Menu costs

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1 Partial equilibrium

- Want to minimize distance for optimal relative price $z$ (in log terms)
- $z$ follows random walk
  \[ z' = z + \epsilon \]
- Loss function
  \[ (p_i - p - z)^2 \]
- Fixed cost $\phi$ of changing $p_i$
- State variable
  \[ x = p + z - p_i \]
- Law of motion if price not adjusted
  \[ x' = p' + z' - p_i = x + \pi + \epsilon' \]
  where $\pi$ inflation
- Let $V(x)$ be the value if no adjustment occurs
- Bellman equation
  \[ V(x) = -x^2 + \beta E \max \{ V(x + \pi + \epsilon'), V^* - \phi \} \]
  where $V^* = \max_x V(x)$
- The function $V$ is quasi-concave and bounded. We can apply contraction mapping theorem to that function space
- There exists two values $\underline{x}, \overline{x}$ such that it is optimal to keep price unchanged if $x \in (\underline{x}, \overline{x})$ and is optimal to set $x = x^*$ otherwise
2 General equilibrium

- Suppose money (or nominal demand) follows geometric random walk (with drift)
  \[ M_t = M_{t-1} e^{\epsilon_t} \]
  with binary shock
  \[ \epsilon_t = \begin{cases} \Delta & \text{with prob. } q \\ 0 & \text{with prob. } 1 - q \end{cases} \]

- Output
  \[ C_t = Y_t = \frac{M_t}{P_t} \]

- Preferences
  \[ \log C_t - N_t \]
  so wages
  \[ \frac{1}{C_t} \frac{W_t}{P_t} = 1 \]

- Production function
  \[ Y_{i,t} = AN_{i,t} \]

- Firms maximize profits
  \[ \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{1}{A} \frac{W_t}{P_t} Y_{i,t} \]
  facing demand function
  \[ Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\sigma} \frac{M_t}{P_t} \]

- So profits are
  \[ \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{1}{A} \frac{W_t}{P_t} Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{1-\sigma} C_t - \frac{1}{A} \left( \frac{P_{i,t}}{P_t} \right)^{-\sigma} C_t^2 \]

- Conjecture, real consumption is constant and unaffected by money shocks
  \[ C = \frac{M_t}{P_t} \]
  so
  \[ \left( \frac{P_{i,t}}{P_t} \right)^{1-\sigma} C - \frac{1}{A} \left( \frac{P_{i,t}}{P_t} \right)^{-\sigma} C^2 \]
Then analyze problem with individual state variable
\[ X = \frac{P_{it}}{P_t} \]

define
\[ \Pi (X) = CX^{1-\sigma} - A^{-1}C^2 (X)^{-\sigma} \]

Bellman equation
\[ V (X) = \Pi (X) + \beta E \max \{ V (X') , V^* - \phi \} \]

Optimal policy
\[ P_{it} = X^* M_t \]
if
\[ P_{it} > X M_t \]
or
\[ P_{it} < X M_t \]
and
\[ P_{it} = P_{it-1} \]
otherwise

Equilibrium uniform distribution of prices
\[ \frac{P_{it}}{M_t} = X^* \]
\[ \frac{P_{it}}{M_t} = X^* e^{-\Delta} \]
\[ \ldots \]
\[ \frac{P_{it}}{M_t} = X^* e^{-(L-1)\Delta} \]
each with probability \( 1/L \)

To find \( L \) just look for smallest integer such that
\[ X^* e^{-L\Delta} < X \]

Aggregating
\[ \frac{1}{C} = \frac{P_t}{M_t} = \left\{ \int \left( \frac{P_{it}}{M_t} \right)^{1-\sigma} \, dt \right\}^{\frac{1}{1-\sigma}} = \left\{ \frac{1}{L} \sum_{l=0}^{L-1} \left( e^{-\Delta l} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \]
from this equation, given \( L \) and \( \Delta \) we get \( C \)

Fixed point problem in \( C \)

To complete equilibrium, check that distribution is uniform
- Yes, because any time there is a money shock $1/L$ prices would go from

$$\frac{P_{t-1}}{M_{t-1}} = X^* e^{-(L-1)\Delta}$$


to

$$\frac{P_{t-1}}{M_t} = \frac{P_{t-1}}{M_{t-1}} \frac{M_{t-1}}{M_t} = X^* e^{-L\Delta} < X$$

so they jump to $X^*$, all other prices stay unchanged