Risk Sharing Externalities

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Question

• Two steps of financial amplification:
  ▶ Balance sheet:

  \[ \text{net worth (20) = assets (100) - liabilities (80)} \]

  assets drop 10%, liabilities not state contingent, net worth drops 50%

  ▶ Financial accelerator: capacity to accumulate assets depends on net worth

• Why balance sheet exposed to aggregate risk? Why not better risk management?

• Is degree of risk management socially efficient?
Why risky balance sheets?

- Many papers make primitive assumptions that make balance sheets “risky”
  - Non-state-contingent debt in Kiyotaki-Moore, Brunnermeier-Sannikov, He-Krishnamurthy, …

- Papers that allow for more risk sharing: Krishnamurthy (2003), Rampini and Vishwanathan (2010), Di Tella (2017)
  - State-contingent debt tends to dampen amplification

- Assuming primitive limits to risk management in models may be a problem when we turn to macropru policy
This paper

- Explore simple explanation: general equilibrium effects
- When balance sheet of some systemic agents is compromised → incomes go down for everyone
- This makes it costly to insure against these shocks
- Start from simple model:
  - neoclassical structure with balance sheet effects
  - with state contingent claims
  - macro spillover: when net worth of banks goes down, incomes go down
  - see how it affects model predictions (amplification)
  - ...and welfare implications (externalities)
Plan

• Model

• Positive part:
  ▶ Analytical results
  ▶ Numerical results

• Normative part:
  ▶ Analytical results
  ▶ Numerical results
Model

- Infinite horizon, discrete time, \( t = 0, 1, \ldots \)

- Two groups of agents
  - Consumers: Epstein-Zin preferences, supply labor to entrepreneurs
  - Entrepreneurs: log utility, accumulate capital, hire labor to produce

- Technology:
  \[ y_t = (u_t k_t)^\alpha l_t^{1-\alpha} \]

- Shock \( u_t \) also affect depreciation \( (1 - \delta)u_t k_t \)

- Markov process \( s_t \) determines \( u_t \)
Financial Markets

• At $s^t$ agents trade state-contingent claims at prices $q(s_{t+1}|s^t)$
  ▶ $a(s^{t+1})$ held by consumers
  ▶ $b(s^{t+1})$ issued by entrepreneurs
  ▶ Market clearing
  \[
  a(s^{t+1}) = b(s^{t+1})
  \]

• Limited enforcement friction:
  ▶ Entrepreneurs can default on the payments $b(s^t)$
  ▶ If they default, they loose a fraction $\theta$ of capital
  ▶ Default entails no exclusion from financial markets
Model: Consumers

- Epstein-Zin/GHH preferences, discount factor $\beta$

$$V_t = \left\{ (1 - \beta) \left( c_t - \chi \frac{l_t^{1+\psi}}{1 + \psi} \right)^{1-\rho} + \beta \left( E_t \left[ V_{t+1}^{1-\sigma} \right] \right)^{\frac{1-\rho}{1-\sigma}} \right\}^{\frac{1}{1-\rho}}$$

- At $s^t = (s_t, s_{t-1}, \ldots)$ trade one period state-contingent claims

- Budget constraint

$$c(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t) a(s_{t+1}^{t+1}) = w(s^t) l(s^t) + a(s^t)$$
Model: Entrepreneurs

- Log preferences, discount factor $\beta_e < \beta$
- Unique access to technology to accumulate capital
- Budget constraint

$$c_e(s^t) + k(s^t) = 
\begin{align*}
\underbrace{y(s^t) - w(s^t)l(s^t) + (1 - \delta)u_t k(s^{t-1}) - b(s^t)}_{\text{n}_t: \text{net worth}} + \\
\sum_{s_{t+1}} q(s_{t+1}|s^t) b(s^{t+1})
\end{align*}$$

- Collateral constraint (state by state)

$$b(s^{t+1}) + \gamma w(s^{t+1}) l(s^{t+1}) \leq \theta (1 - \delta) u_{t+1} k(s^t)$$
Timeline

- Hire labor, borrow to pay fraction $\gamma$ of wages in advance
- Production takes place
- Choose whether to repay $\gamma w + b$ or default
- If default lose $\theta$ of undepreciated capital stock
- Issue $b(s^{t+1})$
Optimal risk management

- Optimality for $b(s^{t+1})$:

$$q(s_{t+1}|s^t) \frac{1}{c_e(s^t)} = \beta_e \pi(s_{t+1}|s_t) \left( \mu(s^{t+1}) + \frac{1}{c_e(s^{t+1})} \right)$$

- $\mu(s^{t+1}) \geq 0$ is Lagrange multiplier on collateral constraint

- Log utility implies usual

$$c_e(s^t) = (1 - \beta_e)n(s^t)$$
Plan

• Model

• Positive part:
  ▶ Analytical results
  ▶ Numerical results

• Normative part:
  ▶ Analytical results
  ▶ Numerical results
Simple special case

- Simplifying assumptions:
  - $\gamma = 0$ (no working capital)
  - $\rho = 0$
  - $\beta_e = \beta$
  - inelastic labor supply $= 1$

- One time temporary shock $u_1$ at $t = 1$, no other shock

- Solve model in two steps, proceeding backward in time
  - Solve for “continuation equilibrium” from $t = 1$ onward
  - Solve for risk sharing at $t = 0$
Continuation equilibrium

- Interest rate is $1/\beta$
- After shock only one state variable
  \[ n_1 = \alpha(u_1k_0) + (1 - \delta)u_1k_0 - b_1(u_1) \]
- If $n_1 \geq n^*$ capital jumps to first best level $k^*$
- If $n_1 < n^*$ capital transitions gradually to $k^*$
- Present value of future labor income
  \[ W(n_1) = \sum_{t=1}^{\infty} \beta^t (1 - \alpha)k_t^\alpha \]
• Focus on equilibrium with non-binding constraints at $t = 0$

• Ex ante risk sharing condition

$$(n_1(u_1))^{-1} = \text{const.} \cdot (Y_1(u_1) - n_1(u_1) + W(n_1(u_1)))^{-\sigma}$$

where $Y_1$ output + non depreciated capital

• It implies

$$n'_1(u_1) = \frac{\omega}{\omega + (1 - \omega) \frac{1}{\sigma} - \omega W'(n_1)} Y'_1(u_1)$$

where $\omega$ is share of $n_1$ over total wealth
Risk sharing (continued)

- If $\sigma = 0$ then $n'_1(u_1) = 0$

- If $W' = 0$ then

$$n'_1(u_1) = \frac{\omega}{\omega + (1 - \omega)\frac{1}{\sigma}} Y'_1(u_1)$$

each bears risk in proportion to their risk tolerance

- If $W' > 0$

$$n'_1(u_1) = \frac{\omega}{\omega + (1 - \omega)\frac{1}{\sigma} - \omega W'(n_1)} Y'_1(u_1)$$

entrepreneurs tend to bear more risk, changes in $n_1$ produce endogenous background risk
Plan

- Model

- Positive part:
  - Analytical results
  - **Numerical results**

- Normative part:
  - Analytical results
  - Numerical results
Back to full model

- How much financial amplification in calibrated economy?

- Compare three versions of model
  - First-best economy (FB)
  - Benchmark economy with state-contingent debt (CM)
  - Economy with non-contingent debt (IM). Additional constraint
    \[
    b(s^t, s_{t+1}) = \bar{b}(s^t) \quad \forall s_{t+1}
    \]

- Main results
  - CM can produce as much amplification as IM
  - ... if enough consumer risk-aversion and GE spillovers
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Concept</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital income share</td>
<td>0.330</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation</td>
<td>0.025</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor, consumers</td>
<td>0.990</td>
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<tr>
<td>$\psi$</td>
<td>Frisch elasticity</td>
<td>1.000</td>
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<tr>
<td>$\chi$</td>
<td>Disutility of labor</td>
<td>1.980</td>
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<tr>
<td>$\rho$</td>
<td>Inverse IES, consumers</td>
<td>1.000</td>
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<tr>
<td>$\gamma$</td>
<td>Fraction of wages paid in advance</td>
<td>0.500</td>
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<tr>
<td>$\beta_e$</td>
<td>Discount factor, entrepreneurs</td>
<td>0.984</td>
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<tr>
<td>$\theta$</td>
<td>Fraction of pledgeable assets</td>
<td>0.818</td>
</tr>
<tr>
<td>$u_L$</td>
<td>Capital quality in low state</td>
<td>0.925</td>
</tr>
<tr>
<td>$\Pr(u' = u_L</td>
<td>u = u_L)$</td>
<td>Transition probability</td>
</tr>
<tr>
<td>$\Pr(u' = u_H</td>
<td>u = u_H)$</td>
<td>Transition probability</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Consumers’ risk aversion</td>
<td>[1, 10]</td>
</tr>
</tbody>
</table>

- $\{\beta_e, \theta\}$ to obtain leverage = 4 and “spread” = 50bp in ss
- Two states for capital quality, $u = \{u_H, u_L\}$. Process as in Gertler and Karadi (2011)
IRFs to negative shock: first best

![Graphs showing IRFs to negative shock for Labor, Investment, and Output.](image)
IRFs to negative shock: distance from first best

![Graph showing IRFs to negative shock for labor, investment, and output.]
IRFs to negative shock: distance from first best

![Graphs showing IRFs for Labor, Investment, and Output](image-url)
IRFs to negative shock: distance from first best

![Graph showing IRFs to negative shock for Labor, Investment, and Output](image-url)
IRFs to negative shock: distance from first best

Labor

Investment

Output

CM ($\sigma=10$)
Consumers’ risk aversion and entrepreneurs’ balance sheet

<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 1$</th>
<th></th>
<th></th>
<th>$\sigma = 10$</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>FB</td>
<td>IM</td>
<td>CM</td>
<td>FB</td>
<td>IM</td>
<td>CM</td>
</tr>
<tr>
<td>Panel A: Quantities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\Delta(\log n_t)$</td>
<td>-25.21</td>
<td>-3.12</td>
<td></td>
<td>-25.29</td>
<td>-16.71</td>
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<tr>
<td>$\Delta(\log \tilde{n}_t)$</td>
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<td>16.26</td>
<td></td>
<td>-97.73</td>
<td>-73.31</td>
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<tr>
<td>$\Delta(\log y_t)$</td>
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<td>-5.73</td>
<td>-3.62</td>
<td>-3.77</td>
<td>-5.30</td>
<td>-9.94</td>
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<tr>
<td>Panel B: Entrepreneurs’ balance sheet</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_{t-1}$</td>
<td>7.89</td>
<td>6.32</td>
<td></td>
<td>7.80</td>
<td>6.69</td>
<td></td>
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<tr>
<td>$\tilde{n}_{t-1}$</td>
<td>2.29</td>
<td>0.95</td>
<td></td>
<td>2.28</td>
<td>1.57</td>
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</tr>
<tr>
<td>$k_{t-1}/n_{t-1}$</td>
<td>3.09</td>
<td>3.95</td>
<td></td>
<td>3.08</td>
<td>3.55</td>
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<tr>
<td>$b_{L,t-1}/b_{H,t-1}$</td>
<td>1.00</td>
<td>0.91</td>
<td></td>
<td>1.00</td>
<td>0.97</td>
<td></td>
</tr>
</tbody>
</table>

- $\tilde{n} = \theta(1 - \delta)k - b$ is borrowing capacity for working capital
- As $\sigma$ increases, entrepreneurs issue more $L$-contingent debt
- Makes their balance sheet more sensitive to aggregate risk
Why entrepreneurs choose riskier balance sheet?

- As $\sigma$ increases, consumers discount more heavily low $u$ states
- Premium for insuring these states increases
The role of macro spillovers

- In theory, two ingredients generate financial amplification
  1. Risk averse consumers
  2. GE spillovers of net worth on labor income

- Ingredient 1 necessary (≈ no amplification with $\sigma$ small)

- To isolate ingredient 2, consider model with no GE spillovers
  - Consumers earn labor income of first best
  - Entrepreneurs hire hand-to-mouth agents

- Compare results to FB and CM ($\sigma = 5$)
The role of macro spillovers

<table>
<thead>
<tr>
<th></th>
<th>First best</th>
<th>Complete markets</th>
<th>No spillover</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Quantities</strong></td>
<td></td>
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</tr>
<tr>
<td>$\Delta (\log n_t)$</td>
<td>-6.75</td>
<td>-3.24</td>
<td>-3.24</td>
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<tr>
<td>$\Delta (\log y_t)$</td>
<td>-3.77</td>
<td>-6.78</td>
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<tr>
<td><strong>Panel B: Prices and entrepreneurs’ balance sheet</strong></td>
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<tr>
<td>$\Delta (\log LL_t)$</td>
<td>-3.77</td>
<td>-12.75</td>
<td>-3.77</td>
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<tr>
<td>$q_{L,t-1}/\pi_{L,t-1}$</td>
<td>1.20</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>$q_{H,t-1}/\pi_{H,t-1}$</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$k_{t-1}/n_{t-1}$</td>
<td>3.94</td>
<td>3.95</td>
<td></td>
</tr>
<tr>
<td>$b_{L,t-1}/b_{H,t-1}$</td>
<td>0.93</td>
<td>0.91</td>
<td></td>
</tr>
</tbody>
</table>

- Labor income doesn’t fall as much in No spillover economy
- Consumers don’t bid up insurance premium, even if risk averse
Plan

• Model

• Positive part:
  ▶ Analytical results
  ▶ Numerical results

• Normative part:
  ▶ Analytical results
  ▶ Numerical results
Welfare Analysis

- So far: entrepreneurs may end up bearing large fraction of aggregate risk

- Is risk exposure too high? What is the optimal policy?

- Three results
  1. Competitive equilibrium is constrained inefficient
  2. Optimal policy affects relative price of debt instruments
  3. Restricting leverage uniformly not optimal because of risk shifting across debt instruments
Planner’s problem

- Consider a planner that intervenes at $t$, after entrepreneurs’ borrow for working capital

- Planner instruments: taxes on debt $\tau_b(u_{t+1})$, capital $\tau_k$, transfers at $t$ $T_e$ and $T_c$

- Intervention lasts only one period

- Sufficient instruments to control $(b_{t+1}, k_t, c_t, c_{e,t})$

- Two steps
  - Identify source of inefficiency in simplified model
  - Optimal policy in calibrated economy
Simple special case

- Back to simple case of positive part
- Focus on policy intervention that only changes $b^s_1$
- Planner problem, maximize

$$E[V_e(n_s, N_s)] + \zeta \left( E \left[ V(Y_s - n_s + W(N_s))^{1-\sigma} \right] \right)^{\frac{1}{1-\sigma}}$$

- Effects of policy:
  - direct effects (internalized by agents)
  - GE effects (not internalized)

- Direct effects only gives

$$(n_s)^{-1} - \text{const.} \cdot (Y_s - n_s + W(n_s))^{-\sigma}$$
Pareto frontiers

Consumers’ utility

Entrepreneurs’ utility
Pareto improvement

- Two states $H$ and $L$, local perturbation
- Effect on consumers proportional to

$$\sum_s \pi_s^s \left( V_1^s \right)^{-\sigma} \left( 1 - \sum_{t=1}^{\infty} \beta^t \frac{dw_t^s}{dn_1^s} \right) db_1^s$$

- Effect on entrepreneurs

$$\sum_s \pi_s^s \left( -\left( c_{e,1}^s \right)^{-1} + \sum_{t=1}^{\infty} \beta^t \left( c_{e,t}^s \right)^{-1} \frac{dw_t^s}{dn_1^s} \right) db_1^s$$

- Two states $s$, in $H$ unconstrained, in $L$ constrained
- Pareto improvement: reduce $b_1^L$ and increase $b_1^H$

$$\sum_{t=2}^{\infty} \beta^t \left( 1 - \frac{\left( c_{e,t}^L \right)^{-1}}{\left( c_{e,1}^L \right)^{-1}} \right) \frac{dw_t^L}{dn_1^L} > 0$$
## Optimal policy in calibrated economy

<table>
<thead>
<tr>
<th>Panel A: Quantities</th>
<th>FB</th>
<th>CE</th>
<th>PL</th>
<th>PL-c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta (\log n_t)$</td>
<td>-16.71</td>
<td>-16.22</td>
<td>-16.52</td>
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<tr>
<td>$\Delta (\log \tilde{n}_t)$</td>
<td>-73.31</td>
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<table>
<thead>
<tr>
<th>Panel B: Taxes</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$1 - \tau_b(u_L)$</td>
<td>1.00</td>
<td>0.80</td>
<td>1.01</td>
</tr>
<tr>
<td>$1 - \tau_b(u_H)$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>$1 + \tau_k$</td>
<td>1.00</td>
<td>0.98</td>
<td>0.99</td>
</tr>
</tbody>
</table>

| Panel C: Entrepreneurs’ balance sheet | |
|--------------------------------------|---|---|---|
| $n_t$ | 5.66 | 5.68 | 5.98 |
| $\tilde{n}_t$ | 0.75 | 0.90 | 0.75 |
| $k_t/n_t$ | 3.55 | 3.56 | 3.56 |
| $b_{L,t-1}/b_{H,t-1}$ | 0.97 | 0.96 | 0.97 |

- Optimal policy makes $L$-debt more expensive. Entrepreneurs’ balance sheet safer in equilibrium
- If planner can only tax debt uniformly, no tax on debt
Why taxing debt uniformly not optimal?

- When planner imposes uniform tax on debt, entrepreneurs substitute toward riskier instruments
- Not effective in reducing risk exposure
Conclusion

- Studied benchmark model with financial frictions and state-contingent claims

- Role of macro spillovers for explaining risk-exposure and understanding its efficiency

- Two main takeaways
  
  1. Possible to study risk-management in models with financial frictions
  
  2. Introducing risk-management considerations critical for understanding effects of macro-prudential regulation