1. Facts

Figure 1. Employment

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2. WEDGES/REAL RIGIDITIES

Consider standard RBC model with preferences

$$\log C_t = \frac{\varepsilon}{1 + \varepsilon} N_t^{1+1/\varepsilon}$$

technology

$$Y_t = A_t F (K_t, N_t)$$
Planner optimality

\[ \frac{1}{C_t} A_t \frac{\partial F(K_t, N_t)}{\partial N} = N_t^{1/\varepsilon} \]

with Cobb-Douglas

\[ (1 - \alpha) \frac{Y_t}{C_t} = N_t^{1 + \frac{1}{\varepsilon}} \]

Decentralized version

\[ \frac{1}{C_t} \frac{W_t}{P_t} = N_t^{1/\varepsilon} \]

Both versions can be tested in the data. They don’t work well.

\[ n_t = \varepsilon (w_t - p_t - c_t) \]

does not work because \( c_t \) more cyclical than \( w_t - p_t \), so not even the sign is right.

You can argue that measured wages don’t capture the relevant margin (wages include insurance and contractual components, not relevant for allocations). Then go to

\[ n_t = \frac{\varepsilon}{1 + \varepsilon} (y_t - c_t) \]

\( y_t - c_t \) tends to be procyclical (consumption less volatile than income). So with very large \( \varepsilon \) it might work. In Great Recession in fact, \( y - c \) in fact, grows minimally, so you need very very large \( \varepsilon \). (In great recession saving rate increases, disposable income/GDP also increases, and the two effects roughly cancel each other). Micro estimates of Frisch elasticity of labor supply.

Baseline new Keynesian model also has trouble with labor markets. New Keynesian Phillips curve

\[ \pi_t = mc_t + \beta E_t \pi_{t+1} \]

where \( mc_t \) is a measure of real marginal costs. In the data, Phillips curve is very flat in \( y_t \). So need flat relation between \( y_t \) and \( mc_t \). Need real wages to move little in equilibrium.

Terminology: some say we need a source of “real rigidity”.

Can it help to turn to frictional labor markets?

3. Canonical DMP

• Matching function

\[ m(u, v) \]

• Assume constant returns to scale

• Probability for worker of finding a vacancy is

\[ \frac{m(u, v)}{u} = m(1, v/u) \]
• Define market tightness

\[ \theta \equiv \frac{v}{u} \]

• Then probability for worker of finding a vacancy is

\[ \mu(\theta) \equiv m(1, \theta) \]

• Probability for firm with vacancy to meet a worker is

\[ \frac{m(u, v)}{v} = \frac{m(u, v)}{v/u} = \frac{\mu(\theta)}{\theta} \]

• Agents are risk neutral, discount factor \( \beta \)

• Values
  – For workers: \( U \) value of being unemployed, \( V \) value of being employed,

\[ U = b + (1 - \mu(\theta)) \beta U + \mu(\theta) \beta V \]
\[ V = w + \beta (1 - s) V + \beta s U \]

  – For firms:

\[ J = y - w + \beta (1 - s) J \]

• Bargaining

• If worker/firm agree on wage \( \tilde{w} \)

\[ \tilde{V} = \frac{\tilde{w} + \beta s U}{1 - \beta (1 - s)} \] to the worker

and

\[ \tilde{J} = \frac{y - \tilde{w}}{1 - \beta (1 - s)} \] to the firm

• So if

\[ \tilde{V} \geq U \text{ and } \tilde{J} \geq 0 \]

a match will be formed

• A necessary and sufficient condition for the existence of a \( \tilde{w} \) such that a match will be formed is

\[ \frac{y + \beta s U}{1 - \beta (1 - s)} \geq U \]

• How do they agree on a \( \tilde{w} \)? Nash bargaining

\[ \max_{\tilde{w}} \left( \frac{\tilde{J}}{\tilde{V} - U} \right)^{\eta} \left( \tilde{V} - U \right)^{1-\eta} \]

• Which implies split of the total surplus \( \tilde{J} + \tilde{V} - U \) proportional to \( \eta \) and \( 1 - \eta \)

• We can now look for symmetric equilibrium where \( w = \tilde{w} \)

• So we have conditions

\[ V - U = (1 - \eta) (J + V - U) \]
\[ J = (1 - \eta) (J + V - U) \]

• Let’s find an expression for equilibrium surplus
• Combining conditions above we get
\[
J + V - U = y - b + \beta (1 - s) (J + V) + \beta s U - (1 - \mu (\theta)) \beta U - \mu (\theta) \beta V
\]
or
\[
J + V - U = y - b + \beta (1 - s) (J + V - U) - \mu (\theta) \beta (V - U)
\]
• Using the surplus split we have
\[
J + V - U = y - b + \beta (1 - s) (J + V - U) - \mu (\theta) \beta (1 - \eta) (J + V - U)
\]
or
\[
J + V - U = \frac{1}{1 - \beta (1 - s) + \mu (\theta) \beta (1 - \eta)} (y - b)
\]
• What determines \( \theta \)?
• Free entry condition for firms
\[
\kappa = \frac{\beta \mu (\theta)}{\theta} J
\]
• Substituting \( J = \eta (J + V - U) \) we obtain a single equation in \( \theta \)
\[ (1) \quad \kappa = \beta \frac{\mu (\theta)}{\theta} \frac{\eta (y - b)}{1 - \beta (1 - s) + \beta (1 - \eta) \mu (\theta)} \]
• Assume
\[
\beta \frac{(1 - \eta) (y - b)}{1 - \beta (1 - s)} > \kappa
\]
then there exists a unique \( \theta > 0 \) that solves (1)
• Unemployment dynamics
\[ u_{t+1} = u_t - \mu (\theta) u_t + s (1 - u_t) \]
• The equilibrium displays no transitional dynamics for \( \theta, w, J, U, V \)
• So dynamics simply follow
\[ u_{t+1} = u_t - \mu (\theta) u_t + s (1 - u_t) \]
and converge to steady state unemployment
\[ u = \frac{s}{s + \mu (\theta)} \]

4. The DMP model and the data

• Vacancies are procyclical
• Unemployment is countercyclical
• Relation between \( v \) and \( u \) is decreasing
• The Beveridge curve
• A decreasing relation is consistent with the (steady state) relation

\[
    u = \frac{s}{s + \mu \left( \frac{v}{u} \right)}
\]

• Debate on the shift of the curve during the recession
• Peter Diamond warned: don’t use it to predict \( u \) won’t fall as the recovery continues
• (Diamond Sahin 2014)
• He was right
• Still, why the shift?
• Some role of unemployment benefits
• Large role of firms being more picky (upskilling in recessions, downskilling in booms)
• Modestino and Shoag 2017
• Now let’s go back to estimating the matching function
• Measuring finding rates
• Monthly data
• \( u_t^s \): number of workers who lost a job during the month
• Finding rate can be computed from

\[
    u_{t+1} = (1 - f_t) u_t + u_t^s
\]

or

\[
    f_t = \frac{u_t + u_t^s - u_{t+1}}{u_t}
\]

• Frequency matters, because workers can lose and find job again, the shorter the better
• The series for \( f_t \) is procyclical
• Then we can plot $f$ against $v/u$ (in logs) and obtain $\alpha$ from regression

$$\log f_t = \log h + (1 - \alpha) \log (v_t/u_t)$$

• This yields $\alpha = 0.72$

• Separation rates are countercyclical, but not very volatile

• So two pieces of the model seem reasonably well grounded in data

$$f_t = \mu(\theta_t)$$

and

$$u_{t+1} = (1 - \mu(\theta_t)) u_t + s (1 - u_t)$$

• Now we come to determining $\theta$
• Go back to equation

$$\kappa = \beta \frac{\mu(\theta)}{\theta} \frac{\eta (y - b)}{1 - \beta (1 - s) + \beta (1 - \eta) \mu(\theta)}$$

• Rewrite it as

$$h\theta^{-\alpha} \beta \eta (y - b) = \kappa \left(1 - \beta (1 - s) + \beta (1 - \eta) h\theta^{1-\alpha}\right)$$

• In logs

$$\log h \beta \eta + \log (y - b) = \log \kappa + \alpha \log \theta + \log \left(1 - \beta (1 - s) + \beta (1 - \eta) h\theta^{1-\alpha}\right)$$

• Differentiate

$$\left(\alpha + (1 - \alpha) \frac{\beta (1 - \eta) f}{1 - \beta (1 - s) + \beta (1 - \eta) f} \frac{0.99 \times 0.72 \times 0.83}{1 - \beta (1 - s) + 0.99 \times 0.72 \times 0.83}\right) d\log \theta = d\log (y - b)$$

• Calibration
  - $\beta = 0.99$
  - $s = 0.1$ (jobs last about 2.5 years, i.e. 10 quarters)
  - for $f = h\theta^{1-\alpha}$ use average finding rate, which is 0.45 monthly which becomes $0.83 = 1 - (1 - 0.45)^3$ quarterly
  - $\alpha = 0.72$ from matching function
  - suppose $\eta = 1 - \alpha$ (Hosios condition)

• Expression in parenthesis is close $\approx 1$

$$\alpha + (1 - \alpha) \frac{\beta (1 - \eta) f}{1 - \beta (1 - s) + \beta (1 - \eta) f} \approx 0.72 + 0.28 \times 0.99 \times 0.72 \times 0.83 = 0.96$$

• So

$$\frac{d\theta}{\theta} \approx \frac{dy}{y - b}$$

• Business cycle volatilities

(2) $\sigma_f \approx (1 - \alpha) \sigma_{v/u}$

(3) $\sigma_{v/u} \approx \frac{y}{y - b} \sigma_y$

• In the data we have

$$\sigma_f = 0.118$$
$$\sigma_{v/u} = 0.382$$
$$\sigma_y = 0.020$$

• (2) works (because the matching function works)

• (3) only works if $b$ is very close to $y$

• If we make more realistic assumptions on preferences... things get worse
• Representative family has continuum of workers, some employed, some unemployed

\[ U(c, l) \]

where \( c \) consumption of family, \( l \) non-work time

• Then \( b \) is replaced by

\[
\frac{U_c(c, l) b + U_l(c, l)}{U_c(c, l)}
\]

value of unemployment benefits plus the value of time

• (Also, need to replace \( \beta \) with appropriate discount factor, but that’s less important)

• Problem: the marginal rate of substitution between time and money

\[
\frac{U_l(c, l)}{U_c(c, l)}
\]

tends to go down in recessions as \( l \) increases and \( c \) decreases

• Blanchard-Gali: in baseline model with log preferences \( \theta \) becomes completely acyclical!

• Chodorow-Reich and Karabarbounis: measure MRS as well as they can, it’s strongly procyclical, which dampens movements in \( \theta \) if wages are determined by standard Nash bargaining

5. Unemployment insurance

• Unemployment insurance

• Typically extended during recessions

• Tradeoffs?

• A simple 2 period search model (no discounting)

• Only 2 periods but richer in 2 dimensions
  – decreasing returns production function

\[ F(n) \]

  – labor search effort

• Initial stock of unemployed and employed \( n_0 \) and \( u_0 \) with \( n_0 + u_0 = 1 \)

• Matching function

\[ m(eu_0, v) \]

where \( e \) is effort per worker

• Define

\[ \theta = \frac{v}{eu_0} \]

and

\[ \mu(\theta) = m(1, \theta) \]
- Flows
\[ u = sn_0 - m(eu_0, v) = sn_0 - eu_0\mu(\theta) \]
- Workers’ effort
\[ \max e\mu(\theta) (U(w) - U(b)) - \psi(e) \]
- Optimality
\[ \psi'(e) = \mu(\theta) \Delta U \]
- So we obtain
\[ 1 - n = sn_0 - \psi'^{-1}(\mu(\theta) \Delta U) u_0\mu(\theta) \]
- Increasing relation between \( \theta \) and \( n \), “labor supply”
- Two reasons, mechanical effect on finding rate + behavioral response
- Firms hire workers, produce labor services 1:1, sell them at flexible price \( \tilde{w} \) to final goods producers, so
\[ F'(n) = \tilde{w} \]
(useful fiction, other approaches possible)
- Free entry
\[ \frac{\mu(\theta)}{\theta} (F'(n) - w) = \kappa \]
- This gives a decreasing relation between \( \theta \) and \( n \), “labor demand”
- For now let’s keep \( w \) completely fixed
- Question: effects of increasing \( b \) on unemployment
- Traditional approach, partial equilibrium, higher \( b \) discourages \( e \)
\[ \psi'^{-1}(\mu(\theta) \Delta U) \]
- Classic moral hazard trade off: insurance vs incentives (Baily)
- What happens when you include GE effects?
- Let’s first go back to the linear DMP model
- In that model there is no search effort, so if we measured incentive effects at micro level we would conclude that effects are zero
- However in that model there are important GE effects via \( U \) (value of unemployment)
- Higher \( b \), higher \( U \), higher wages, low incentive to create jobs, low \( \theta \)
- Take away: in the standard DMP model with Nash bargaining, GE effects amplify the individual incentive effects
- Welfare: unemployment benefits are more costly than Baily formula would suggest
- Now consider the model here
- Consider extreme case with \( w \) completely rigid
• Combine labor demand and labor supply relation

\[ \frac{\mu(\theta)}{\theta} (F'(n) - w) = \kappa \]

\[ 1 - n = s n_0 - \psi^{-1} (\mu(\theta) \Delta U) u_0 \mu(\theta) \]

• Increasing \( b \) reduces \( \Delta U \)

• But now GE effect, if all workers search less \( \theta \) endogenously increases

• The less elastic the labor demand relation, the bigger the GE effect

• Take away: in search-matching model with rigid wages and decreasing returns, GE effects dampen the individual incentive effects