which people enter and exit a group (the various $\gamma^i_t$’s). At a broad level, the calibration strategy I adopt is the following. First, I fix the retirement rate, $\gamma^R_t$, at a level consistent with an expected working life of 45 years. Then, I calibrate the values of the three remaining rates—i.e., the death rate, $\gamma^D_t$, the birth rate, $\gamma^B_t$, and the net growth rate of domestic debt holders, $\gamma^H_t$ (recall that this last group of voters lies behind the calibration strategy for the default cost $k^H_t$ that I described at the end of Section 3.1)—so that the population shares implied by the three equations that govern the demographic dynamics for the three population groups match their corresponding values in the data over the period covered in my analysis. Appendix D.3 presents the calibration strategy in details.

I obtain the shares of young and old individuals from the historical data and projections in the World Population Prospects, United Nations (2019). I consider population age 20-64 for the young, and population age 65 and above for the old. From these two groups, I subtract
a fraction of individuals that belong to the group of domestic debt holders. I infer the share of domestic debt holders from the Bank of Italy’s Survey on Household Income and Wealth. In particular, the guiding principle is to identify those young and old individuals who will be hurt in case of a sovereign default. This can happen either because they directly hold Italian government bonds, or because they hold assets issued by entities highly exposed to Italian debt, such as domestic financial institutions. The exact classification criteria I adopt are described in Appendix D.5. The resulting share of domestic debt holders, $s^H_t$, is about 7% and remains relatively stable over the simulation period.

Finally, my model solution (see Appendix F.1) requires the aging process to reach a steady state at some date. I assume that this happens in 2070, which is consistent with the United Nations (2019) data. Indeed, UN projections show the bulk of the demographic shift happening before the mid 2040’s, with a leveling starting around 2050 (see the top panels of Figure A1) and continuing over the following decades. I therefore choose 2050 as the end point for the simulations, and set the steady state for the population dynamics at $T = 2070$. Having population reaching a steady state a few years after the end of the simulations ensures that results for the last simulation years are not unduly influenced by the choice of $T$, which is ultimately arbitrary. A robustness exercise presented in Figure A6 in Appendix G.4 shows that the main results are not affected by setting $T = 2100$.

### 3.2.2 Economic and Political Parameters

The risk free rate is set at $r = 1.7\%$, the average yield of German government bonds with residual maturity of one year, over the period 2000-2017. The coefficients or risk aversion for $Y$ and $O$ take the value of 2, which is standard in the sovereign default literature. Parameters of the productivity process (13) are estimated from Italian data for (log) labor productivity for the period 1970-2017, in deviations from a quadratic trend. The result are $\rho = 0.733$ and $\sigma_\varepsilon = 0.014$. The probability of re-accessing financial markets after a default is $\chi = 0.25$, implying a mean exclusion of 4 years, in line with the experience of Greece during the European Debt Crisis. The share of sovereign debt held domestically is $\delta = 0.637$, which is the mean value for the period 2000-2017 for Italy.

The parameters $\omega^J$, capturing the political bias toward group $J$, are calibrated by considering the fraction of directed government spending (i.e., total government spending net of public goods and expenditures for agents outside of the model) that group $J$ was able to command on average over the period 2000-2017.$^{17}$ Appendix D.6 provides more details on the

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$^{17}$Some previous works used voters’ turnout by age as target for the calibration of the $\omega^J$’s (e.g., Song et al., 2012). However, this choice is not totally consistent with what the $\omega^J$’s capture in the political game—i.e., the different salience of fiscal policy to different (age) groups, ultimately shaping policies in favor of the most responsive ones. In fact, a country may show a turnout rate comparable across voters’ age groups (as it seems to be the case for Italy, see Tuorto, 2014), but if there are no candidates proposing pro-young platforms (at least for what concerns fiscal policy), then the final result might still be policies biased toward the preferences of other voters.
procedure. The final results reported in Table A2 in that appendix are \( \omega^Y = 0.072, \omega^O = 0.605, \) and \( \omega^H = 0.323. \) They clearly show a disproportionate relative power of the old and, to a lesser extent, domestic debt holder vis-à-vis the young. In Appendix G.8, I show the effects of setting \( \omega^Y = \omega^O = \omega^H = 1/3 \) on the simulations.

When solving and simulating the model, I add additional term to the budget constraint, calibrated so to match the elements of the real-world government balance sheet that are not present in the model (notably, non-entitlements spending and non-labor-income tax revenues). That is in the order of -6% of GDP. See Appendix D.2 for details.

Finally, the remaining parameters, i.e., \( \Theta \equiv [\beta, d_0, d_1, a_0, a, \varphi] \), are jointly calibrated with the method of simulated moments. This technique consists in choosing parameter values so to achieve the best match between some moments in the data and their model’s analogues. In my work, I choose the moments with the aim of drawing the most accurate future simulations for the following variables: old-age spending, income tax revenues, government debt (all three as a share of GDP), and the interest rate spread between Italian and German sovereign bonds.

When thinking about the simulations, a first objective is to have them starting at values that are reasonable vis-à-vis the data. This is achieved by targeting the level of available historical data for the last year available, i.e., 2017. The second objective is to match the past time-dynamics for these variables, with the understanding that some demographic shift has already occurred, although at a substantially lower scale than what is expected to lie ahead (as documented in Figure 1). The sensible span of data to consider for this exercise is arguably the one running from the introduction of the Euro to the beginning of the financial crisis. Those were events that severely affected the dynamics of the targeted fiscal variables for contingent reasons that likely dwarfed population aging, while not being captured in the model.

In light of this discussion, the following are the final calibration targets: the 2017 levels of old-age spending, income tax revenues, debt (all as a share of GDP), and the interest rate spread; and the change between 2000 and 2008 in those four variables, as predicted by a linear time trend. Old-age spending encompasses both pensions and health spending for old people. Income tax revenues in the model are mapped to taxes on income and wealth plus social contributions in the data. As for debt, the model considers only one-period bonds. Therefore, the spread is calculated considering only Italian and German bonds with one year residual maturity. Moreover, as in D’Erasmo and Mendoza (2018), I map debt in the model to a maturity-adjusted version of the debt stock in the data, computed by dividing the outstanding stock by the average maturity observed over the period 2000-2017 (6.9 years). This is important to properly match the amount of debt due each period in the data, and the amount of new debt issuances subject to the current interest rates. The maturity adjustment translates the 2017 debt-to-GDP ratio of 132% in the data to a target level for calibration purposes of 19%.18 Appendix D.2 reports model details on the exact matching of debt and all the other fiscal variables.

18See page 28 in D’Erasmo and Mendoza (2018) for a discussion of the consistency of this approach with respect to the long-term debt structure assumed in sovereign default models that follow Chatterjee and Eyigungor (2012).
in the model with the data.

The actual implementation of the method of simulated moments to calibrate $\Theta$ follows D’Erasmo and Mendoza (2018) and Bocola and Dogis (2019). In particular, the preferred parameter configuration is chosen as the one minimizing the distance between the data targets and their model analogues, according to the loss function $L(\Theta) = [M^d - M^m(\Theta)]'W[M^d - M^m(\Theta)]$. $M^d$ and $M^m(\Theta)$ are the vectors of data targets and model analogues (medians across simulations), respectively. The weighting matrix $W$ is the identity matrix, so to assign equal weight to all targets.\footnote{To make the targets entering $L(\cdot)$ more homogeneous in terms of units of measurement, the two targets related to spreads are multiplied by a factor of ten.}

The final results of the calibration of economic and political parameters are summarized in Table 1. Table A3 in Appendix G.1 performs some comparative statics exercises on the dependence of calibration targets on the parameters in $\Theta$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate $r$</td>
<td>0.017</td>
<td>German bonds yield, mean 2000-2017</td>
</tr>
<tr>
<td>Risk Aversion $\gamma$</td>
<td>2</td>
<td>Conventional Value</td>
</tr>
<tr>
<td>Autocorrelation $\ln A$</td>
<td>$\rho$</td>
<td>Labor productivity, 1970-2017</td>
</tr>
<tr>
<td>Std dev innovations $\varepsilon$</td>
<td>$\sigma_\varepsilon$</td>
<td>Labor productivity, 1970-2017</td>
</tr>
<tr>
<td>Share domestic debt $\delta$</td>
<td>0.65</td>
<td>Mean 2000-2017</td>
</tr>
<tr>
<td>Re-entry probability $\chi$</td>
<td>0.25</td>
<td>Mean Exclusion of 4 years (Greece)</td>
</tr>
<tr>
<td>Political Bias $Y$</td>
<td>$\omega^Y$</td>
<td>0.072</td>
</tr>
<tr>
<td>Political Bias $O$</td>
<td>$\omega^O$</td>
<td>0.605</td>
</tr>
<tr>
<td>Political Bias $H$</td>
<td>$\omega^H$</td>
<td>0.323</td>
</tr>
<tr>
<td>Discount Factor $\beta$</td>
<td>0.7</td>
<td>Method of Simulated Moments</td>
</tr>
<tr>
<td>Frisch Elasticity $\varphi$</td>
<td>2</td>
<td>Method of Simulated Moments</td>
</tr>
<tr>
<td>Debt Default Costs $\tilde{d}_0; \tilde{d}_1$</td>
<td>0.095; 2.3</td>
<td>Method of Simulated Moments</td>
</tr>
<tr>
<td>Ent Default Costs $\alpha_0; \alpha$</td>
<td>0.75; 2.8</td>
<td>Method of Simulated Moments</td>
</tr>
</tbody>
</table>

Notes: Debt default cost parameters have been rescaled to ease the comparison with the literature. $\tilde{d}_0$ corresponds to the percentage drop in $A$ when the latter is at its mean. $\tilde{d}_1$ is the ratio of the drops when $A$ is four standard deviations above the mean, as opposed to its mean.

### 3.3 Model Fit

The model under the preferred parameter configuration is able to match the data targets well, as documented in Table 2. The 2017 values are very close, and time trends in the model match the ones in the data in terms of sign, with a bit of discrepancy for some of the magnitudes.

Given the aim of drawing simulations for future dynamics, it is more important in my setting to match time-trends and final historical values rather than the business cycle statistics.
Table 2: Calibration Targets

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entitlement Spending/GDP, 2017</td>
<td>19.5%</td>
<td>20.2%</td>
</tr>
<tr>
<td>Income Tax Revenues/GDP, 2017</td>
<td>27.8%</td>
<td>27.5%</td>
</tr>
<tr>
<td>Debt/GDP, 2017</td>
<td>19.0%</td>
<td>19.1%</td>
</tr>
<tr>
<td>Spread, 2017</td>
<td>0.38%</td>
<td>0.24%</td>
</tr>
<tr>
<td>Trend-predicted change 2000-08 in Entit/GDP</td>
<td>1.1%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Trend-predicted change 2000-08 in Tax Rev/GDP</td>
<td>1.7%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Trend-predicted change 2000-08 in Debt/GDP</td>
<td>-0.4%</td>
<td>-0.3%</td>
</tr>
<tr>
<td>Trend-predicted change 2000-08 in Spreads</td>
<td>0.06%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

Notes: Model targets are medians across 20,000 simulation runs over the period 2000-2017, derived as described in Appendix F.2. Trend-predicted changes are the difference between the 2008 value of a variable and its 2000 value, as predicted by a linear time-trend estimated over the period 2000-2008.

usually targeted in the sovereign debt literature. Nonetheless, in Table 3 I report results on some of the statistics typically considered in previous works, as a matter of comparison.

Table 3: Additional Data Matching

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of Variation of Spread</td>
<td>1.53</td>
<td>1.67</td>
</tr>
<tr>
<td>Skewness of Spread</td>
<td>1.78</td>
<td>2.08</td>
</tr>
<tr>
<td>Correlation debt and output</td>
<td>-0.41</td>
<td>-0.86</td>
</tr>
<tr>
<td>Correlation spreads and output</td>
<td>-0.10</td>
<td>-0.71</td>
</tr>
<tr>
<td>Correlation primary balance and output</td>
<td>-0.10</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

Notes: Data moments are calculated over the period 2000-2017. Output is linearly de-trended (in logarithms). Debt and primary balance are divided by output.

Bearing in mind that they should be interpreted with a grain of salt, given the relatively short span of annual data over which they are calculated, the untargeted moments in Table 3 line up relatively well with the data. The skewness and the coefficient of variation for the spreads are close to their data counterparts. The cyclicality of debt, spreads, and primary balance is aligned in terms of sign, while the model tends to overshoot magnitudes.

One interesting comparison can be drawn with the results in Bocola et al. (2019) (BBD, from now on). They emphasize how their model, in contrast with the standard ones in the sovereign default literature, is able to deliver countercyclical debt issuances, and spreads that are both volatile (with a coefficient of variation above one) and highly right-skewed. These results are in line with data for European countries. They ascribe their success to a higher discount factor,
coupled with the introduction of a fixed lower bound for (government) consumption in the utility function. These elements lessen the incentives to front-load consumption, which leads to opposite results in typical models in the literature.

My model displays similar quantitative results. The correlation of output to debt-issuances-to-GDP is -0.84 (-0.13 if \( b' \) is not scaled by \( y \)). Moreover, Table 3 reports a large coefficient of variation and a significant right skewness for the spread.\(^{20}\) Yet, those outcomes are obtained, in line with the rest of the literature, with a low discount factor and without an ad-hoc lower bound to consumption in the utility function. In fact, one can see entitlements in my setting as another, more micro-founded way to attenuate the incentive to front-load consumption of the standard models. Furthermore, they appear to be potent enough to generate countercyclical debt issuances even with a low discount factor.

4 Characterization of the Equilibrium

Before presenting the simulation results, it is instructive to analyze some of the properties of the equilibrium solution. I focus on the main new elements in my model compared to existing works: first, the relationship between entitlements and both the debt-default decision and sovereign risk; then, the evolution of the equilibrium under the demographic shift, with an emphasis on the aging-induced market discipline through debt prices and the pressure that the aging population puts on spending.

Debt-default decision. Start by considering the decision on whether to default on debt, \( d^{B^t}(S) \). I present its dependence on the state variables with the help of Figure 2. This shows the optimal default decisions on debt, \( d^{B^t}(S) \), and entitlements, \( d^{E^t}(S) \), over some cross-sections of the state space. I plot the \( t = 2017 \) functions, although the same findings apply qualitatively to any \( t \). In particular, Figure 2 uses different colors to highlight whether in a given state \( s \) there is: debt repaid and current entitlement promise kept (RK, \( d^{B^t}(s) = d^{E^t}(s) = 0 \)); debt repaid and entitlements broken (RB, \( d^{B^t}(s) = 0, d^{E^t}(s) = 1 \)); debt defaulted and entitlements kept (DK, \( d^{B^t}(s) = 1, d^{E^t}(s) = 0 \)); or debt defaulted and entitlements broken (DB, \( d^{B^t}(s) = d^{E^t}(s) = 1 \)).

As in typical sovereign default models, \( d^{B^t}(S) \) is increasing in the level of debt currently due and decreasing in productivity. Note that the monotonicity in debt was not obvious a priori in my context, given the new additional cost of defaulting on domestic debt, which is increasing in debt due. The numerical results suggest that the strength of this new force is not enough to alter the usual positive relationship between defaults and debt levels.

The dependence of the optimal debt default decision on the level of entitlements can be

\(^{20}\)The share of very low realizations of the spread matches the data also in my model. In particular, the fraction of spreads below 10% of their median is 38% in my model and 44% in the data. However, note that those numbers are a little less informative in my setting, given that they are calculated from annual data that cover the entire European debt crisis period, as compared to the ones in BBD, which are quarterly and up to 2012Q2.
Figure 2: State-Space Default Regions

Notes: Result from the solution at $t = 2017$. The regions are: debt repaid and entitlements kept (RK), debt repaid and entitlements broken (RB), debt defaulted and entitlements kept (DK), and debt defaulted and entitlements broken (DB). Debt is divided by mean output. Entitlements are per capita amounts multiplied by the size of the $O$ population and divided by mean output. Productivity is reported in terms of standard deviations above/below its mean. Fixed variables in each panel are set at their median value in the baseline simulations.

appreciated by considering the left and right panels of Figure 2. These show that, first, for low enough levels of debt and/or high enough levels of productivity, debt obligations are always honored, regardless of the promised entitlements. The converse is true for combinations of low productivity and high debt. Spending promises can affect debt default decisions at intermediate values of debt and productivity. In particular, if defaulting on debt is optimal at some level of entitlements, then it remains so at any higher level. This monotonicity is prevalent over most of the state space, with the exception of very few points, not covered in the regions depicted in Figure 2, where an increase in $e$-promises moves the optimal choice on debt, ceteris paribus, from default to repayment.\footnote{I verify numerically that these “entitlement-disciplining-default” regions encompass states where spending promises are broken under both debt repayment and default. What the former allows is not a less drastic cut of current transfers $p$ below the promised $e$, but rather the possibility of no (or less extreme) reforms for future entitlements. That is because, by avoiding the productivity cost of default, the economy can enjoy higher (expected) future output, ceteris paribus. This allows for higher entitlements to be honored at $t + 1$, thereby making them desirable as promises at $t.$}

Interestingly, note that even very high levels of entitlements need not trigger a default on debt. In fact, only small enough values of old-age spending promises have an effect on $d^{B_t}_t$. That is because, once entitlements reach a certain level, then it becomes optimal, ceteris paribus, to
renege on and reform them, both under debt repayment and default. When that happens, $d^B_t(b, A, e)$ becomes independent on $e.$

**Debt pricing schedule.** The shape of the debt-default policy for period $t + 1$ described in Figure 2 maps into sovereign risk perceived at $t$ and hence, according to equation (11), into debt prices at $t$. Figure 3 displays cross-sections for the equilibrium pricing schedule for two periods, $t = 2017$ and $t = 2050$.

![Figure 3: Pricing Schedule](image)

*Notes: This figure presents some cross-sections for the equilibrium pricing schedule in 2017 (equation (11)). Debt Issuance, $b'$, is divided by mean output in 2017. Productivity, $A$, is reported in terms of standard deviations above/below its mean. Future Entitlements, $e'$, are per capita amounts multiplied by the 2017 size of the O population and divided by mean output in 2017. The fixed coordinates in each panel are set at the 2017 median values.*

Start by analyzing the monotonicity of the curves in Figure 3. Debt prices are decreasing in the amount of debt issuances (left panel) and increasing in productivity (middle panel), two features shared with standard sovereign default models.

In terms of the dependence on future entitlements ($e'$, right panel), consider the 2050 curve first. Consistently with the discussion of Figure 2, higher values of $e'$ make future defaults weakly more likely, depressing $q_t$. However, this effect is at work only for small enough future entitlements. Once the latter become large enough, then they will be reneged upon and reformed in $t + 1$, effectively becoming irrelevant for the debt-default decision. Debt prices then settle at the level consistent with the expected probability of repayment in $t + 1$, which need not be zero, and is constant in $e'.$

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22 In fact, the specific functional forms for the entitlement default and reform costs assumed in this paper (in particular, their linearity in $u()$, see equation (15)) imply that once $e$ gets so high that there is entitlement reneging and reform under both debt repayment and default, then the difference between $V^R_t$ and $V^D_t$ becomes exactly constant in $e$. This is because all terms involving $e$ then cancel out. Hence, $d^B_t$ no longer changes with $e$.

23 Consistently with the discussion in footnote 21, there are some small portions of the $q_t$ function that are increasing in $e'$. The careful reader may spot one minor example of those in the 2017 curve, around an entitlement promise level of 0.15.
Debt prices for 2017 are instead shown to be independent from \( e' \) (right panel of Figure 3), and consistent with an expected probability of default equal to zero. That is because the government in \( t + 1 \) will repay the debt level of \( b' \) (that is fixed in the right panel of Figure 3) at any level of future productivity with a non-negligible chance of realization, regardless of the choice of whether there will be reneging on entitlements. In terms of Figure 2, this correspond to a situation where the only regions with non-negligible probabilities of being visited at \( t + 1 \) are those in the top portion of the left panel, where debt defaults never occur at any level of \( e \), regardless of the choice of entitlement default.

**Sovereign risk and the aging-induced market discipline.** Importantly, Figure 3 also allows to appreciate the evolution of sovereign risk under the demographic shift. The lower price in \( t = 2050 \) compared to \( t = 2017 \) is consistent with a higher probability of default in the older economy at any given triplet of debt issued, productivity, and promised future entitlements. Intuitively, as the population ages, the tax base shrinks, while the pool of old-age entitlement receivers increases. The distortions associated with higher tax rates and the costs of defaulting on entitlements then imply that, for the same level of total promises in terms of debt and spending, higher values of productivity \( A \) are necessary to optimally sustain debt repayment in the older economy. That is, sovereign risk is higher for any given level of newly issued debt \( b' \) and promised entitlements \( e' \), i.e., \( q_t(b', A, e') \) is lower at any given triplet \( (b', A, e') \). This shift in the pricing schedule is the core of the aging-induced market discipline effect.

**Aging-induced pressure on spending.** The pressure on spending arising in the model with population aging can be appreciated in Figure 4, which compares policy functions related to old-age spending in year 2017 and 2050.\(^{24}\) The top panel (a) displays the level of optimal per capita old-age transfers, \( p^*_t(b, A, e) \), as a function of each of the three state variables. The bottom panel (b) displays the total optimal transfer spending as a share of output, \( s^O_t p^*_t(b, A, e)/\gamma^*_t(S) \), which can be interpreted as a measure of the total share of resources that the old are able to command. The different line patterns in the figure denote the different combinations of default choices on debt and entitlements (already discussed in Figure 2) in a given state.

Overall, Figure 4 shows that old individuals in 2050 enjoy lower per capita transfers compared to old individuals in 2017, at any given state (the blue line is below the black line in panel (a)). At the same time, they are able to appropriate a strictly larger fraction of total output as a group (the blue line is above the black line panel (b)).\(^{25}\) Again, the reason is that the ratio of old-age transfers recipients to taxpayers is almost twice as large in 2050 than in 2017. This puts enormous pressure on public finances, which cannot be eased with an increase in borrowing, given the less favorable debt prices at issuance already discussed. The final result

\(^{24}\)Figure A2 in Appendix G.2 shows all the policy functions of the model.

\(^{25}\)The result in panel (a) holds numerically across the entire state-space. The one in panel (b) is reversed in very few states where, by transferring a lower fraction of resources to the \( O \), the government in 2017 is able to keep some of the promises that the government in 2050 would break.
Figure 4: Optimal Policies - Old-Age Transfers

Panel (a) - Transfers Per-Capita

Panel (b) - Total Transfers over GDP

Notes: Panel (a) reports cross-section along the three inputs of optimal per capita transfers, $p^*(S)$. Panel (b) reports cross-section of total transfer payments over output, $s^*_t(S,y^*_t(S))$. Line styles correspond to different default-regions of the state-space in terms of default choices (see the notes to Figure 2 for an explanation of the acronyms). Debt is divided by mean output. Productivity is reported in terms of standard deviations above/below its mean. Entitlements are per capita amounts multiplied by the size of the $O$ population and divided by mean output. Fixed variables in each panel are set at their median value in the simulations.

is that per capita transfer cuts (together with tax rate increases, documented in Figure A2) are unavoidable at any given state in the 2050 economy with respect to the 2017 one. Nonetheless, the larger group of old voters in the future is still able to leverage upon its higher political weight, so to secure a strictly larger share of output in the majority of states of the economy.

A final remark on the evolution of policies with population aging. It might sound surprising that an electorate featuring an increasing share of older voters allows the government to cut entitlements per capita so drastically at any given state. It is important, however, to remember that: (i) the probabilistic voting scheme I assume maps into the government maximizing the political objective function (3) subject to the budget constraint. Therefore, it is as if voters are given the option to choose a fiscal package only among the set of feasible ones (i.e., those satisfying the budget constraint). The aging of the population, with its budget consequences, has the effect of lowering the feasible amount of per capita entitlements among which voters can choose; (ii) voters are rational and forward looking. They realize that too high levels of spending promises will be defaulted upon. Therefore, they refrain from voting for them. I provide more evidence on the role of rational expectations with the exercise in Section 5.3.
5 Simulations of Fiscal Dynamics

In this section, I present the results of the simulations of future fiscal dynamics derived under the baseline calibration (Section 5.1). I then move to the description of the counterfactual exercises that examine the various forces that population aging induces in my model (Section 5.2). Finally, I present the results of a model that allows for irrational expectations about future fiscal policy (Section 5.3).

All the simulations of fiscal dynamics derived in this and the following section are based on the algorithm that I detail in Appendix F.2. Here I highlight only some of its main features. First, the general simulation strategy consists of deriving many paths for the endogenous variables under different realizations of the exogenous shocks. The results of interest are (some moments of) the distributions of the endogenous fiscal variables obtained for each year in the simulation. Second, the simulation period runs from 2018 to 2050. I chose 2050 as the final year because the shares of young and old individual stabilize afterwards in the population projections for Italy. Third, the baseline simulations are initialized several periods before 2018, to avoid the influence of the initial state. Finally, the simulations for all the alternative scenarios derived from Section 5.2 onward share the same set of initial states and realizations of the exogenous shocks as the baseline ones in Section 5.1.

5.1 Baseline Simulations

The dynamics of the fiscal variables of interest under the baseline calibration of the model are presented in Figure 5. The four panels show actual data up to 2017 (solid lines) and then simulation results. In particular, I report the medians (thicker dotted lines) and 5th and 95th percentiles (thinner dotted lines) obtained in each year of the simulations for entitlement spending over GDP, tax revenues over GDP, debt over GDP, and the interest rate spread on government debt with respect to the world risk-free rate.

Figure 5 shows that the model predicts an increase in the ratio of old-age transfers to GDP of about 5 percentage points over the simulation period. This is accompanied by a rise in the labor income tax revenues as a share of GDP (i.e., the tax rate τ in the model) by almost 4 percentage points. At the same time, maturity-adjusted debt-to-GDP drops by almost 6 percentage points. The declining debt reduces sovereign risk and leads to a decreasing median spread.

Three remarks are important when analyzing the findings in Figure 5. First, the trajectory of debt hints at a prominent role of the aging-induced market discipline. Results in sections 5.2 and 5.3 will confirm this intuition.

Second, note that the dynamics of spending and taxes tend to stabilize after 2040. This is because the bulk of the demographic transition happens before that year.

Third, the second panel in Figure 5 displays an increase in tax rates. It is up for debate whether labor income taxes can be further raised in Italy before moving to the wrong side of the Laffer curve. Trabandt and Uhlig (2013) provide some evidence in favor of the existence of
some room of maneuver, at least in comparative terms vis-à-vis several other European countries. In fact, most of the fiscal adjustment in my model happens through spending cuts rather than tax hikes (as it will be apparent shortly when I present the welfare costs of the changes in fiscal policy for future generations). This limited adjustment of tax rates is consistent with the model economy being relatively close to the top of the tax Laffer curve. The position of the economy on this curve is fundamentally governed by the Frisch elasticity of labor supply $\phi$. When conducting comparative statics exercises in which I vary $\phi$, I find that, intuitively, the lower is $\phi$, the more tax rates are raised over the simulation period (see Table A3 in Appendix G.1).

The frequency of default and reform episodes in each year in the simulations are presented in Figure 6. The left panel shows that the likelihood of a sovereign default remains low and constant over time, if anything slightly declining from 2% to about 1.5%. The middle and right panels of Figure 6 show that entitlement cuts are instead more frequent. The likelihood of defaulting on spending promises grows from 9% in 2018 to 19% in mid 2040’s. Entitlement reforms are observed in around 15-20% of the simulations during the first two decades. They become less frequent afterwards, given the lower incentive to adjust future entitlements once the aging process has slowed down after 2040.

The noticeable difference in magnitudes between the frequency of defaults on debt and entitlements is only partly due to the different structures assumed for the default costs (equations (14) and (15)), as well as the calibrated values of the parameters that enter these cost functions (see Table A3 in Appendix G.1). In fact, the difference in the frequency stems for a more

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26 In terms of differential current promise breaking, episodes of default on debt but not on entitlements are very rare, accounting for less than 3% of all sovereign default episodes. This results is consistent with sovereign defaults happening in exceptionally dire periods for the economy (in terms of productivity), which require adjustments not only to debt but also to entitlements.

27 Some recent strategic sovereign debt papers model the cost of debt defaults in terms of (one-time) exogenous...
fundamental reason: debt can be freely reduced each period, whereas entitlements can only be lowered through defaults and reforms. Although there is some leeway in the choice of the spending cut to adopt (with model parameters having an impact on this choice, as shown in Table A3), population aging does not allow to defer the reductions for too long. The result is that defaults and reforms on entitlements are always more frequent than defaults on debt.

**Figure 6: Baseline — Frequency of Episodes**

*Notes:* Panels report the frequency of events (debt defaults, entitlement default, and entitlement reforms) observed in the simulations in each period. Defaults are episodes of reneging on current obligations. Reforms are situations in which the entitlement promises for the next period are lower than the current period ones.

Dynamics for other fiscal variables in the model are reported Figure A3 in Appendix G.3. One interesting result displayed in that figure is that old-age transfers per capita are cut by about 47% by 2050 with respect to 2017. This fall generates large reductions in welfare for future retirees compared to current ones. I quantify these welfare changes relying on the unconditional compensating variation (CV), as defined in Lester et al. (2014). Intuitively, the CV is the percentage change in consumption that an agent alive in period $t$ must be offered from $t$ onward to experience, ceteris paribus, the same level of expected lifetime utility of an individual of the same age group alive in 2017. Appendix C provides the precise definition and derivation of the welfare metrics in my model.

The welfare implications of the fiscal changes induced by the aging population are presented in Figure 7. This figure shows the CVs for both old (left panel) and young (middle panel) individuals, as well as for all voters alive in period $t$ (right panel). The CV in the latter case is computed as the change in consumption—common across all voters—that equates the expected total welfare for generations alive in period $t$ to the one of 2017’s generations. This total welfare is the sum of the lifetime utility of all individuals alive in $t$, weighted only by their shares in the population (see equation (24) in Appendix C). There are sizable and growing welfare reductions that the demographic shift entails for future generations are sizable. The decrease is particularly large for future retirees, with CVs up to 80%. These large CVs

utility drops, rather than income/productivity reductions. That would allow for a bit more parallelism in the costs for debt and spending promise-breaking in the current setting. However, the nature of the government problem in this context would make this choice not ideal, as it would require to find a reasonable way to split this utility cost and impute it to the various groups of voters.
Figure 7: Baseline — Welfare Effects of Aging

Notes: CV is the unconditional compensating variation, calculate for old, young, and all voters. The CV for all voters (right panel), is based on total welfare of individuals alive in a given year, calculated as the sum of their lifetime utility weighted only by the groups’ population shares.

are the result of the large cut to per capita transfers to the elderly. There are reductions in welfare also for future workers, with CVs reaching 7.5%. The main driver of the CVs for the young is the increase in taxes documented in the top right panel of Figure 5. The lower future entitlements do not play a decisive role, since they are heavily discounted under the stochastic lifetime structure I assume. The large difference in the CVs for young and old is consistent with fiscal adjustments being more severe on spending rather than taxes.

The right panel in Figure 7 shows that the CVs for the entire electorate reaches almost 10% by 2050. The magnitude of this loss is a weighted average of the losses for young and old, with more weight attached to the young for the following reasons: (i) there are always more young than old individuals in the economy. In particular, the ratio of workers-to-retirees shrinks over time, but it remains always above one (it reaches 1.4 by 2050); (ii) the concavity in consumption of the flow utility (equation (12)) is globally higher for young voters as compared to old voters once the optimal labor supply is factored in. As a result, changes in consumption of the young have a larger impact on the total welfare of the electorate, and so they are weighted more heavily when deriving the CV for all voters.

The baseline results presented in this section appear consistent with a strong role of market discipline. The counterfactual exercises in the next two sections help shed more light on this and the other main forces in my model.

5.2 Inspecting the Mechanisms

The evolution of fiscal variables presented in Section 5.1 is the result of the balancing of three main forces arising from population aging in my setting: pure demographics, politics, and market discipline. Each of these forces maps into specific elements in the model. The “purely demographic” effect of population aging works through: (i) the shares of young and old individuals in the government budget constraint ($s_t^Y$ and $s_t^O$ in inequalities (4) and (5)), which determine tax revenues and total entitlement spending; and (ii) the life expectancy of the el-
The political effect of the aging population stems from the populations shares entering the weights in the political objective function of the government ($s_i$ in equation (3)). Finally, the market discipline effect is due to the movement of the debt pricing schedule in response to the evolution of sovereign risk induced by the aging of the population, as discussed in Section 4.

In this section, I examine the contribution of each of the three forces in shaping the baseline results. I do so by analyzing counterfactual scenarios in which I allow different combinations of the forces to affect fiscal dynamics. Specifically, I conduct two exercises, whose results I then compare the baseline’s ones. First, I simulate the model under the effects of the sole demographic pressure induced by population aging. Namely, I let the demographic variables in the budget constraint and in the lifetime utility evolve as in the baseline scenario. I instead keep constant the pricing schedule and the weights in the political objective function, fixing them at their 2017 values. The second exercise is analogous to the first, but it allows also the political weights to vary over time, keeping only the pricing schedule fixed at its 2017 level.

Before presenting the findings, let me stress that I conduct these exercises with the only purpose of shedding more light on the forces in the model. Each of these counterfactual experiments violates some of the assumptions that ensure the consistency of the political process and/or the rationality of the lenders. None of them should therefore be seen as describing a valid alternative to the baseline scenario.

The results of the counterfactual exercises are summarized in Figure 8. There, I report the percentage change in the median value for several fiscal variables between the beginning and the end of the simulations (panels in the first two rows). The figure also shows the average yearly frequency of defaults on debt and entitlements over the simulation period (third row). The blue bars (**Dem**) correspond to the first counterfactual exercise where only the demographic force is at play. The orange bars (**DemPol**) are for the case of both demographic and political forces, i.e., only market discipline through debt prices is absent. Finally, the yellow bars (**DemPolMkt**) correspond to the baseline findings already analyzed in Section 5.1, where all the three forces (demographics, politics, and market discipline) are present.

The exercise **Dem** in Figure 8 displays a decline in output over the transition: the tax rate, and hence individual labor supply, is not changed, but the labor force has contracted. A constant tax rate applied to a smaller tax base (i.e., output) leads to a decline in tax revenues. Debt levels are also relatively constant, resulting in a significant increase in the debt-to-output ratio. With falling tax revenues and without an increase in debt revenues, per capita transfers to the elderly are cut significantly.

When there is rising political power of the elderly on top of demographic forces (exercise **DemPol**), a pressure on spending is generated. Older voters are able to obtain a smaller reduc-

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28“Pure demographics” affects also the $n_t$ that adjusts the per-capita level of debt by population growth (see inequalities (4) and (5)). The demographic variables pertaining to the domestic debt holders drop from the problem under the assumptions I adopt when calibrating the political cost of default. See Proposition 1 in Appendix B.
Notes: Panels in the first two rows report the percentage change in the median value of each variable in the simulation, comparing the last year (2050) to the first (2018). The panels in the last row report the average annual frequency of episodes (defaults on debt and entitlements) across simulations. The blue bars (Dem) correspond to a counterfactual exercise where only the demographic force is at play. The orange bars (DemPol) are for the case of both political and demographic forces. The purple bars (DemPolMkt) correspond to the baseline results already analyzed in Section 5.1, where all the three forces (demographics, politics, and market discipline) are present. Initial states $S_{2018}$ and productivity shocks in the simulations are identical in the three scenarios.

As a result, the government has to finance this higher spending relying on taxes or debt. Now, it is important to realize that the former hurt only the current workers, while the latter impacts all future generations: by increasing the government’s future financing needs, higher debt levels heighten the risk of a sovereign default, whose economic costs affect the whole economy. Since retirees in the model have a sizable probability of surviving to next period (which is growing over the demographic transition), they prefer to use their rising political power to raise tax rates rather than debt.
Finally, when markets exert their pressure through the shrinking pricing schedule on top of the other forces (exercise DemPolMkt), then taxes are raised even more over the transition, to finance a large reduction in the debt level. Debt now declines over time as a share of output and some of the savings from the lower interest payments are appropriated by the elderly to further reduce the decline in per capita entitlements compared to the other scenarios. The significant reduction in the debt level leads to a lower likelihood of sovereign default. At the same time, the less favorable pricing schedule and the higher tax rates lower the ability of the government to raise funds to honor the entitlement promises in the face of negative productivity shocks, which leads to a higher probability of entitlement reneging.

5.3 Irrational Expectations About Fiscal Policy

One crucial assumption for market discipline to lead to a declining debt-to-GDP ratio and a low frequency of sovereign default is that voters correctly internalize the future optimal policies. In particular, in order to undertake the costly entitlement cuts and tax hikes observed in the baseline scenario, voters need to realize that the current fiscal policy will not be sustainable in the future. A concern one might have is that real voters might not be as rational. Will a departure from full rationality alter the dynamics of fiscal variables and, in particular, government debt observed in my baseline results?

I answer this question relying on a modified version of my model where I assume that a share of voters expect entitlements and taxes to remain at their current level forever. The rest of the voters and the lenders are instead fully rational, correctly internalizing the impact that the irrational voters will exert on future equilibrium choices. The exact expressions for the lifetime indirect utilities of rational and irrational voters are reported in Appendix G.5.

Figure 9 shows the evolution of the main fiscal variables in this counterfactual exercise (blue lines), compared to the baseline results (dotted black lines). Figure 10 shows the comparison of default and reform frequency in the two scenarios. I set the share of irrational voters in each age-group at 20%, but I revisit this assumption at the end of this section.

Intuitively, when there are myopic expectations about future fiscal policy, the government tends to make promises that are harder to keep: it pledges higher levels of entitlements and it issues more debt (see Figure A7 in Appendix G.5). The higher reliance on debt compared to the baseline scenario is consistent with: (i) the need to rise more revenues to finance higher levels of promised spending; (ii) irrational agents not internalizing the impact that the irrational voters will exert on future equilibrium choices. The result is a debt-to-GDP ratio that increases over time (third panel in Figure 9), despite the rising sovereign spreads (right panel in Figure 9).

The higher level of per capita entitlement promises compared to the baseline scenario are again consistent with irrational agents not properly internalizing the future consequences of current choices. In particular, irrational agents fail to internalize that too high levels of per capita entitlements will be defaulted upon in the future. Therefore, they vote for higher spend-
Figure 9: Irrational Fiscal Expectations — Main Fiscal Variables

Notes: Solid lines are data for the period 2000-2017 (see the Notes to Figure 5 for details on the data series). Dotted black line is median of distributions in each simulation period for baseline scenario. Thick dashed blue line is median and thinner dashed lines are 10th and 90th (75th for spreads) percentiles of distributions in each simulation period for alternative scenario. Initial states $S_{2018}$ and productivity shocks in the simulations are identical in the baseline and alternative scenario.

Future governments are then forced to raise tax rates by more. However, despite the higher tax rates and debt issuances, government obligations are often so high that more defaults on both debt and, especially, entitlements are observed.

Figure 10: Irrational Fiscal Expectations — Crises And Reform Frequency

Notes: Panels report the frequency of events (debt defaults, entitlement default, and entitlement reforms) observed in the simulations in each period. Defaults are episodes of reneging on current obligations. Reforms are situations in which the entitlement promises for the next period are lower than the current period ones. Black lines are for the baseline scenario, blue ones for the alternative. Initial states $S_{2018}$ and productivity shocks are identical in the baseline and counterfactual exercise.

Finally, how large should the fraction of irrational voters be to have debt-to-GDP ratios rising instead of falling over the demographic transition? I provide an answer to this question in Figure 11. The left panel in this figure displays the change in the median debt-to-GDP ratios between the initial and final simulation year (vertical axis) for different values of the share of irrational voters (horizontal axis). A share of zero correspond the baseline scenario with fully rational voters. Figure 11 shows that a fraction of irrational voters sightly above 10% would be enough to observe an increasing debt-to-GDP ratio over the transition.

The right panel in Figure 11 shows that the average yearly frequency of sovereign defaults
Figure 11: Share of Irrational Voters and Debt Dynamics

Notes: The left panel reports the change in the median debt-to-GDP ratio across simulations between 2050 and 2018. The right panel reports the average yearly frequency of debt default episodes in the simulations over the period 2018-2050.

in the simulations is increasing in the fraction of irrational voters, with sizable changes with respect to fully rational voters starting to appear for shares again in the order of 10%.

6 Policy Interventions

This section presents the simulations obtained under two policy interventions. Section 6.1 examines the effects of a lending mechanism that allows for external funding at convenient prices, though under conditionality. Section 6.2 discusses the outcomes of a reform that increases the minimum age required to receive old-age transfer.

As it was the case in the counterfactual exercises in Sections 5.2 and 5.3, the scenarios presented in this section are also derived by first solving the model under the new setting—which is assumed to be introduced unexpectedly and permanently—and then using the same initial states and shock realizations as in the baseline to derive the simulations. This ensures comparability in the variables’ trajectories.

6.1 Spread Break With Conditionality

This section presents the effects of the introduction of a lending scheme that allows the government to issue debt at a high, fixed price (thus providing a break to the spread). The access to this facility is granted only under the commitment to a prudent fiscal policy that ensures debt sustainability. This lending arrangement resembles the Primary Market Support Facility offered by the European Stability Mechanism (ESM), the financial institution set up by euro area states to help member countries in financial distress.

The rationale of such an emergency lending scheme is the same in the model and in the ESM’s intentions: it provides fiscal relief to governments that experience sudden shocks that raise the interest rates on government debt. In particular, the facility is meant to be used in those situations in which the shock is expected to have only transitory effects, not a long-lasting
impact that impairs the solvency of the country. At the same time, this lending arrangement relaxes the link between debt prices and the probability of future repayments, i.e., it constraints market discipline. This is always a paramount concern when evaluating crisis resolution mechanisms for the eurozone (Corsetti et al., 2015, Andritzky et al., 2019).

I introduce a few modifications to my model that allow to study the effects of this lending arrangement in my aging economy. In particular, the specifics of the lending scheme that I assume are the following. First, the government can issue debt at a fixed price $q$. Second, the lending facility can be accessed only under the following conditions: (i) market access has been re-gained in case of a previous default; (ii) the country cannot increase its stock of debt (i.e., $b' \leq b$); (iii) future per capita entitlements cannot be increased above the current ones (i.e., $e' \leq e$); (iv) the expected probability of debt repayment in $t+1$ must be at least $P$. The latter requirement is an operationalization of the notion of sustainable debt.\[30\]

**Figure 12:** Spread Break — Main Fiscal Variables

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Notes: Solid lines are data for the period 2000-2017 (see the Notes to Figure 5 for details on the data series). Dotted black line is median of distributions in each simulation period for baseline scenario. Thick dashed blue line is median and thinner dashed lines are 10th and 90th (75th for spreads) percentiles of distributions in each simulation period for alternative scenario. Initial states $S_{2018}$ and productivity shocks in the simulations are identical in the baseline and alternative scenario.

Figures 12 and A8 in Appendix G.6 display the dynamics for fiscal variables after the introduction of the facility. There are two distinct phases. In the first one, that lasts for approximately 5 years, the government lowers taxes and rises entitlements. These fiscal changes are financed with debt. This is a clear manifestation of the politico-demographic forces in the model that indeed push for higher debt accumulation when market discipline is constrained. Note that the mere possibility to access the emergency lending arrangement in case of bad shocks reduces sovereign risk. Therefore, the pricing schedule offered by lenders is now higher than in the baseline, allowing the government to borrow more without increasing the spreads.

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29In deriving the results in this section, I set $q$ consistently with the average interest rate spread of 34 basis points observed between ESM/EFSF and German bonds with 1-year residual maturity over the period 2012-2018.

30ESM guidelines do not provide a precise threshold for debt sustainability. $P$ is set at 75% in what follows. The Primary Market Support Facility also requires a minimum participation of private investors to the issuance, in the order of 50%. The assumption of a fraction $\delta = 0.65$ of debt remaining domestic in this model implicitly guarantees that this condition is always satisfied.
Moreover, Figure 13 shows that the government is initially able to avoid defaults on both debt and entitlements, and to defer reforms. In fact, the favorable movement of the debt pricing schedule allows the government to respond to any shock by issuing more debt, without defaulting and without having to resort to the lending facility (right panel of Figure 13, which shows the frequency of simulations in which the government asks for the spread break).

**Figure 13: Spread Break — Frequency of Episodes**

Notes: Panels report the frequency of events (debt defaults, entitlement default, entitlement reforms, and access to the spread break arrangement) observed in the simulations in each period. Defaults are episodes of reneging on current obligations. Reforms are situations in which the entitlement promises for the next period are lower than the current period ones. Black lines are for the baseline scenario, blue ones for the alternative. Initial states $S_{2018}$ and productivity shocks are identical in the baseline and counterfactual exercise.

However, the first phase of fiscal profligacy ends quickly and ruinously. After about five years since the introduction of the lending mechanism, sovereign debt has reached levels so high that the government has to rely on the lending facility in about 70-80% of the simulations. The economy is stuck with high levels of debt that both make it hard to leave the special financing program. There is also a non-trivial fraction of the simulations in which the government cannot (or finds too costly to) guarantee the sustainability of its debt, thus ending up defaulting. Tax rates are raised to levels similar to the no-reform scenario and per capita transfers are set at even lower levels. By the end of the simulations, even higher tax rates are set, depressing output and therefore increasing all the GDP ratios. Yet, these tax are still not enough to cover the total old-age spending promises, and a rising frequency of entitlement cuts is observed. As already discussed, demographic dynamics lower reform incentives in the last decade of the simulations, hence most of the entitlement cuts take the form of defaults.

Finally, Figure 14 presents the welfare effects associated with the introduction of the lending facility. It displays the CVs for old, young, and all voters. The black dotted lines are the CVs for the baseline economy (identical to the ones in Figure 7). The blue dashed lines are analogous CVs calculated for the economy with the spread break, where the reference is the 2017 utility (which is the same in the baseline and in the alternative scenario, given that the policy is assumed to be introduced unexpectedly). Finally, the solid orange line represents the CVs that would equate the expected lifetime utility of young, old, and all voters at each $t$ under the new policy with respect to the baseline.
Figure 14: Spread Break — Welfare Effects of Aging

Notes: Unconditional compensating variation (CV) for old, young, and all voters. For the latter (right panel), CVs are based on total welfare of individuals alive at $t$, weighted by pure population shares. The black dotted line shows the CVs with respect to the 2017 utility under baseline (same as Figure 7). The blue dashed line are the CVs in the alternative scenario, with respect to 2017 utility (which is the same as in the baseline). The orange solid “Comp” line refers to the CV at each time period $t$ that equates the expected lifetime utility of agents under the alternative scenario with respect to the baseline.

Figure 14 shows that, intuitively, the economy benefits from the initial spending rises and tax cuts under the new policy. The negative CVs mean that current generations favor the introduction of the lending mechanism. However, future generations (or their future selves), suffer from it. As a matter of fact, as soon as the political profligacy reaches unsustainable levels in mid 2020’s, the necessary drastic fiscal adjustments prove very painful, making the welfare impact of aging even direr than in the baseline scenario.

Finally, note that the introduction of the facility has the effect of breaking the expected zero-profit condition for lenders. As committed as they could be to this policy, it is hard to predict for how long they would remain willing to incur ex-post losses on their investment. If they were to withdraw their support, the consequences would likely be severe, at least in the short run, with sovereign defaults becoming probably unavoidable given the high stock of debt and exorbitant issue prices the economy would face.

6.2 Increasing the Retirement Age

This exercise examines the effect of a reform that gradually increases the age threshold (which is exogenous in the model) to receive entitlements. In particular I gradually increase the minimum age requirement from 65 in 2017 to 70 in 2058, in accordance with the specifics of

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31The reason for the very marginally negative CVs for the old under the spread break is the following. Although per capita transfers are higher in the first few years after the introduction of the lending facility vis-à-vis the baseline scenario, reducing the consumption of a retiree (i.e., her transfer) from $t$ onward entails breaking the entitlement promise in each period from $t$ onward. As a result, even small reductions in consumption can lead to significant utility drops, resulting in CVs for the old that are never too negative. I refer the interested reader to Appendix C, where the treatment of the entitlement default cost appears explicitly in the definition of the CVs.
the pension reform adopted by Italy in 2011. More methodological details and the new demographic patterns are presented in Appendix G.7. Figures 15, 16, and A10 (the latter in Appendix G.7) show the results of this exercise.

**Figure 15:** Increasing Retirement Age — Main Fiscal Variables

![Graph showing Old-Age Spending, GDP share, Tax Revenues, GDP share, Debt, GDP share, Spread, basis pts over time](image)

*Notes:* Solid lines are data for the period 2000-2017 (see the Notes to Figure 5 for details on the data series). Dotted black line is median of distributions in each simulation period for baseline scenario. Dotted and thinner dashed lines are 10th and 90th (75th for spreads) percentiles of distributions in each simulation period for alternative scenario. Initial states $S_{2018}$ and productivity shocks in the simulations are identical in the baseline and alternative scenario.

Intuitively, the main effect of the reform is to ease the fiscal pressure that aging generates on public finances. The result is sovereign risk rising less over time, translating into smaller movements in the debt pricing schedule. Therefore, the government can increase its reliance on borrowing, both in level (lower left panel in Figure A10) and as a share of GDP (lower left panel in Figure 15), without this translating into an increase in neither spreads nor the frequency of sovereign defaults.

**Figure 16:** Increasing Retirement Age — Crises And Reform Frequency

![Graph showing Debt-Default, Entitlement-Default, Entitlement-Reform over time](image)

*Notes:* Panels report the frequency of events (debt defaults, entitlement default, and entitlement reforms) observed in the simulations in each period. Defaults are episodes of reneging on current obligations. Reforms are situations in which the entitlement promises for the next period are lower than the current period ones. Black lines are for the baseline scenario, blue ones for the alternative. Initial states $S_{2018}$ and productivity shocks are identical in the baseline and counterfactual exercise.

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Recall that old-age transfers in the model should be interpreted in a broad sense. They encompass not only pensions, but also other forms of government support for the elderly, including public health care. In light of this remark, the parallelism with the 2011 reform is only partial.
As far as the other fiscal variables are concerned, being the pool of entitlement receivers growing more slowly than in the no-reform scenario, per capita transfers can actually be cut less over time (top left panel in Figure A10). Symmetrically, the lower pace at which the labor force shrinks, coupled with the additional space on the borrowing side, allows for a milder reduction of tax rates early on, followed by lower tax hikes from the mid 2020’s onward. All these fiscal changes happen without increasing the likelihood of entitlement reneging episodes and reforms can be implemented less frequently.

Experiencing lower tax hikes and per capita entitlement cuts, the economy fares better in welfare terms. Figure 17 shows that the reform reduces the welfare costs for all future generations.

Figure 17: Increasing Retirement Age — Welfare Effects of Aging

Notes: Unconditional compensating variation (CV) for old, young, and all voters. For the latter (right panel), CVs are based on total welfare of individuals alive at \( t \), weighted by pure population shares. The black dotted line shows the CVs with respect to the 2017 utility under baseline (same as Figure 7). The blue dashed line are the CVs in the alternative scenario, with respect to 2017 utility (which is the same as in the baseline). The orange solid “Comp” line refers to the CV at each time period \( t \) that equates the expected lifetime utility of agents under the alternative scenario with respect to the baseline.

Finally, a remark about the implementation of a similar reform in practice. Increasing the working life negatively impacts the welfare of those people who are close to retirement. If the latter have enough political power, they might succeed in hindering or dampening such a reform. This effect is not captured in my model, where young voters constitute a homogeneous group with an expected working life that is long enough to make the marginal increase introduced by the reform almost immaterial. Studying the political feasibility of reforms that increase the minimum retirement age is a particularly relevant area for further research, given the prominence of this kind of reform in the policy debate.

7 Conclusion

This paper studies the effects of an aging population on the dynamics of fiscal policy and government debt. I propose a sovereign debt model with defaultable debt and old-age spending promises, where overlapping generations of workers and retirees vote on fiscal policy. The
pressure to increase debt associated with an aging population is opposed by the discipline that markets impose via interest rates to an economy that presents a higher propensity to default as it ages.

Simulations of the model calibrated to Italian data display fiscal dynamics largely shaped by market discipline: debt declines as a share of GDP and remains sustainable over the transition. At the same time, welfare-reducing fiscal adjustments are implemented on taxes and, especially, spending for the elderly. In counterfactual exercises in which either market discipline is absent or some voters have myopic expectations about future fiscal policies, sovereign defaults are more frequent and the debt-to-GDP ratio rises over the demographic shift.

Finally, I show that an emergency lending facility that allows the government to borrow at a low interest rate in exchange for fiscal policies that enhance debt sustainability produces welfare-detrimental effects soon after its introduction. On the contrary, a reform that increases the minimum age to access old-age transfers improves the welfare of future generations in the model.

There are a number of possible directions for future research stemming from this work, beyond those already presented in the paper. First, it would be interesting to study the optimal evolution of debt maturity in aging societies, which I abstract from in my model by focusing on one-period debt. Second, immigration, insofar as it eases the demographic pressure stemming from an aging population, might have beneficial effects analogous to those that I obtained for the increase in the minimum retirement age. It would be interesting to study the fiscal and welfare effects of an immigration that happens at rates different from those used in United Nations (2019). Third, the substantial welfare costs that I derive for future old individuals suggest an important scope for private savings for retirement. Introducing them would be an interesting extension of my analysis. Finally, allowing for private savings for retirement in the form (at least partially and/or indirectly) of risky domestic sovereign debt could be a first step in the investigation of the effects of population aging on the aggregate demand for defaultable government debt.
References


Appendix

A Probabilistic Voting

This appendix describes the probabilistic voting mechanism à la Lindbeck and Weibull (1987) adopted in this work. It closely follows Chapter 3.4 in Persson and Tabellini (2000) and Appendix B.1 in Song et al. (2012).

The political game is identical in each period, so time subscripts are dropped. There are different groups of voters, indexed by $J$, with corresponding shares in the population, denoted by $s_J$. There are two candidates (or, equivalently, parties), $A$ and $B$, seeking to win elections. Citizens base their voting decision on policy proposals, as well as their ideology and the average popularity of candidate $J$. In particular, voter $i$ in group $J$ will vote for candidate $A$ if and only if:

\[ U^J(g_A) > U^J(g_B) + \sigma^{ij} + \delta, \]

where $U^J(\cdot)$ is the (indirect) utility that an individual belonging to group $J$ derives from the policy vectors $g_A$ and $g_B$ proposed by the two candidates. These vectors encompass all the fiscal policy choices to be made, which in the current model are $d, \tau, p, b'$, and $c'$. The variable $\sigma^{ij}$ represents $i$’s ideological bias toward candidate $B$. It is random across $i$’s, and drawn from a $J$-group specific uniform distribution:

\[ U\left[ -\frac{1}{2\phi^J}, \frac{1}{2\phi^J} \right]. \]  

(16)

Intuitively, the lower is $\phi^J$, the higher is the likelihood for a voter in the $J$ group to have a strong ideological preference toward candidate $B$ (if $\sigma^{ij} > 0$) or candidate $A$ (if $\sigma^{ij} < 0$), making the fiscal policy proposals $g_A, g_B$ less relevant for winning her vote. On the contrary, very high values of $\phi^J$ imply draws $\sigma^{ij} \approx 0$. In this case, fiscal policy proposals $g_A, g_B$ have a much higher weight in $i$’s final voting decision. Finally, $\delta$ is an average popularity shock for candidate $B$ (capturing, for instance, the relative success of her electoral campaign). It is uniformly distributed according to:

\[ U\left[ -\frac{1}{2\psi}, \frac{1}{2\psi} \right]. \]  

(17)

Ultimately, $\sigma^{ij} + \delta$ is the inherent bias of voter $i$ toward candidate $B$, irrespectively of her policy proposal. The assumed functional forms are for analytical tractability. They can be somewhat generalized (see Persson and Tabellini, 2000).

The timing of the game is the following: (i) the two candidates announce their policy platforms $g_A$ and $g_B$. This is done simultaneously and non-cooperatively, while knowing voters preferences and distributions (16) and (17), but not the actual realizations of the random variables; (ii) All uncertainty in terms of $\delta$ and $\sigma^{ij}$’s is resolved; (iii) Elections take place; (iv) The

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33 The results in this Appendix do not require to be specific about the identity of the $J$ groups.
winning candidate implements her announced platform (no deviations are allowed after elections).

Note that voters are forward-looking in their assessing of the policy platforms $g$. Although political competition takes place every period, they internalize the fact that current choices can become state variables for the next period, and so they vote taking them strategically into account.\textsuperscript{34}

To derive the optimal proposals by the candidates, it is useful to define the swing voter in group $J$. She is the $i$ with $\sigma^J = \sigma^J$, where:

$$\sigma^J \equiv U^J(g_A) - U^J(g_B) - \delta.$$  

The vote share for candidate $A$, $\pi_A$, can then be derived as the sum of voters in each group $J$ with $\sigma^J < \sigma^J$. Formally:

$$\pi_A = \sum_J \pi^J_A = \sum_J s^J \phi^J \left( \sigma^J - \left( -\frac{1}{2} \phi^J \right) \right) = \sum_J (s^J \phi^J \sigma^J + \frac{s^J}{2}).$$

Then, the probability of $A$ winning the election, $p_A$, can be shown to be equal to:

$$p_A = \Pr\left[ \pi_A \geq \frac{1}{2} \right] = \Pr\left[ \sum_J s^J \phi^J \left( U^J(g_A) - U^J(g_B) - \delta \right) \geq 0 \right] = \frac{1}{2} + \psi \sum_J \left( s^J \phi^J \left( U^J(g_A) - U^J(g_B) \right) \right).$$

Symmetry for $B$ and simultaneous announcement imply a unique Nash equilibrium with both candidates announcing the same policy $g^*$, specifically, the one defined as:

$$g^* \equiv \arg \max_g \sum_J s^J \phi^J U^J(g),$$

or, equivalently,

$$g^* \equiv \arg \max_g \sum_J s^J \omega^J U^J(g),$$

where $\omega^J \equiv \frac{\phi^J}{\sum \phi^J}$. This last expression defines the objective function to be ultimately maximized when setting fiscal policy optimally in each period. Note how it reflects the crucial features of the political game. First, to win elections, a candidate needs to take into account

\textsuperscript{34}As highlighted in Song et al. (2012), a crucial assumption to make probabilistic voting working in the current model with Markov perfect equilibrium is to have agents conditioning their voting strategy only on payoff-relevant state variables. Moreover, the popularity shock $\delta$ must be i.i.d. in order not to become an additional state variable.
the relative size of each group of voters, \( s^J \), and how that group values the proposed policy platform \( g, U^J(g) \). However, there is more to consider if there is heterogeneity across groups in the stochastic ideological element of their voting, which is captured by the \( \omega^J \)'s. In particular, the lower is the dispersion of ideological bias of a group \( J \), i.e., the higher \( \phi^J \), the higher is the “electoral responsiveness” to fiscal policy for that group. This means that the candidates want to assign more weight to those groups with the higher \( \phi^J \). The \( \omega^J \)'s in the political objective function (which recall are defined as \( \omega^J \equiv \frac{\phi^J}{\sum J \phi^J} \)), and so \( \frac{\partial \omega^J}{\partial \phi^J} > 0 \)) capture exactly this incentive. This argument justifies the interpretation of \( \omega^J \) in terms of government’s political bias toward group \( J \) provided in the paper.

B Domestic Debt and Private Savings

Section 2.1.4 elaborates on the reasons why it is desirable to have domestic debt in the model. This appendix discusses the precise assumptions made in this respect. It also elaborates on the challenges that private savings would introduce in the model, and it provides a possible micro-foundation and calibration strategy for the reduced-form cost of default \( k_H t \) introduced in Section 2.

Before getting into the details, a premise is worth some discussion. In principle, the substantial fraction of Italian debt held domestically could be captured by a model where \( Y \) and \( O \) are allowed to directly save in sovereign debt (among other assets). However, to bring this model to the data, one would have to confront the fact that sovereign debt holdings and, in fact, financial assets holdings in general appear to be negligible for the bulk of the Italian population (data supporting this claim are presented in Appendix D.5). This would require having heterogeneity among households in asset positions. The result would be an OLG model with heterogeneity (in age and assets) and aggregate shocks, whose limited tractability is widely recognized in the literature.\(^{35}\) The problem would be even more severe in the political context of the current model, where the entire distribution of voters is needed to derive the political objective function. Therefore, this paper proposes a different approach, making assumptions that allow to retain tractability, while remaining consistent with the salient features of the data. They are described next.

Suppose there is a third group in the population, denoted by \( H \). Those are individuals belonging to rich dynasties living off their wealth, who are willing to invest in sovereign debt. As a group of voters, they will enjoy some political weight in the decisions of the government,

\(^{35}\) A list of existing solution methods for heterogeneous-agent OLG models, with and without aggregate uncertainty, but with risk-free debt, is provided in Nishiyama and Smetters (2014). The sovereign default model in D’Erasmo and Mendoza (2018) features domestic investors who are subject to idiosyncratic shocks and are allowed to optimally save in government bonds at the price set by foreigners. Their key for tractability comes from assuming that the government aggregates households’ preferences according to some ad-hoc welfare weights, independent of the distribution of assets (and idiosyncratic shocks) in the economy.
in analogy to workers and retirees. The same derivations carried out in Appendix A will lead
to a political objective function for the government of the form:
\[ s^Y \omega^Y U^Y(.) + s^O \omega^O U^O(.) + s^H \omega^H U^H(.), \]
where \( U^Y \) and \( U^O \) are exactly (9) and (8), while \( U^H \) is the indirect utility function of \( H \).

Before formally describing \( U^H \), the assumptions made with respect to \( H \) and domestic debt
are introduced and discussed. They are:

(i) the \( H \) group is of size \( N_t^H \), so that total population amounts to:
\[ N_t = N_t^Y + N_t^O + N_t^H. \] (18)

The size of group \( H \) grows at rate \( \gamma_t^H \) at the end of each period. This can be decomposed
into a death rate \( \gamma_t^{HD} \) and a birth rate \( \gamma_t^{HB} \), leading to:
\[ N_{t+1}^H = (1 + \gamma_t^H)N_t^H = (1 - \gamma_t^{HD} + \gamma_t^{HB})N_t^H. \] (19)

Perfect foresight by all agents is assumed also for these aggregate demographic variables;

(ii) \( H \) are risk neutral and deep-pocketed;

(iii) \( H \) can access the same financial markets as the foreign investors, hence facing the same
opportunity cost of funds, \( r \);

(iv) no selective default is allowed based on the residency of the holders;

(v) a constant fraction \( \delta \) of total debt \( b' \) issued each period is bought by \( H \);

(vi) \( H \) do not overlap with neither \( Y \) nor \( O \);

(vii) all dynasties within the \( H \) group are assumed to hold the same position in government
debt at each period, effectively reaching perfect risk-sharing. One can think of this as
the result of all \( H \) households investing in a common sovereign-debt mutual fund that
redistributes the proceeds equally across all of its (alive) shareholders.

Assumption (i) allows for the \( H \) group to grow at rate \( \gamma_t^H \) each period, which is useful
for matching the data. The choice of the other assumptions was guided by the ultimate goal
of retaining tractability, by avoiding to introduce new choice and, especially, state variables.
Specifically, assumptions (ii)-(iv) allow for a unique pricing schedule in equilibrium, given by
(11). Assumption (v) resolves this indeterminacy without introducing any new choice variable,
and corresponding state.\textsuperscript{36}

\textsuperscript{36}Bocola et al. (2019) face a similar indeterminacy. They break it by allowing investors to access both risky
government debt and state-and-policy-contingent Arrow securities, showing that the latter can be omitted from
the state vector.
Finally, assumptions (vi) and (vii) allow to retain homogeneity within age-groups, which is the key for model’s tractability. One implication is that a large fraction of the population (the Y and the O) is left without private savings in the model. In fact, this appears to be consistent with Italian data, as described in Appendix D.4.

The value function $U^H$ resulting from assumptions (i)-(vii) can now be presented. It is:

$$U_t^H(d, b', e'; b, A) = (1 - d) \left( \frac{\delta b}{(1 + n_t)s_t^H} - q_t(b', A, e') \frac{\delta b'}{s_t^H} \right) + \beta_t^H \mathbb{E}_{A|A} \left[ U_{t+1}^H(d_{t+1}'(S'), bp_{t+1}'(S'), ep_{t+1}'(S'); b', A') \right],$$

where the term in the top row is the flow utility, $\beta_t^H$ is the discount factor, and starred functions again denote optimal policies. $H$-agents derive utility (linearly, by assumption (ii)) from the additional net wealth accrued from the investment in government debt. This is the difference between what they individually receive as debt repayment, $\frac{\delta b}{(1 + n_t)s_t^H}$, and the funds invested in new debt, $q(b', A, e') \frac{\delta b'}{s_t^H}$, subject to the possibility of default ($d = 1$, in which case the additional net wealth is zero). Under assumptions (i) and (iii), the discount factor can be show to be $\beta_t^H = \frac{(1 + n_t + 1) \delta s_t^H}{(1 + r)(1 - r) \delta s_t^H}$. This expression shows that the discount factor of foreign investors (i.e., $1/(1 + r)$) is adjusted for the possibility for dilution/increase of the investment due to demographics and perfect risk-sharing (assumptions (i) and (vii)).

Another useful result can be derived under the specified assumptions: $H$ agents expect to make zero profits on their sovereign debt investments starting from today and for any future period. This means that $U^H$ in case of debt repayment effectively collapses to:

$$U_t^H(0, b', e'; b, A) = \frac{\delta b}{(1 + n_t)s_t^H},$$

whereas $U_t^H(1, b', e'; b, A) = 0$ in case of default.

Now think about the default decision of the government at time $t$. The value function under repayment will be:

$$\bar{V}_t^R(b, A, e) = \max_{\tau \in (0, 1], p \in \mathbb{E}, b' \in B, e' \in E} \quad s_t^O \omega^O U_O^O(p, b', e'; A, e) + s_t^Y \omega^Y U_t^Y(\tau, b', e'; A, e) + \omega^H \frac{\delta b}{1 + n_t}$$

s.t. $\tau s_t^Y A' = q(b', A, e') b' - \frac{b}{1 + n_t} + s_t^O p,$

\[\text{To derive this result, consider the problem at period } t \text{ of an } H \text{ agent under the assumptions (ii)-(iv). Allow her discount factor to be denoted by } \bar{\beta}. \text{ Assume that the agent is allowed to optimally choose her debt purchases at the price } q \text{ set by foreigners. The Euler equation will be:}\]

$$\frac{\delta}{s_t^H} = \bar{\beta}(1 - \gamma^H) \mathbb{E}_{A|A} \left[ (1 - d'(S')) - \frac{\delta}{(1 + n_t + 1)s_{t+1}^H} \right].$$

Now substitute the expression for the optimal price set by foreigners (i.e., equation (11)). The expression above can then be rearranged to $\bar{\beta} = \frac{(1 + n_t + 1) \delta s_t^H}{(1 + r)(1 - \gamma^H)(1 + n_t)s_t^H}$. This shows that, as long as the discount factor of $H$ satisfies this expression at each $t$, $H$ is willing to trade any amount of debt at the price set by foreigners according to (11).
which is related to the value function $V_t^R$ defined in (7) by the equation:

$$\bar{V}_t^R (b,A,e) = V_t^R (b,A,e) + \omega H \frac{\delta b}{1 + n_t}.$$ 

Analogously, the value function under default, $\bar{V}_t^D (A,e)$, is related to $V_t^D$ defined in (10) by the equation:

$$\bar{V}_t^D (A,e) = V_t^D (b,A,e) + k H (\delta b).$$

Hence, the final default decision of the problem described in this appendix is governed by:

$$\bar{V} (A,e) = \max_{d \in \{0,1\}} (1 - d) \bar{V}_t^R (b,A,e) + d \bar{V}_t^D (A,e)$$

which is identical to (6) when setting $k_t H (\cdot) = \omega H (\cdot + 1 + n_t)$. Namely, under this functional form for $k_t H (\cdot)$, the model presented in this appendix is isomorphic to the one described in Section 2.38

Therefore, the results in this appendix can be summarized in the following proposition:

**Proposition 1 (Equivalence of Problems)**

Under assumptions (i)-(vii) and $k_t H (\cdot) = \omega H (\cdot + 1 + n_t)$, the problems described in Appendix B and Section 2 are equivalent.

### C Derivation of the Welfare Metric

This appendix derives the formulas to calculate the unconditional compensating of variation (CV), largely following Lester et al. (2014). I derive CVs obtained in a general setting, cast in terms of two alternative scenarios for the economy: a baseline (A) and a counterfactual (B). These scenarios can be interpreted as either different versions of the economy at a given time period $t$, or, alternatively—net of some notational adjustment—, two different points in time for the same economy, with A being the reference year. The CV derived in the latter case corresponds to the counterfactual comparison of the expected lifetime utility of a $J$-individual alive in period B with respect to the one of a $J$-individual alive at A.

At the most general level, the CV for an individual belonging to group $J$ in $t$ is the $\lambda_t^J$ solving:

$$E \left[ U_{t,A}^* (S_t) \right] = E \left[ \sum_{k=0}^{\infty} \bar{\beta}^k \bar{u}_t^{l_h+k} \left((1 + \lambda_t^J) c_{t+k,B}\bar{r}_{t+k,B} \bar{s}_{t+k,B} e_{t+k,B} \right) \right].$$ (21)

---

38Note that the equivalence requires all the demographic variables to be the same in the two settings.
In this formula, $\mathbb{E}[.]$ are unconditional expectations operators with respect to the distribution of the state. On the left-hand side of (21), $U^O_t(S)$ is the indirect utility that an individual belonging to group $J$ at time $t$ would derive in scenario $A$ (see equations (8), (9), (20)), evaluated at the optimal choices for the policy variables. On the right-hand side of (21) there is the lifetime utility that the individual would obtain under scenario $B$. $\hat{u}^O_t$ is the proper flow-utility to consider in each period $t$: for an $O$ individual, $\hat{u}^O_t(c, l, e'; e) = u(c, l) - k(c; e) - \beta(1 - \gamma^D_t)k(e'; e)$ (recall $l = 0$ for the old); for a $Y$ individual, $\hat{u}^Y_t(c, l, e'; e) = u(c, l) - \beta\gamma^B_t k(e'; e)$. The discount factor $\hat{\beta}$ appearing on the right-hand-side of (21) is also adjusted so to properly consider the probability of demographic transitions happening in the future. Functional form and demographic assumptions assumed in this paper simplify (21) along several dimensions, as detailed in the reminder of this appendix.

### C.1 Compensating Variation for Old

First, consider an $O$ individual alive at $t$. Start by focusing on $t = T$, i.e., the time-period in which the demographic transition reaches the steady state. Her expected utility is constant at any $t \geq T$, implying that:

$$
\mathbb{E}[U^O_t] = \mathbb{E}\left[\frac{u(c_t^*, l_t^*) - k(c_t^*; e_t) - \beta(1 - \gamma^D_T)k(e^*_t; e_T)}{1 - \beta(1 - \gamma^D_T)}\right],
$$

where the dependence of $U^O_t$ on $S_t$ is dropped to ease the notation. Intuitively, the lifetime utility of an $O$ is given by the sum of the (flow) utility she can get while alive, properly discounted and weighted by the probability of dying in the future.

Therefore, $\lambda^O_T$ can be found by (numerically) solving:

$$
\mathbb{E}[U^O_{T,A}] = \mathbb{E}\left[\frac{u((1 + \lambda^O_T)c^*_{T,B,T}; l^*_T) - k((1 + \lambda^O_T)c^*_{T,B,T}; e_{T,B}) - \beta(1 - \gamma^D_T)k(e^*_T, e_{T,B})}{1 - \beta(1 - \gamma^D_T)}\right].
$$

(22)

Denote by $B^O_T(\lambda^O_T)$ the term on the right-hand side of (22).

Consider now $t = T - 1$. $\lambda^O_{T-1}$ can be found by solving:

$$
\mathbb{E}[U^O_{T-1,A}] = \mathbb{E}\left[\frac{u((1 + \lambda^O_{T-1})c^*_{T-1,B,T}; l^*_{T-1}) - k((1 + \lambda^O_{T-1})c^*_{T-1,B,T}; e_{T-1,B}) - \beta(1 - \gamma^D_{T-1})k(e^*_{T-1,B}, e_{T-1,B})}{1 - \beta(1 - \gamma^D_{T-1})}\right] + 
\beta(1 - \gamma^D_{T-1})B^O_T(\lambda^O_{T-1}).
$$

By defining the generic $B^O_T(\lambda^O) \equiv \mathbb{E}\left[\frac{u((1 + \lambda^O)c^*_{t,B,T}; l^*_T) - k((1 + \lambda^O)c^*_{t,B,T}; e_{t,B}) - \beta(1 - \gamma^D_t)k(e^*_{t,B}, e_{t,B})}{1 - \beta(1 - \gamma^D_t)}\right] + 
\beta(1 - \gamma^D_t)B^O_{t+1}(\lambda^O)$, for $t = T-1, T-2, \ldots$, all the previous $\lambda^O_t, t = T-1, T-2, \ldots$, can then be found recursively, by solving:

$$
\mathbb{E}[U^O_{t,A}] = \mathbb{E}\left[\frac{u((1 + \lambda^O_t)c^*_{t,B,t}; l^*_t) - k((1 + \lambda^O_t)c^*_{t,B,t}; e_{t,B}) - \beta(1 - \gamma^D_t)k(e^*_{t,B}, e_{t,B})}{1 - \beta(1 - \gamma^D_t)}\right] + \beta(1 - \gamma^D_t)B^O_{t+1}(\lambda^O_t).
$$

39When deriving the numerical results in the paper, all the expectations appearing in the various CV formulas derived in this section are approximated using their empirical analogues obtained in the simulations.
C.2 Compensating Variation for Young

A logic analogous to the one in the previous section guides the derivation of the formula for the CV for \( Y \), \( \lambda_T^Y \). Start again by focusing on \( t = T \). The following holds:

\[
\mathbb{E}\left[U_T^{Y*}\right] = \frac{\mathbb{E}\left[u(c^*_T,i^*_T) - \beta \gamma_T^R k(e^*_T;e_T)\right]}{1 - \beta(1 - \gamma_T^R)} + \frac{\beta \gamma_T^R}{1 - \beta(1 - \gamma_T^R)} \mathbb{E}\left[U_T^{O*}\right].
\]

Intuitively, the lifetime utility of a \( Y \) is given by the sum of the (flow) utility she can get while in the labor force plus the (lifetime) utility she can get from the moment of retirement onward, both of them properly discounted and weighted by the probability of the demographic transition happening at any point in the future.

Therefore, \( \lambda_T^Y \) can be found by (numerically) solving:

\[
\mathbb{E}\left[U_{T,A}^{Y*}\right] = \frac{\mathbb{E}\left[u((1 + \lambda_T^Y)c^*_{T,B}i^*_T;e^*_T) - \beta \gamma_T^R k(e^*_T;e_T)\right]}{1 - \beta(1 - \gamma_T^R)} + \frac{\beta \gamma_T^R}{1 - \beta(1 - \gamma_T^R)} \mathbb{E}\left[U_{T,B}^{O*}\right](\lambda_T^Y). \tag{23}
\]

Denote by \( B_T^Y(\lambda_T^Y) \) the term on the right-hand side of (23).\(^{40}\)

Consider now \( t = T - 1 \). \( \lambda_{T-1}^Y \) can be found by solving:

\[
\mathbb{E}\left[U_{T-1,A}^{Y*}\right] = \mathbb{E}\left[u((1 + \lambda_{T-1}^Y)c^*_{T-1,B}i^*_{T-1,B}) - \beta \gamma_T^R k(e^*_{T-1,B};e_{T-1,B})\right] + \beta(1 - \gamma_T^R)B_T^Y(\lambda_{T-1}^Y) + \beta \gamma_T^R B_T^O(\lambda_{T-1}^Y).
\]

By defining the generic \( B_T^Y(\lambda) \equiv \mathbb{E}\left[u((1 + \lambda)c^*_{T,B}i^*_B;e^*_T) - \beta \gamma_T^R k(e^*_T;e_T)\right] + \beta(1 - \gamma_T^R)B_T^Y(\lambda) + \beta \gamma_T^R B_T^O(\lambda) \), for \( t = T - 1, T - 2, \ldots \), all the previous \( \lambda_t^O, t = T - 1, T - 2, \ldots \), can then be found recursively, by solving:

\[
\mathbb{E}\left[U_{t,A}^{Y*}\right] = \mathbb{E}\left[u((1 + \lambda_t^Y)c^*_{t,B}i^*_B;e^*_B) - \beta \gamma_T^R k(e^*_B;e_B)\right] + \beta(1 - \gamma_T^R)B_{t+1}^Y(\lambda_t^Y) + \beta \gamma_T^R B_{t+1}^O(\lambda_t^Y).
\]

C.3 Compensating Variation for Domestic Debt Holders

The CV can be derived also for \( J = H \), while considering only the portion of their consumption coming from domestic debt repayments. Recalling that they have linear utility, their CV is simply given by:

\[
\lambda_t^H = \frac{\mathbb{E}\left[U_{t,A}^{H*}(S_t)\right]}{\mathbb{E}\left[U_{t,B}^{H*}(S_t)\right]} - 1.
\]

\(^{40}\)The following property of the GHH preferences assumed for \( u \) (equation (12)) is useful to calculate the solution to (23). First, note that the optimal equilibrium decisions for consumption and labor under the specified functional form for \( u \) are \( c^* = ((1 - \tau)A)^{1-\gamma} \) and \( l^* = ((1 - \tau)A)^{\gamma} = (c^*)^{1-\gamma} \). Then, the following holds:

\[
u(a^*,l^*) = u(a^*,(c^*)^{1-\gamma}) = \frac{1}{1 - \gamma} \left( a^* - \frac{c^*}{1 + \frac{1}{\phi}} \right)^{1-\gamma} = \frac{1}{1 - \gamma} \left( \frac{(\alpha(1 + \phi) - \phi)}{1 + \phi} c^* \right)^{1-\gamma} = (\alpha(1 + \phi) - \phi)^{1-\gamma} u(c^*,(c^*)^{1-\gamma}) = (\alpha(1 + \phi) - \phi)^{1-\gamma} u(c^*,l^*).\]
C.4 Compensating Variation for All Voters

Finally, I show how to calculate a CV for all voters alive in a given $t$. This is denoted by $\lambda_t$, representing the change in consumption, common to all $J$ groups, that sets total welfare of people alive at $t$ equal in A and B:

$$
E[s_t^{O} U_{t,A}^{O} + s_t^{Y} U_{t,A}^{Y} + s_t^{H} U_{t,A}^{H}] = \\
= s_t^{O} \left[ E\left[u((1 + \lambda_t)c_{t,B}^{*},0) - k((1 + \lambda_t)c_{t,B}^{*}e_{t,B}) - \beta \gamma_D^{t}(k(e_{t,B}^{*}e_{t,B})) + \beta(1 - \gamma_D^{t})B_{t+1}^{O}(\lambda_t)\right] + \\
+ s_t^{Y} \left[ E\left[u((1 + \lambda_t)c_{t,B}^{*},l_{t,B}^{*}) - \beta \gamma_R^{t} k(e_{t,B}^{*}e_{t,B}) + \beta(1 - \gamma_R^{t})B_{t+1}^{Y}(\lambda_t) + \beta \gamma_R^{t} B_{t+1}^{O}(\lambda_t)\right] + \\
+ s_t^{H} (1 + \lambda_t)E[U_{t,B}^{H*}],
\right]
$$

(24)

where the total welfare is calculated as the sum of the lifetime utility of all groups, weighted by their size.\footnote{This total welfare measure excludes yet-to-be-born individuals, who might want to be considered by an utilitarian social planner. Yet, for the purposes that CV serves in this paper, the measure adopted in the paper seems preferred.}

D Data Sources and Methods

This appendix describes the data sources, the mapping from model quantities to data adopted in this work, and the details and results of the empirical methods used to calibrate some of the parameters.

D.1 Data Sources

Population. Data from the latest release of the World Population Prospects, United Nations (2019). This dataset includes historical data from 1950, and projections up to 2100 for each country in the world with a nontrivial current population. The medium fertility scenario has been used in this work. Projections by (5-year) age groups are provided at 5-year intervals. Annual values are derived applying a cubic spline interpolation.

Interest Rates and Spreads. Data from Bloomberg, gross yields for generic one-year government bonds (GBOTG12M Index for Italy, and GDBR1 Index for Germany). Averages across monthly values for the period 2000m1-2017m12. Spreads are then calculated as the difference between the Italian and German series at the annual frequency.\footnote{The spread was negative by a few basis points in early 2000’s for bonds with 1-year maturity (but not with any other longer once). Negative values are inconsistent with the model, so they are set to zero.} A similar strategy is followed to obtain the data used to calibrate $q$ in the spread-break exercise in Section 6.1, where the ESM/EFSF series considered are ESM GB 1Y Corp and EFSF GB 1Y Corp, over the period 2012m10-2018m10.


Government Spending by Function. Data from Eurostat, COFOG dataset. Annual values for the period 2000-2017. Data for sub-categories start from 2001. For the variables needed for earlier periods, the following imputations have been performed: pension spending (old-age-survivors) recovered assuming the same growth rate of total social protection (of which they constitute more than 80%); spending for public debt transactions recovered assuming the same growth rate of net interest payments (the correlation is 95% among the two variables).

Sovereign Debt by Holding Sector. Data for general government debt by holding sector from the Bank of Italy’s statistical database (code TCCE0200). Averages across monthly values for the period 2000m1-2017m12.

Domestic Debt Exposure. Data from the Bank of Italy’s Survey on Household Income and Wealth dataset. This is a survey representative of the Italian population that is carried out every two years. Waves covering the span 1987 to 2016 have been considered in this work.

D.2 Fiscal Variables and Government Budget Constraint: Model Versus Data

This appendix describes how variables in the model are mapped to quantities in the data. In order to do so, start by considering a general formulation of the government budget constraint:

\[ \text{rev}_t - \text{exp}_t = \text{bal}_t, \]

where \( \text{exp}_t \) are total expenditures at time \( t \), \( \text{rev}_t \) are total revenues, and \( \text{bal}_t \) is the government balance (also called net lending). \( \text{bal}_t > 0(<0) \) corresponds to a government surplus (deficit) in period \( t \).

The government balance is linked to government debt according to:

\[ -\text{bal}_t = \text{drev}_t - \text{drep}_t, \]

where the term on the right-hand side corresponds to net debt revenues at \( t \), i.e., the difference between proceeds from debt issuance, \( \text{drev}_t \), and debt repayments, \( \text{drep}_t \).

I can then combine (25) and (26) to write the government budget constraint as:

\[ \text{rev}_t - \text{exp}_t = -\text{drev}_t + \text{drep}_t. \]

I now rewrite (27) so to make it more suitable for a direct mapping to the variables in my model. In so doing, I also provide more details on the exact variables from the National Accounts (ESA 2010) that I consider.
First, I split expenditures into old-age entitlement spending \((\text{ent}_t)\), interest payments on the debt \((\text{intpay}_t)\), and the rest \((\text{nnent}_t)\):

\[
\text{exp}_t = \text{ent}_t + \text{intpay}_t + \text{nnent}_t. \tag{28}
\]

I include in \(\text{ent}_t\): (i) old-age pensions, encompassing the classes “old age” and “survivors” in the Classification of the functions of government (COFOG) data. They constitute about 82% of \(\text{ent}_t\), on average between 2000 and 2017; (ii) the fraction of the COFOG spending class “health” going to the old, estimated at 49.5%.\(^{43}\)

Second, I divide total revenues into proceeds from taxes on income and wealth plus social contributions \((\text{incwsoc}_t)\), interest received \((\text{intrec}_t)\), and other tax revenues \((\text{othr}_t)\):

\[
\text{rev}_t = \text{incwsoc}_t + \text{intrec}_t + \text{othr}_t. \tag{29}
\]

Third, I combine (27), (28), and (29) to obtain:

\[
\text{incwsoc}_t + \text{intrec}_t + \text{othr}_t - (\text{ent}_t + \text{intpay}_t + \text{nnent}_t) = -\text{drev}_t + \text{drep}_t. \tag{30}
\]

The assumption of one-period debt in the model requires some extra care in matching debt-related variables in the data and in the model. In particular, all the outstanding stock of debt in the model matures each period. This is not the case in the data, where a fraction of debt has longer maturity. For this reason, I map debt in the model to a maturity-adjusted version of debt in the data, i.e., total government debt divided by the average maturity of outstanding debt between 2000-2017 (6.9 years).

This adjustment strategy allows also to better capture the observation that only new debt issuances—which in the model correspond to the entire debt stock, but in the data are only a fraction of the stock—are subject to current interest rates. Matching debt in the model to the actual level in the data would therefore overstate the dependence of the model’s interest payments to the current state of the economy with respect to what is observed in the data. Yet, this implies that the portion of interest payments on long-term debt is not captured by variables in my model, and will be included in the residual term in the budget constraint I will shortly define. But before doing so, let me split \(\text{intpay}_t\) into \(\text{intpay}_t^{ST}\) and \(\text{intpay}_t^{LT}\), the portions of interest payments arising from short and long-term debt, respectively. In analogy to the maturity-adjustment for the stock of debt, I match \(\text{intpay}_t^{ST}\) to 1/6.9 of the entire interest payments of year \(t\), and \(\text{intpay}_t^{LT}\) to the complementary (1-1/6.9).

Then, I can rearrange (30), using the split of \(\text{intpay}_t\), to provide the final mapping of model’s variables to the data:

\[
\frac{(\text{othr}_t - \text{nnent}_t + \text{intrec}_t - \text{intpay}_t^{LT}) + \text{incwsoc}_t}{\Delta_t} - \frac{\text{ent}_t}{\tau_t Y_t} = \frac{\text{drep}_t}{B_t} - \frac{(\text{drev}_t - \text{intpay}_t^{ST})}{q_t B_{t+1}}. \tag{31}
\]

\(^{43}\) I calculate this share from official statistics on spending for acute care by age groups in 2017, as reported by the Italian Ministry of Economics and Finance (MEF, 2017) in Box 3.1, Fig. A.1.
The numbers above the quantities in equation (31) are from the 2017 National Accounts for Italy (all reported as a fraction of GDP) and are shown to provide a sense of the magnitude of each term. Braces below the equation instead define the matching between data and model variables. In particular, upper case $B_t$, $B_{t+1}$, $Y_t$ represent the level of, respectively, debt due at $t$, debt issued at $t$ (due in $t+1$), and aggregate income (in the paper they appear in lower case, which is the notation adopted to denote per capita levels). The term $\Delta_t$ is included in the government budget constraint when solving and simulating the model numerically, to ensure comparability of magnitudes between data and model results. Finally, primary balance in the data (0.016 in 2017) is matched to the left-hand-side terms in (31) plus $\text{intpay}_t^{LT}$.

### D.3 Calibration of Demographics

The demographic process in the 3-agent version of the model (see Appendix B) is governed by the following system of equations:

\[
\begin{align*}
N_{t+1}^Y &= (1 - \gamma_t^R)N_t^Y + \gamma_t^B N_t^Y, \\
N_{t+1}^O &= (1 - \gamma_t^D)N_t^O + \gamma_t^R N_t^Y, \\
N_{t+1}^H &= (1 + \gamma_t^H)N_t^H, \\
N_t &= N_t^Y + N_t^O + N_t^H,
\end{align*}
\]  

(32)

where $N_t^J$ is the mass of individual in group $J, J \in \{Y, O, H\}$, alive at $t$. The following strategy has been followed to calibrate the demographic variables:

(i) $N_t$ is matched to population aged 20 and above in data for all $t$;\(^{45}\)

(ii) $N_t^Y$ is matched to $(1 - e^Y)\mathcal{Y}_t$ for all $t$, where $\mathcal{Y}_t$ is the number of people aged 20-64 in the data in year $t$ and $e^Y$ is the share of people aged 20-64 with a significant exposure to domestic sovereign debt (calculated as described in Appendix D.5). Analogously, $N_t^O$ is matched to $(1 - e^O)\mathcal{O}_t$ for all $t$, where $\mathcal{O}_t$ is the number of people aged 65 and above in the data in year $t$ and $e^O$ is the share of people aged 65 an above with a significant exposure to domestic sovereign debt.

(iii) $N_t^H = N_t - N_t^O - N_t^Y$ for all $t$;

(iv) $\gamma_t^R = 0.022$ for all $t$, so to target and average working life of 45 years;

(v) the other transition probabilities, $\gamma_t^B, \gamma_t^D, \gamma_t^H$, are then derived as the solution to the first three equations in the system (32) for each $t$;

---

\(^{44}\)For simulations purposes, I keep the value of $\Delta_t/Y_t$ constant at -0.107, its mean over the period 2000-2017. In fact, this quantity has been relatively stable over this period, with 10th-90th quantile range $[-0.115, -0.102]$.

\(^{45}\)Considering people that are at least 20 years old is consistent with the minimum voting age in Italy, which is set at 18 years for the Lower Chamber and at 25 for the Senate.
(vi) Population shares are then $s^j_t = N^j_t / N_t$, $j = O, Y, H$ and $n_t$ is the growth rate of $N_t$.

The population is assumed to reach a steady state in $t = T$. Namely, for each $t = T+1, T+2, ...$: $N^j_t = N^j_{t-1}$, $n_t = 0$, and transition probabilities are again set as described in (iii),(iv) above. The steady state is set at $T = 2070$ in the data. Section 3.2.1 in the paper elaborates on this choice.

The following figures shows the results that this procedures yields for demographic variables over the period considered in the paper.

**Figure A1:** Estimated Population Dynamics

![Graphs showing population dynamics](image)

**Notes:** Author’s calculations based on data from the World Population Prospects, United Nations (2019).

### D.4 Private Savings for Retirement in Italy

An important assumption in the environment assumed in this paper is the absence of private savings for workers and retirees. In particular, there is no private accumulation of assets to finance consumption in retirement. This prevents the agents to possibly insure against the only idiosyncratic shock they face in the model, namely the transition to retirement, above and beyond what the government offers them on aggregate through the social security/old-age transfer scheme. Appendix B already explained in details the tractability considerations behind this modeling choice. The role of this appendix is to provide empirical evidence showing that this assumption is in fact in line with Italian data.

First of all, note that the pension system is largely public in Italy. The OECD Global Pension Statistics show that the assets of all private retirement vehicles as a share of GDP where around...
10% for Italy, well below the OECD average of 56%. Moreover, the latest report by the COVIP, the Italian regulator of private pensions funds, reports that the share of Italian workers with accrued capital above 100,000 Euro was less than 3% in 2018.

Other pieces of evidence can be found by looking at the Bank of Italy’s Survey on Household Income and Wealth (SHIW). The first observation is that also these data confirm the limited role of private pensions. Over the last two decades, less than 2% of Italian retirees’ annual pension income has come from private savings, and only around 1% of retirees reported to have higher pension income from private rather than public sources.

More broadly, financial asset holdings are very limited for the majority of Italian households. Over the last 30 years, only 16% (6%) of them reported to have net financial assets higher than one time (two times) their annual income.

Household wealth is substantially higher if one adopts a definition that encompasses also housing properties. The share of families with assets above one time (two times) their annual income would be 71% (67%) in that case. Yet, the extent to which homeownership represents an actual form of saving for retirement is a point up for discussion.

Many recent comparative studies find very low residential mobility both over the life-cycle and at older ages in particular, especially in Southern European countries (see, e.g., Angelini et al., 2014). In fact, SHIW data show that only around 17% of families with at least one pension recipient has moved to the current house (either owned or rented) when the retiree was 60 or older. This is a necessary condition to have retirement consumption (partially) financed by either moving from owning to renting or downsizing. As such, 17% represents an upper bound for the number of families entering a new house close or during the retirement period of a family member. It overestimates the actual number of interest, given that it also encompasses families who might: (i) have sold the previous house for a price lower than the one of the new house; (ii) have been renting before purchasing the currently owned house; (iii) have been renting also before entering the currently rented house.

Although this analysis does not do full justice to this topic—which is definitely worth more systematic research—it provides prima facie evidence of a limited historical role for investment in housing as a form of saving for retirement in Italy.

D.5 Exposure to Domestic Sovereign Debt in the Data

The ultimate role played in the model by the \(H\) group of voters is to impose an additional cost to the government in case of default, as stemming from debt held domestically. This observation guided the design of the empirical methodology to estimate the size of \(H\) in the data. That

\[\text{\textsuperscript{46}}\text{The necessity of this condition assumes that retirement happens after 60 and the selling-to-cash-in strategy is performed closed to the beginning of the retirement period, in accordance with consumption smooth considerations. In fact, the validity of the latter in the data has been somewhat questioned (see, e.g., Chiuri and Jappelli, 2010).}\]
relies on data from the Italian Survey on Household Income and Wealth, and consists of three steps.

First, a household $h$ at time $t$ is classified as significantly at risk in case of a domestic debt default according to the following criterion:

$$sov_{h,t} + \max\{0; dep_{h,t} - g\} + other_{h,t} > y_{h,t}. \quad (33)$$

The first term, $sov_{h,t}$, is the amount of government securities directly held by $h$ at $t$. However, a household is likely to incur financial losses in case of a default also indirectly, through the drop in value of other assets whose issuers are highly exposed to government debt. In the Italian case, those are mainly the domestic financial institutions.\(^{47}\) Therefore, two other terms are considered when calculating the exposure. First, the amount of bank deposits, $dep_{h,t}$, above the level of deposit insurance, $g = €100,000$.\(^{48}\) Second, investment in other assets issued by financial institutions. Unfortunately, those are not directly available in the dataset. Therefore, $other_{h,t}$ is set to the amount of assets other than deposits, sovereign bonds, and credits toward family and friends held by $h$ at $t$, with the understanding that this provides an upper bound for the quantity of interest. Finally, the value of annual income, $y_{h,t}$, is used to assess the economic significance of the (potential) default losses for a household.\(^{49}\) A few robustness checks along the latter two dimensions are reported in Table A1.

The second step involves moving from surveyed households to the entire population of voters. This is achieved by considering the reported number of members of the household and the household’s population weight provided in the survey. Individuals are then split according to their reported age into young (20-64) and old (65 and above).

The third and final step consists in taking the average share of exposed individuals in the two age classes. This gives the shares $e^Y$ and $e^O$ eventually used in the calibration of the demographic process described in Appendix D.3. The following table reports the results of this whole exercise, together with some robustness checks.

\(^{47}\)As of December 2018, 48% of the Italian stock of public debt was held by domestic financial institutions. The remaining was split between foreigners (29%), central bank (17%) and domestic households and firms (6%).

\(^{48}\)Whether the deposit insurance scheme will effectively remain solvent in case of a default on Italian debt that severely impacts the entire financial system is an interesting question from which this work abstracts.

\(^{49}\)This criterion implicitly abstracts from the possibility of default haircuts lower than 100%. Those should be viewed as proportional reductions of the left-hand side of (33), effectively making the criterion for inclusion in the $H$ group more stringent. Column $2\bar{y}$ in Table A1 can be thought as presenting the results for a case of a 50% haircut and right-hand side still at $y_{h,t}$.
Table A1: Share of Domestic Debt Holders

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>From 2000</th>
<th>0.5 y</th>
<th>2 y</th>
<th>0.5 other</th>
<th>sov</th>
<th>sov2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>7.2 (1.3)</td>
<td>6.8 (0.9)</td>
<td>13.5</td>
<td>2.9</td>
<td>5.1 (1.6)</td>
<td>3.0</td>
<td>6.6</td>
</tr>
<tr>
<td>Young - eY</td>
<td>6.7 (1.5)</td>
<td>6.0 (1.0)</td>
<td>13.0</td>
<td>2.5</td>
<td>4.6 (1.8)</td>
<td>2.6</td>
<td>6.1</td>
</tr>
<tr>
<td>Old - eO</td>
<td>8.8 (1.5)</td>
<td>9.2 (1.1)</td>
<td>14.9</td>
<td>4.0</td>
<td>6.7 (1.4)</td>
<td>4.2</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Notes: Numbers are in percentage points. They are average shares across the survey waves 1987-2016, unless differently specified. Standard deviations in parentheses. Total reports the share of H-individuals in the population aged 20 and above, rows Young-eY and Old-eO the ones within a specific age group. The values eY and eO in column Baseline are the shares used in the baseline calibration (see Appendix D.3). Column From 2000 shows statistics calculated only for waves 2000-2016. Columns 0.5 y and 2 y change the income criterion in the right-hand-side of (33) to, respectively, 0.5 and 2 times yh,t. Column 0.5 other considers only half of the amount of other assets as coming from issuers significantly exposed to domestic sovereign debt. Columns sov and sov2 consider only direct exposure to sovereign debt (i.e., sovh,t in the left-hand-side of (33)), with the income criterion set to, respectively, 1 and 0.5 y.

D.6 Shares of Directed Spending and Calibration of the Political Bias

The methodology adopted to calibrate the political bias toward voter groups, \( \omega^J, J = Y, O, H, \) is the following. First, total public expenditure by function (COFOG) at time \( t, G_t \), is split the following categories:

\[
G_t = G_t^O + G_t^Y + G_t^H + G_t^R.
\]

Here, \( G_t^O \) encompasses spending directed to \( O \), namely old-age entitlements \( ent_t \), as defined in Appendix D.2. The term \( G_t^Y \) captures the spending for working-age population \( Y \), i.e., social protection other than pensions and the fraction of health spending for people aged 20-64 (42% in the data described in footnote 43). \( G_t^H \) corresponds to spending in public domestic debt transactions (largely, interest payments). The remaining COFOG classes are lumped into \( G_t^R \), encompassing spending going to children, to purely public goods, and to foreigners. This forth class should be seen as the one upon which voter groups in the model do not exert differential political influence.

The fractions of directed spending going to each group are then calculated as \( g_t^J = \frac{G_t^J}{\sum_{J=Y,O,H} G_t^J} \) and their mean, \( \bar{g}_t^J \), are computed for the period 2000-2017. These are used as input in the following equations:

\[
\begin{align*}
\bar{g}_t^Y \omega^Y + \bar{g}_t^O \omega^O + \bar{g}_t^H \omega^H &= \bar{g}_t, \\
\bar{g}_t^Y \omega^Y + \bar{g}_t^O &= \bar{g}_t^Y, \\
\bar{g}_t^O \omega^O + \bar{g}_t^H &= \bar{g}_t^O, \\
\bar{g}_t^H \omega^H &= \bar{g}_t^H, \\
\omega^Y + \omega^O + \omega^H &= 1,
\end{align*}
\]
where $s^J$ are average population shares for 2000-2017. The values of $\omega^J$ solving this system are finally used as parameters for the quantitative model. The following table summarizes the inputs and outputs of this procedure.

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>O</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^J$</td>
<td>0.699</td>
<td>0.230</td>
<td>0.071</td>
</tr>
<tr>
<td>$g^J$</td>
<td>0.236</td>
<td>0.656</td>
<td>0.108</td>
</tr>
<tr>
<td>$\omega^J$</td>
<td>0.072</td>
<td>0.605</td>
<td>0.323</td>
</tr>
</tbody>
</table>

**Notes:** Variables denoted as $(\cdot)$ are means over the period 2000-2017. The average population shares, $s^J$, and the average share of directed spending, $g^J$, are inputs for the calibration of the political bias, $\omega^J$.

### E Fiscal Situation in Italy and Japan

As mentioned in the main text, Italy is an ideal target for the analysis, given the magnitude of its aging phenomenon and its fragile fiscal situation. Another natural candidate would have been Japan, given that it leads the G-7 economies (see Figure 1), and in fact the world, in terms of the aging phenomenon.

However, there are a number of reasons suggesting that the current fiscal situation might be more fragile in Italy than in Japan, making the former the preferred target for the analysis. First, Japan has direct control over its monetary policy while Italy—being a member of the Euro Area—has not. This in principle gives Japan the possibility to “inflate away” its debt, while that is hardly an option for Italy under European Central Bank policies. My model is not suitable to accommodate inflation as a policy tool for fiscal stabilization.

Second, consider the sovereign debt position of the two countries.\(^{50}\) It is true that Japan has the highest gross debt-to-GDP ratio in the world, 237%, even higher than the 130% of Italy. Yet, the difference in net debt between the two countries is much smaller, with Japan at 153% while Italy at 119%. The large discrepancy between gross and net debt in Japan stems from central bank’s holdings, which are over 40% of the entire stock (17% in Italy). Moreover, only around 10% of Japanese debt is held by foreigners (30% in Italy).

Third, in terms of other fiscal variables, the size of the government sector is smaller in Japan. Total expenditure is 37% of GDP (49% in Italy), with social security payments at 13% of GDP (20% in Italy). Government revenues are 34% (46% in Italy).

Finally, the difference in the 10-year government bond yield (−0.19% in Japan, 0.93% in Italy, according to Bloomberg data, October 2019) might be interpreted as yet another signal of the more fragile fiscal situation in Italy compared to Japan.

\(^{50}\)All the fiscal data presented in this appendix are from the World Economic Outlook and the OECD database. They refer to 2018.
F Solution and Simulation Algorithm

This appendix describes in detail the algorithms followed to solve and simulate the model.

F.1 Solution Algorithm

The algorithm used to solve the model builds upon standard global approximation methods used in the sovereign default literature.\(^{51}\) These are its steps:

1. Discretize into grids the domain of each state variable in \( S = [b, A, e] \) and of the endogenous variables \( b' \) and \( e' \). The latter, together with \( A \), constitute the arguments of the pricing schedule, \( S^q = [b', A, e'] \).

2. Solve the problem at period \( t = T \), i.e., the period at which population reaches the steady-state. In particular:
   
   (a) Specify an initial guess for the government value functions, \( V_{R,T,0}^R(S), V_{D,T,0}^D(S) \), voters’ optimal indirect utility functions, \( U^*_T(S), J = Y, O, H \), and pricing schedule \( q_{T,0}(S^q) \). Good candidates are the ones that would emerge in the last period of a finite-horizon version of the economy. Derive also the optimal policy functions consistent with the guesses.
   
   (b) Update \( q_{T,i} \) and the continuation values in \( U^*_{T,i} \), according to the initial guess (at iteration \( i = 1 \)) or the previous iteration (at iterations \( i = 2, 3, \ldots \)).
   
   (c) Solve the maximization problems defining \( V_{R,T,i}^R \) and \( V_{D,T,i}^D \) and recover the updated policy functions.
   
   (d) Check the convergence criterion for value and pricing functions. If it is met, then continue to Step 3. Otherwise repeat Steps 2.b-2.d.

3. Solve the model backwards for all \( t = T - 1, T - 2, \ldots, 0 \). In particular, at each \( t \):
   
   (a) Update the population parameters to their values at \( t \).
   
   (b) Perform the Steps 2.b and 2.c, using the optimal policy functions derived in the previous iteration, \( t + 1 \), to compute \( q_t \) and the continuation values in \( U^*_{I,t}(.) \).

The end result is a collection of value, optimal indirect utility, policy, and pricing functions, jointly denoted by \( \Gamma \equiv \{\Gamma_t\}_{t=0}^{T} \), which can then be used to perform simulations as described in Section F.2.

The specifics of the algorithm are as follows. The grid for the state-space \( (b, A, e) \) has \( [30 \times 15 \times 30] \) nodes, while the number of points in the \( b' \) and \( e' \) grids are 150 and 234, respectively. No lending is allowed, consistently with the data. The \( A \)-grid covers \( +/-4 \) standard

\(^{51}\)See Hatchondo et al. (2010) for a comparison of alternatives. This paper uses a particular version of interpolation, described in this Appendix.
deviations of the stochastic process. The expectations entailed in the pricing schedule and utility continuations are approximated using Gauss-Hermite quadrature with 50 nodes. Linear interpolation of $V^R, V^D$ is used when updating $q_t$, while the $U^{J_\alpha}$ for continuations are interpolated linearly in $b, A$ and with cubic spline in $e$. The latter helps in smoothing the kink induced by the entitlement constraint. The maximization problems in $V^R_t, V^D_t$ are solved with a grid-search method that ensures the respect of the government budget constraint. The convergence criterion in Step 2.d is defined as having absolute maximum discrepancies over the entire state space between two consecutive iterations to be less than $10^{-4}$ for value functions and the pricing schedule.

### F.2 Simulation Algorithm

The general strategy for obtaining one simulation from the model is the following: (i) choose an initial realization of state vector, $S_0$, at $t = 0$; (ii) obtain a sequence of realizations of the random component of the exogenous productivity process, $\{\varepsilon_t\}_{t=0,\ldots,T}$; (iii) feed $S_0$ and $\{\varepsilon_t\}_{t=0,\ldots,T}$ into the model’s solution $\Gamma$ to derive a full dynamic path for all the variables. This will effectively cover the span:

$$t = 0, \ldots, h_1, \ldots, h_H, f_1, \ldots, f_F, \ldots, T,$$

where $h(.)$ are periods corresponding to historical data, and $f(.)$ the ones of future simulations. This procedure can be run $N$ times for $N$ different paths of $\{\varepsilon_t\}$, to derive a distribution for each variable at each time $t$.\(^{52}\) Moments of these distributions can then be used to assess the model’s fit to historical data and evaluate the likelihood of paths for future dynamics.

The above strategy constitutes the basis for the simulation algorithm. Before detailing the actual steps followed, two crucial choices are worth some more discussion. First, the selection of $S_0$. In principle, an ideal simulation strategy would have the model solved for a large number of periods before $h_1$, so large that the influence of $S_0$ in the simulations would have vanished by the time each run reaches the historical data period (and, a fortiori, the projection one). However, the necessary demographic data are not available before 1950, and even setting $t = 0$ to 1950 would be too computationally costly.\(^{53}\) The solution adopted is to move from a single $S_0$ to a set of $N$ draws from the (stationary) distribution of the state at $t_{h_1}$, and then obtain $N$ simulation runs starting from each of them. Details on how this is operationalized are provided later in this section.

The second point worth of discussion is the way the solution $\Gamma$ is used to derive optimal choices at each simulation period. A common approach in the literature would be to perform policy function interpolation. However, this could give very inaccurate results in the current

\(^{52}\)The state vector in this model is a time-inhomogeneous first order Markov Chain. This prevents the use of the long-simulation method usually adopted in the sovereign default literature, which requires time-homogeneous Markov Chains.

\(^{53}\)This strategy was performed only once, as a robustness check (see Figure A5 in Appendix G.4).
setting, where important non-linearities in the model (notably, the debt default and entitlement reneging decisions) give rise to policy functions that are non-smooth over some regions of the state space. A more refined method is therefore adopted, which combines interpolation with constrained maximization. In particular, these are the steps followed:

S.1 Construct a grid of points for \( b' \times e' \).

S.2 Evaluate the optimal utility continuation values and pricing function on each grid node, together with the realized productivity. Do this by interpolating the relevant objects in \( \Gamma \).

S.3 Solve the government problem for the remaining variables, consistently with the current state and the \((b', e')\)-grid nodes; this gives the optimal policy choices.

The actual simulation algorithm can now be presented in details. It is composed by the following steps:

1. Obtain \( N \) draws from the distribution of the state at \( t = h1 \). This is performed with the following procedure:

   (a) Provide an initial value for the state vector, \( s_{h1,0} \).

   (b) Fix the current-period population parameters at their \( t = h1 \) values.

   (c) Draw a long simulation of length \( M \geq N \). Specifically:

      i. Under the current state \( s_{h1,i} \), follow the steps S.1-S.3 detailed above, with continuations and pricing schedule taken from \( \Gamma_{h1} \).

      ii. Obtain “next period” state \( s_{h1,i+1} \) from the optimal choices at \( i \) together with a random draw for the productivity shock \( \varepsilon_{i+1} \).

      iii. Iterate \( M \) times on 1.c.1-1.c.2.

   (d) Discard a large enough number of initial simulations, so to avoid any influence of the initial \( s_{h1,0} \). Furthermore, discard the observations that involve debt or entitlements defaults and recoveries, given that none has been observed for Italy in the period covered by the available historical data.

   (e) Randomly select \( \{s_{h1,i}\}_{i=1}^{N} \) of the remaining realizations.

---

54 This grid is chosen so to include the points \( b' \) and \( e' \) that would result from pure interpolation of the policy functions. This ensures that the final policy choices with the refined simulation method are always weakly preferred to the ones that can be derived with standard policy function interpolation.

55 \( M \) should be set at a value large enough so that the selection procedure leaves at least \( N \) realizations of the state vector.

56 Technically, these are not draws from the stationary distribution at \( t = h1 \), given the time-inhomogeneity of the state Markov chain. To partially control for this, a few “burn-in” periods are allowed by setting \( h1 \) to a few years before the first one used for calibration. In all the main simulation results in the paper, \( h1 \) was set to 1994. Moving it to 1990 would not alter the baseline results (see Figure A4). Figure A5 in Appendix G.4 shows how the main results are also robust to having longer simulation runs, all starting from a single state in 1950.
2. Use the \( \{s_{t_1,i}\}_{i=1,...,N} \) realizations as initial states for \( N \) simulations spanning the period \( t = h1,...,fF \). Specifically, for each simulation run \( i \):

   (a) Given a state \( s_{t,i} \) for period \( t \) in simulation run \( i \), follow the steps S.1-S.3 detailed above, with continuations and pricing schedule taken from \( \Gamma_t \).

   (b) Obtain next period state \( s_{t+1,i} \) from the optimal choices at \( \{t,i\} \), together with a random draw for the productivity shock \( \epsilon_{t+1,i} \).

   (c) Iterate on 2.a.i-2.a.ii for the required number of time periods.

   (d) For the calibration period, discard any run with defaults on either debt or entitlements defaults, given that none have been observed in the data for the relevant period. As for future simulations, only discard the periods following a debt default.\(^{57}\)

Finally, the specifics for the implementation of this algorithm are the following. The number of simulations is \( N = 25,000 \), unless differently specified. The evaluations of the continuation and pricing functions in step S.2 are derived via linear interpolation. The maximization in step S.3 is performed with a grid-search method analogous to the one used for solving the model. All the grids for the choice variables in the simulations are \( \{i,t\} \)-specific, optimized so to cover the relevant range for each simulation-period.

G Additional Results

This section presents additional results discussed in various sections of the main text, to which the reader is referred to for more details.

G.1 Dependence of Calibration Targets on Parameters

The following Table A3 shows comparative static exercises for the calibration targets and average yearly frequency of default and reforms episodes as parameters in \( \Theta \) are varied, one at a time. The first line reports the same results for the baseline calibration.

\(^{57}\)The exclusion of the recovery after a debt default is standard in the sovereign debt literature, to avoid having results affected by the peculiar assumptions made for those periods. Excluding the entire run in which a sovereign default occurs instead of discarding only the post-default period does not alter the results.
Table A3: Parameter Dependence

<table>
<thead>
<tr>
<th>Targets</th>
<th>Data</th>
<th>Baseline</th>
<th>Varying β - Baseline 0.7</th>
<th>Varying d_δ - Baseline 0.095</th>
<th>Varying d_δ - Baseline 2.3</th>
<th>Varying α - Baseline 0.75</th>
<th>Varying α - Baseline 2.8</th>
<th>Varying ϕ - Baseline 2.6</th>
<th>Varying ϕ - Baseline 2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ent'17</td>
<td>0.195</td>
<td>0.199</td>
<td>0.65</td>
<td>0.075</td>
<td>0.085</td>
<td>0.105</td>
<td>0.115</td>
<td>0.187</td>
<td>0.199</td>
</tr>
<tr>
<td>Tax'17</td>
<td>0.278</td>
<td>0.272</td>
<td>0.202</td>
<td>0.196</td>
<td>0.202</td>
<td>0.197</td>
<td>0.194</td>
<td>0.187</td>
<td>0.199</td>
</tr>
<tr>
<td>Debt'17</td>
<td>0.190</td>
<td>0.188</td>
<td>0.198</td>
<td>0.174</td>
<td>0.164</td>
<td>0.201</td>
<td>0.205</td>
<td>0.282</td>
<td>0.277</td>
</tr>
<tr>
<td>Spr'17</td>
<td>0.0038</td>
<td>0.0021</td>
<td>0.0035</td>
<td>0.0006</td>
<td>0.0020</td>
<td>0.0013</td>
<td>0.0021</td>
<td>0.009</td>
<td>0.004</td>
</tr>
<tr>
<td>En Trd</td>
<td>0.011</td>
<td>0.004</td>
<td>0.003</td>
<td>0.023</td>
<td>0.018</td>
<td>0.012</td>
<td>0.011</td>
<td>0.003</td>
<td>0.007</td>
</tr>
<tr>
<td>Tax Trd</td>
<td>0.017</td>
<td>0.007</td>
<td>0.003</td>
<td>0.024</td>
<td>0.014</td>
<td>0.015</td>
<td>0.013</td>
<td>0.008</td>
<td>0.010</td>
</tr>
<tr>
<td>Debt Trd</td>
<td>-0.004</td>
<td>-0.005</td>
<td>-0.009</td>
<td>0.014</td>
<td>0.009</td>
<td>0.001</td>
<td>0.004</td>
<td>-0.002</td>
<td>-0.003</td>
</tr>
<tr>
<td>Spr Trd</td>
<td>0.006</td>
<td>0.001</td>
<td>-0.003</td>
<td>0.002</td>
<td>0.002</td>
<td>0.000</td>
<td>0.004</td>
<td>-0.001</td>
<td>-0.003</td>
</tr>
<tr>
<td>Freq b-D</td>
<td></td>
<td>0.019</td>
<td>0.022</td>
<td>0.015</td>
<td>0.018</td>
<td>0.017</td>
<td>0.019</td>
<td>0.020</td>
<td>0.028</td>
</tr>
<tr>
<td>Freq e-D</td>
<td></td>
<td>0.164</td>
<td>0.200</td>
<td>0.158</td>
<td>0.129</td>
<td>0.151</td>
<td>0.173</td>
<td>0.285</td>
<td>0.201</td>
</tr>
<tr>
<td>Freq e-R</td>
<td></td>
<td>0.153</td>
<td>0.158</td>
<td>0.050</td>
<td>0.144</td>
<td>0.150</td>
<td>0.156</td>
<td>0.146</td>
<td>0.209</td>
</tr>
<tr>
<td>Debt Trd</td>
<td>0.054</td>
<td>0.016</td>
<td>0.051</td>
<td>0.051</td>
<td>0.051</td>
<td>0.050</td>
<td>0.044</td>
<td>0.048</td>
<td>0.059</td>
</tr>
<tr>
<td>Spr Trd</td>
<td>0.051</td>
<td>0.017</td>
<td>-0.053</td>
<td>-0.053</td>
<td>-0.053</td>
<td>-0.054</td>
<td>-0.053</td>
<td>-0.055</td>
<td>-0.032</td>
</tr>
</tbody>
</table>
| Notes: Results for 15,000 runs of simulations over the period 2000-2050. Trd variables are changes as predicted by a linear time trend over the period 2000-2008 for “Targets”, and 2018-2050 for “Simulations 2018-2050”. Ent is old-age spending over GDP, Tax is income tax revenues over GDP, Debt is maturity-adjusted debt over GDP, and Spr is the spread between Italian and German government bonds with one year residual maturity. Freq is the average yearly frequency of episodes: debt-default (b-D), entitlements-default (e-D), and entitlements reform (e-R).

G.2 Optimal Policies

Optimal policies for the baseline model in $t = 2017$ and $t = 2050$ are presented in Figure A2. First, some remarks on their shape are offered. Then, the discussion will move to the analysis of how aging affects them.

In terms of the behavior of policy functions across the state-space, panel (a) shows that old-age transfers are equal to entitlements, $e$, as long as the latter are kept. When that is not the case, they decline in debt and increase in productivity. Panels (b) and (c) display tax rates and debt issuances that are generally increasing in current promises, both in terms of debt repayments (left figures) and entitlements (right figures), at least whenever the latter are honored (left figures). These results are consistent with the need of more resources to finance

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58Zero borrowing under DB in Panel (c) is consistent with the assumption of market exclusion under debt de-
higher promises. Higher productivity allows for lower tax rates under debt repayment and are associated with somewhat higher borrowing, whenever possible (middle figures). Finally, panel (d) shows that future entitlements generally decrease in debt increase in productivity, while the monotonicity is not uniform over the level of current entitlements.

A note on some of the flat portions of the optimal choices. Focusing on left figures in each panel, policies are constant in $b$ when debt promises are broken, given that the 100% haircut assumed makes $b$ effectively irrelevant under debt-default. With respect to middle figures, a tax policy constant in $A$ under debt-default (panel (b), middle) is the result of the functional forms for the flow utility (equation (12)). Then, the budget constraint in (10) implies old-age transfers slightly decreasing in productivity (panel (a), middle) due to the fact that the calibration of the parameters in the default cost (equation (14)) effectively makes $h(A)$ slightly decreasing in $A$.\(^\text{59}\) Finally, looking at right figures in each panel, policies are constant in $e$ once entitlements are reneged and reformed (i.e., for high enough $e$) because of the functional form assumed for the cost (15), which effectively makes the optimal policy choices independent on defaulted current entitlements.

In terms of the effects of aging on policies, the economy in 2050 presents strictly lower per capita old-age transfers (panel (a)) and future entitlements (panel (d)), for the reasons already discussed with respect to Figure 4. These results are confirmed numerically over the entire state space. As already anticipated, panel (b) shows that the demographic shifts brings along higher tax rates, as an attempt to compensate for the shrinking tax base.\(^\text{60}\) The effect of aging on debt issuance is instead non-monotonic over the state-space (panel (c)).

Finally, Figure A2 provides some evidence also on the evolution of the optimal default and reneging choices. Numerical results confirm that both the debt-default and entitlement-reneging regions in 2017 are a subset of the 2050. These findings are consistent with the higher risk of promise-breaking in the older economy, ceteris paribus, due to the fiscal pressure generated by aging.

\(^{59}\)The flat portion of $b''$ under RK is a numerical result holding in the particular combination of states represented in Figure A2. It is not a global feature over the entire state-space.

\(^{60}\)This is true over the entire-state space, with the exception of very few points where a higher tax rate allows the younger economy to honor promises that are instead broken in the older one.
**Figure A2: Optimal Policies**

Panel (a) - Transfers Per-Capita

Panel (b) - Tax Rates

Panel (c) - Debt Issuance

Panel (d) - Future Entitlements

Notes: Panel (a) reports optimal per capita transfers, \( p^*_t(S) \); panel (b) tax rates, \( \tau^*_t(S) \); panel (c) debt issuance, \( bp^*_t(S) \); panel (d) promises of future entitlements, \( ep^*_t(S) \). The different line styles correspond to the state-space regions: debt repaid and entitlements kept (RK), debt repaid and entitlements broken (RB), debt defaulted and entitlements kept (DK), and debt defaulted and entitlements broken (DB). Policy functions are not rescaled. Debt is divided by mean output. Productivity is reported in terms of standard deviations above/below its mean. Entitlements are per capita amounts multiplied by the size of the \( O \) population and divided by mean output. Fixed variables in each panel are set at their median value in the simulations.
G.3 Additional Results for the Baseline Scenario

Figure A3 shows the projected dynamics for some additional model variables. They are old-age transfers ($p^*$), current-period entitlements ($e$), debt repayments per capita ($b$), and output per capita ($y^*$).

Figure A3: Baseline — Additional Variables

Notes: Thicker dotted lines are medians in each simulation period. Thinner lines are 5th and 95th percentiles (75th for spreads).

G.4 Robustness of the Baseline Results

The following figures report some robustness checks for the baseline results. The next two are the outcomes under modifications in the simulation method. Figure A4 shows results obtained with the same algorithm used throughout the paper and described in Appendix F.2, with the only difference of a longer “burn-in” period that starts in 1990. The dynamics for the main variables of interest are very close, supporting the validity of the (faster) strategy used in the paper.
**Figure A4:** Simulations From 1990 — Main Fiscal Variables

![Graphs showing Old-Age Spending, Tax Revenues, Debt, and Spread](image)

*Notes:* Simulations corresponding to runs over the period 1990-2050. Solid lines are data for the period 2000-2017 (see the Notes to Figure 5 for details on the data series). In the four upper panels, thicker dotted and dashed lines are medians in each simulation period. Thinner lines are 5th and 95th percentiles (75th for spreads).

Figure A5 reports the results from a more computationally demanding exercise. It consists in obtaining “long” simulations runs covering the period 1950-2050, all initialized at a unique initial state in 1950. Results are similar to the baseline ones derived with the main simulation technique adopted in the paper and described in Appendix F.2, again reassuring about the validity of the latter.
**Figure A5:** “Long” Simulations — Main Fiscal Variables

![Graphs of Old-Age Spending, GDP share, Tax Revenues, GDP share, Debt, GDP share, and Spread, basis pts.](image)

**Notes:** Long simulations correspond to 75,000 runs over the period 1950-2050, starting from a unique initial state $S_{1950}$. Solid lines are data for the period 2000-2017 (see the Notes to Figure 5 for details on the data series). In the four upper panels, thicker dotted and dashed lines are medians in each simulation period. Thinner lines are 5th and 95th percentiles (75th for spreads).

Finally, Figure A6 shows that assuming that the steady state for demographics is reached in 2100, rather than the baseline 2070, is also inconsequential for the results.
G.5 Additional Results with Irrational Expectations over Fiscal Policy

The lifetime utility functions (8) and (9) are modified as follows under the irrational expectations scenario:

\[
U_t^O(p, b', e'; A, e) = u(p, 0) - k_t^O(p, e'; e) + \\
\quad + \beta \theta^O \left(1 - \gamma_t^D\right) \bar{D}_t^O(e', e') + \\
\quad + \beta(1 - \theta^O) E_{A|A} \left[ \left(1 - \gamma_t^D\right) U_{t+1}^O \left(p_{t+1}^*, b_{t+1}^*, e_{t+1}^*; A', e'\right) \right],
\]

(34)

\[
U_t^Y(\tau, b', e'; A, e) = u((1 - \tau)A^*(\tau, A), I^*(\tau, A)) - k_t^Y(e'; e) + \\
\quad + \beta(1 - \theta^Y) E_{A|A} \left[ \left(1 - \gamma_t^R\right) \bar{D}_t^Y(\tau, e'; A', e') + \gamma_t^R \bar{D}_{t+1}^O(e', e') \right] + \\
\quad + \beta \theta^Y E_{A|A} \left[ \left(1 - \gamma_t^R\right) U_{t+1}^Y \left(\tau_{t+1}^*, b_{t+1}^*, e_{t+1}^*; A', e'\right) + \gamma_t^R U_{t+1}^O \left(p_{t+1}^*, b_{t+1}^*, e_{t+1}^*; A', e'\right) \right],
\]

(35)

where \( \theta^O \) and \( \theta^Y \) are the shares of young and old voters, respectively. The “irrational lifetime utility” for \( O \) and \( Y \) take the form:

\[
\bar{U}_t^O(p, e'; e) = u(p, 0) - k_t^O(p, e'; e) + \beta \left(1 - \gamma_t^D\right) \bar{D}_t^O(e', e'),
\]

\[
\bar{Y}(\tau, e'; A, e) = u((1 - \tau)A^*(\tau, A), I^*(\tau, A)) - k_t^Y(e'; e) + \beta \left(1 - \gamma_t^R\right) E_{A|A} \left[ \bar{Y}(\tau, e'; A', e') \right].
\]
Figure A7 shows the projected dynamics for some additional model variables under the baseline and the no-market-discipline counterfactual scenario described in Section 5.2. They are old-age transfers \( p^* \), current-period entitlements \( e \), debt repayments per capita \( b \), and output per capita \( y^* \).

**Figure A7: Irrational Fiscal Expectations — Additional Variables**

![Graph showing projected dynamics for additional variables](image)

*Notes:* Black dotted lines are medians in each simulation period under the baseline scenario. Blue dashed lines correspond to medians (thicker) and 10th-90th percentiles (thinner) under the alternative scenario.

G.6 Additional Results Under Spread-Break With Conditionality

Figure A8 shows the projected dynamics for some additional variables under the spread-break-with-conditionality scenario described in Section 6.1. They are old-age transfers \( p^* \), current-period entitlements \( e \), debt repayments per capita \( b \), and output per capita \( y^* \).

![Graph showing projected dynamics for additional variables](image)
G.7 Additional Results Under Minimum Age Increase

The specifics of the reform exercise described in Section 6.2 are the following. With reference to Appendix D.3, the age threshold to split the data into $N_{t}^{O}$ and $N_{t}^{Y}$ is linearly increased from 65 in 2017 to 70 in 2058, and kept constant afterwards. This is broadly in line with the pension reform adopted by Italy in 2011. The probability of retiring, $\gamma_{t}^{R}$, is adjusted accordingly, so to reflect an increase in the average working life from 45 to 50 years (the assumption of people starting to work at 20 is kept). Results for these adjusted variables are reported in Figure A9.

Figure A9: Estimated Population Dynamics Under Reform

Notes: Author’s calculations based on data from the World Population Prospects, United Nations (2019).

Figure A10 shows the projected dynamics for old-age transfers ($p^{*}$), current-period entitle-
ments ($e$), debt repayments per capita ($b$), and output per capita ($y^*$).

**Figure A10:** Increasing Retirement Age — Additional Variables

![Graphs showing Old-Age Spending, Entitlement Promises, Debt, and Output over time from 2020 to 2050.](image)

*Notes:* Black dotted lines are medians in each simulation period under the baseline scenario. Blue dashed lines correspond to medians (thicker) and 10th-90th percentiles (thinner) under the alternative scenario.

### G.8 Political Weights Re-Alignment

What if young voters were given more say in fiscal policy decisions? Would this attenuate some of the negative effects of population aging presented in the baseline environment? In particular, with direct reference to the model, the exercise performed in this section consists in having individual lifetime utilities weighted only by the size of each group of voters, without the political bias toward the elderly underlying baseline results. This situation would move fiscal decisions closer to those of an utilitarian social planner—net of the consideration of yet-to-be-born generations—, shedding more light on the role of politics in the model.

Rather than seeing the results in this section as purely counterfactual, they will be interpreted as the consequence of some information campaigns on the future prospects of public finances. Those are assumed to be specifically targeted toward young voters, re-aligning their responsiveness to fiscal policy announcements to the one of old (and the domestic debt holders). Operationally speaking, this is achieved by setting the political bias parameters, $\omega^j$, equal across groups.

Figures A11 and A12 show that, as one might expect, the shift of political power from retirees to workers leads to a sharp and immediate cut in tax rates and drastic entitlement reforms, as confirmed also in the right panel of Figure A13.
Figure A11: Re-Aligned Political Power — Main Fiscal Variables

Notes: Initial states $S_{2018}$ and productivity shocks are identical in the baseline and counterfactual exercise. Solid lines are data for the period 2000-2017 (see the Notes to Figure 5 for details on the data series). Thicker dotted and dashed lines are medians in each simulation period. Thinner lines are 10th and 90th percentiles (75th for spreads).

Figure A12: Re-Aligned Political Power — Additional Variables

Notes: Black dotted lines are medians in each simulation period under the baseline scenario. Blue dashed lines correspond to medians (thicker) and 10th-90th percentiles (thinner) under the alternative scenario.
Moreover, there is a re-balancing of the government financing sources from taxes to debt. This is favored by a lower sovereign risk after the reform, which translates into weakly higher debt prices at issuance, ceteris paribus. The latter is the result of the government having more resources to pay back its debt, given the lower levels of promises on the entitlements side at any given state. The general equilibrium effect is that the debt level is higher in any given year after the reform (lower-left panel in Figure A12). It actually grows as a share of GDP over the last decade of the simulations.

The difference in the level of fiscal promises affects the likelihood of default episodes, reported in Figure A13. The higher reliance on borrowing (lower-left panel in Figure A12) leads to slightly more frequent sovereign defaults, while the lower level of spending promises (top-right panel in Figure A12) translates into a lower chance of entitlement reneging.

Figure A13: Re-Aligned Political Power — Crises And Reform Frequency

![Figure A13](image)

Notes: Panels report the frequency of events (debt defaults, entitlement default, and entitlement reforms) observed in the simulations in each period. Defaults are episodes of reneging on current obligations. Reforms are situations in which the entitlement promises for the next period are lower than the current period ones. Black lines are for the baseline scenario, blue ones for the alternative. Initial states $S_{2018}$ and productivity shocks are identical in the baseline and counterfactual exercise.

In terms of welfare, Figure A14 shows that the drastic initial cut and subsequent lower level of the entitlements are highly costly for the old. The opposite is true for the young. They greatly benefit from the tax cuts they could get with the re-alignment of the political power. Results in the right panel suggest that the benefits for young more than compensate the costs for the old, with the economy as a whole benefiting from the reform at any future period (the red line is always negative). Therefore, the political feasibility of this new, arguably desirable order hinges upon the possibility to overcome the resistance that retirees would be likely showing.62

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62The model features homogeneous age-groups. One can expect that, in fact, also workers close enough to retirement would be harmed by the reform, hence joining the old in opposing it.
Figure A14: Re-Aligned Political Power — Welfare Effects of Aging

Notes: Initial states $S_{2018}$ and productivity shocks are identical in the baseline and counterfactual exercise. CV stands for unconditional compensating variation. For the economy as a whole (right panel), total welfare of individuals alive at $t$, weighted by pure population shares, is considered. The solid “Comp” line refers to the CV at each time period $t$ that equates the counterfactual scenario to the baseline one in terms of agent’s expected lifetime utility.

Finally, note that the time-paths of the variables over time are broadly aligned in the two settings, at least after the short-run adjustments following the reform. This translates into dynamics of welfare costs that are similar in the two scenarios: the dashed blue lines in Figure A14 (CVs under the reform) look almost as parallel shifts of the corresponding dashed black ones (CVs under the baseline), except for a somewhat lower growth in the cost for retirees in the new environment, due to the milder cuts in per capita spending (top-left panel in Figure A12). These results suggest that politics might affect the levels, but not the dynamics of the aging-induced welfare losses derived in the baseline.